Control synthesis for an unmanned helicopter with time-delay under uncertain external disturbances

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Abstract

This paper presents the controller synthesis for an unmanned helicopter with minimum initial information about the parameters of its mathematical model with time-delays of measured and control signals. The unknown parameters, wind disturbances, and system nonlinearity are considered as external disturbances that are estimated using a multi-gap observer. The estimates obtained are used in the control law to improve the stability rate for flight regimes.

Keywords: unnamed helicopter, control law, time-delay, external disturbances, disturbances compensation, observers, uncertain.

Nomenclature

a_s, b_s - Longitudinal and lateral flapping angle of main rotor (rad);

g - Local acceleration of gravity (m/s^2);

p, q, r - Vehicle roll, pitch and yaw rates ($\mathrm{deg/s}$);

u, v, w - Longitudinal, lateral and normal velocity components of vehicle C.G. (m/s);

W - Wind actions in the body coordinate system (m/s);

x, y, z - Vehicle position coordinates in local north-east-down frame (m);

$\delta_{col}, \delta_{ped}$ - Normalized collective pitch and rudder servo input $[-1, 1]$;

$\delta_{lon}, \delta_{lat}$ - Normalized elevator and aileron servo input $[-1, 1]$;

$\delta_{\mathrm{ped,int}}$ - Intermediate state in yaw rate feedback controller dynamics (rad);

θ, ϕ, ψ - Vehicle pitch, roll and yaw angles (deg);

Δ – Deviation from the trim values.

Introduction

Influences of measurement time-delays, control forming time, and delays in actuators response must be accounted when designing the helicopter digital Automatic Control System.

Loss of helicopter control effectiveness and handing degradation are possible if these delays are not taken into account [1]. Consequenly, the control laws must be robust in relation to delays and modified to allow for their presence.

Many works are nowadays published on this problem and the most popular are methods based on predictive control [2]. Careful analysis of these methods and their modifications show that they all use, in an implicit or explicit manner, predictions of the system state to achieve its control. A common drawback of these methods is linked to internal instabilities of the prediction, is that they fail to stabilize unstable systems [3].

In recent years, there has been a significant increase of research works dedicated to the study of small-size helicopters. This is due to their availability, low cost, and some dynamic similarities with full-size helicopters. In [4], for example, the predictive state controller based on LQ optimal control is developed for the yaw axis of a tethered helicopter with 1-DOF. A predictive control model for a small-size helicopter, taking into account time-delays actuators and main rotor aerodynamics, is presented in [5]. The developed controller has tested in flight conditions using a 6-DOF helicopter rig. For the inner control loop, a P-controller was used and for outer a PID-controller. Experimental results showed good performance of the controllers.

The main time-delay when using a visual system for helicopter control is linked to image processing, and in [6] an example is shown of a quad-rotor mini-helicopter where on-board cameras are used to detect the vehicle's position. The image processing is introducing a time-delay. To calculate the control using a nonlinear algorithm based on derived dynamic model. A similar work is presented in [7], where for reducing the influence of the time-delay a Kalman filter is used.

Like several works, related to the unnamed "Raptor" helicopter, this platform is used in this paper. The problem of controller synthesis for unmanned helicopters with minimum initial information about their parameters of mathematical models, for cases with time-delays of the measured and control signals, and under wind disturbances is considered.

Problem Statement

The unmanned Raptor helicopter is used in this work. Its non-linear mathematical model and its parameters are presented in [8, 9]. As shown in [10], to simplify the procedure of the navigational controller design, its dynamics equations are presented in the following form:

$$\dot{x}_1 = R(X_3)x_2, \tag{1}$$

$$\dot{x}_2 = B_1(X_{31})u_1 + f_1, \tag{2}$$

$$\dot{x}_3 = S^{-1}(X_{32})x_4, \tag{3}$$

$$\dot{x}_4 = A_2 x_5 + f_2, \tag{4}$$

$$\dot{x}_5 = B_2 u_2 + f_3, \tag{5}$$

Where: $x = X - X^* = \left[x_1^T \;\; x_2^T \;\; x_3^T \;\; x_4^T \;\; x_5^T \right]^T$, X is the n-order state vector, $X^* = \left[x_0 \;\; y_0 \;\; z_0 \;\; 0 \;\; 0 \;\; 0 \;\; \theta^* \;\; \phi^* \;\; 0 \;\; 0 \;\; 0 \;\; 0 \, a_s^* \;\; b_s^* \;\; 0 \right]^T$ is the initial and trimming parameters vector, $x_1 = [\Delta x \;\; \Delta y \;\; \Delta z]^T$, $x_2 = [u \;\; v \;\; w]^T$, $x_3 = [\Delta\theta \;\; \Delta\phi \;\; \psi]^T$, $x_4 = [q \;\; p \;\; r]^T$, $x_5 = \left[\Delta a_s \;\; \Delta b_s \;\; \delta_{\text{ped,int}} \right]^T$; $\mathrm{u} = U - U^* = \left[\Delta\delta_{\text{lon}} \;\; \Delta\delta_{\text{lat}} \;\; \Delta\delta_{\text{ped}} \;\; \Delta\delta_{\text{col}} \right]^T$, $U = \left[\delta_{\text{lon}} \;\; \delta_{\text{lat}} \;\; \delta_{\text{ped}} \;\; \delta_{\text{col}} \right]^T$ are the control signals vector, each of which lies in the range from -1 to 1 [8], U^* is the trimming parameters vectors; $X_3 = X_3^* + x_3$, $X_{31} = \theta^* + \Delta\theta$, $X_{32} = X_{32}^* + x_{32}$, $x_{32} = [\Delta\theta \; \Delta\phi]^T$; $u_1 = \left[\bar{u}_1^T \;\; \Delta\delta_{col} \right]^T$, $\bar{u}_1 = [\sin\theta \;\; \sin\phi]^T$ is the virtual control for the outer loop to move the helicopter relative to the earth coordinate system [10]; $u_2 = \left[\Delta\delta_{\text{lon}} \;\; \Delta\delta_{\text{lat}} \;\; \Delta\delta_{\text{ped}} \right]^T$; f_i $(i = \overline{1,3})$ are vectors of specified disturbances derived from the original equations after isolation of the terms $B_1(X_{31})u_1$, $A_2 x_5$, $B_2 u_2$, where $B_1(X_{31}) = diag\left(-g, g\cos\theta, b_1 \right)$, b_1 is a model parameter; A_2, B_2 are diagonal matrices of the model parameter; $R\left(X_3 \right)$ and $S\left(X_{32} \right)$ are the rotation and kinematic transformations matrices respectively.

We suppose that vectors x_i $(i = \overline{1,4})$ are available to measure discrete moments in time t_i, $i = 0,1,2,...$ with noise v, which is limited in amplitude. A measurement of the vector $y^{(1)} = \left[x_1^T \;\; x_2^T \right]^T$ is performed with sampling period $T_0 = t_{i+1} - t_i$ of time-delay:

$$y^{(1)}(t_i) = \left[x_1^T(t_{i-1}) \;\; x_2^T(t_{i-1}) \right]^T + v^{(1)}(t_{i-1}).$$

Significant measurement delays $y(t_i)$ there occur, for example, when the optical sensors are used. They require significant time for processing by a video-controller.

Assume the controller output is fed to the actuator with a time-delay corresponding to the sampling period.

The problem of developing a robust discrete-time controller is posed for the system (1)-(5). The controller must stabilize the helicopter's motion under wind disturbances.

To achieve the desired control quality, it is necessary to provide compensation for the specified disturbances f_i $(i = \overline{1,3})$. However, as follows from equations (1)-(5), one cannot realize the full compensation of the disturbances f_1 and f_2. Therefore, the control law is constructed so that it suppresses the specified disturbances which affect the dynamics of the state vectors x_i $(i = \overline{1,3})$. To do this, an observer is used to construct estimates of the specified disturbances.

State observers and disturbances synthesis

Let us consider the state vector, and a disturbance estimation method with time-delay of the measured signals, separately, for each subsystem.

1. The first subsystem (1), (2) can be rewritten:

$$\dot{x}^{(1)} = \begin{bmatrix} A^{(1)} & D^{(1)} \\ 0_{3\times 6} & 0_3 \end{bmatrix} x^{(1)} + \begin{bmatrix} B^{(1)}(t) \\ 0_3 \end{bmatrix} u_1 + \begin{bmatrix} 0_{6\times 3} \\ I_3 \end{bmatrix} \dot{\mathrm{w}}_1, \tag{6}$$

where $x^{(1)} = \left[x_1^T \;\; \tilde{x}_2^T \;\; \mathrm{w}_1^T \right]^T$, $\tilde{x}_2 = R\left(X_3(t)\right) x_2$ are calculated results using measurements, $\mathrm{w}_1 = R\left(X_3(t)\right) f_1 + \frac{d}{dt}\left(R\left(X_3(t)\right)\right) x_2$ are generalized disturbances; $A^{(1)} = \begin{bmatrix} 0_3 & I_3 \\ 0_3 & 0_3 \end{bmatrix}$, $D^{(1)} = \begin{bmatrix} 0_3 \\ I_3 \end{bmatrix}$, $B^{(1)}(t) = \begin{bmatrix} 0_3 \\ B_1^{(1)}(t) \end{bmatrix}$, $B_1^{(1)}(t) = R\left(X_3(t)\right) B_1\left(X_{31}(t)\right)$.

Then, taking into account the approximation on the time interval $t_i \le t < t_{i+1}$:

$$u_1(t) = u_1(t_i), \;\; \dot{\mathrm{w}}_1(t) = \dot{\mathrm{w}}_1(t_i), \;\; B_1^{(1)}(t) = B_1^{(1)}(t_i), \tag{7}$$

for the subsystem (6) the obtained discrete model is:

$$x^{(1)}(t_{i+1}) = Ax^{(1)}(t_i) + B(t_i)u_1(t_i) + D\dot{\mathrm{w}}_1(t_i) + w^{(1)}(t_i),$$
$$y^{(1)}(t_i) = Cx^{(1)}(t_{i-1}) + v^{(1)}(t_{i-1}), \tag{8}$$

where $A = \begin{bmatrix} A_d & D_d \\ 0_{3\times6} & I_3 \end{bmatrix}$, $B(t_i) = \begin{bmatrix} B_d \\ 0_3 \end{bmatrix} B_1^{(1)}(t_i)$, $D = \begin{bmatrix} T_0^3/6I_3 \\ B_d \end{bmatrix}$, $A_d = \begin{bmatrix} I_3 & T_0 I_3 \\ 0_3 & I_3 \end{bmatrix}$, $B_d = \begin{bmatrix} 0.5T_0^2 I_3 \\ T_0 I_3 \end{bmatrix}$, $D_d = B_d$, $C = [C_0 \ \ 0_{l\times3}]$, l is the number of measured states; in the disturbance vector $w^{(1)}$ approximation error is counted (7). In this case, there are various types of the output coordinates measurements: $C_0 = I_6$ when changing the vectors x_1 and x_2, using a GPS system and airspeed-measuring sensors, for example; $C_0 = [I_3 \ \ 0_3]$ is when the measurement x_2 is not conducted; $C_0 = [0_3 \ \ I_3]$ – when the measurement x_1 is not conducted.

For estimating the disturbances $\mathrm{w}_1(t_i)$ discrete analog of continuous observers can be used, as proposed in reference [10]. However, the delay of the measured signals (6) increases the estimate error. In this regard, a multi-gap observer can be used, consisting of two observers.

For the subsystem (8), using first observer

$$\tilde{x}^{(1)}(t_i) = A\tilde{x}^{(1)}(t_{i-1}) + B(t_{i-1})u_1(t_{i-1}) + HL\left(y^{(1)}(t_i) - C\tilde{x}^{(1)}(t_{i-1})\right), \tag{9}$$

and finding the estimates $\tilde{y}^{(1)}(t_i) = [I_6 \ \ 0_{6\times3}]\tilde{x}^{(1)}(t_i)$ and $\tilde{w}^{(1)}(t_i) = [0_3 \ \ 0_3 \ \ I_3]\tilde{x}^{(1)}(t_i)$, that are used in the second observer we have:

$$\hat{x}^{(1)}(t_{i+1}) = A\hat{x}^{(1)}(t_i) + B(t_i)u_1(t_i) + D\tilde{w}^{(1)}(t_i) + H\tilde{L}\left(\tilde{y}^{(1)}(t_i) - [I_6 \ \ 0_{6\times3}]\hat{x}^{(1)}(t_i)\right). \tag{10}$$

The final disturbance estimation is calculated by:

$$\hat{\mathrm{w}}_1(t_i) = \tilde{w}^{(1)}(t_i) + [0_3 \ \ 0_3 \ \ I_3]\hat{x}^{(1)}(t_i), \tag{11}$$

where $H = M\tilde{H}$, $M = diag\{I_6,\ \mu^{-1}I_3\}$, $\tilde{H} = \begin{bmatrix} I_6 & 0_{6\times3} \\ D_d^+ & I_3 \end{bmatrix}$, $D_d^+ = \left(D_d^T D_d\right)^{-1} D_d^T$, μ – is a tunning parameter; L, $\tilde{L}$ are the unknown required gain matrices; $\tilde{x}^{(1)}(t_{-1}) = \hat{x}^{(1)}(t_0) = 0$.

Thus, using the observer (9) the state vector and distribution estimates are received, and after using the observer (10) the corrections are performed.

To determine the matrix L, the method proposed in [11] can be used. First, we accept that $H = I_9$. Thus if the limiting action $\dot{\mathrm{w}}_1(t_i)\dot{\mathrm{w}}_1^T(t_i) \le Q_{\dot{\mathrm{w}}_1}$, $v^{(1)}(t_i)v^{(1)T}(t_i) \le Q_v$ (where

Q_w, Q_v, are determined positive definite matrices) and stability conditions are applied then for systems (8) and (9) the estimate $\left(x^{(1)}(t_i)-\tilde{x}^{(1)}(t_i)\right)\left(x^{(1)}(t_i)-\tilde{x}^{(1)}(t_i)\right)^T \le \tilde{X}$ will be correct. In this case the matrix $\tilde{X}$ is satisfies for the matrix inequality:

$$\begin{aligned} &\tilde{X}-\beta_1(A-LC)\tilde{X}(A-LC)^T-\beta_2 DQ_{\dot{w}_1}D^T-\beta_3 LQ_v L^T \ge 0 \\ &\beta_1=(1+\alpha_1)(1+\alpha_2),\ \beta_2=\left(1+\alpha_1^{-1}\right)(1+\alpha_2),\ \beta_3=\left(1+\alpha_2^{-1}\right),\ \alpha_1>0,\ \alpha_2>0. \end{aligned} \tag{12}$$

In [12] that the parameter β_1 is linked with the damping time t_n of the transient processes of the system (9) by the equation $t_n \le 3T_0/\ln\sqrt{\beta_1}$. So, if we set the parameter β_1 using the notation $X=\tilde{X}^{-1}$, $Y=XL$, and the Shura lemma then (12) can be rewritten in the equivalent form:

$$\begin{bmatrix} X & XA-YC & XD & Y \\ A^T X-C^T Y^T & \beta_1^{-1}X & 0 & 0 \\ D^T X & 0 & \alpha_1\beta_1^{-1}Q_{\dot{w}_1}^{-1} & 0 \\ Y^T & 0 & 0 & \beta_1^{-1}(\beta_1-1-\alpha_1)Q_v^{-1} \end{bmatrix} \ge 0,\ X>0,\ \alpha_1>0. \tag{13}$$

We must find a matrix L which allows to obtain a minimal sum of the diagonal matrix $\tilde{X}$ elements and a necessary decay time of step response t_n.

Here, the following statement is put forward:

Statement 1. To determine a matrix L in (8) considering the damping time t_n of transient processes it is sufficient to solve:

$$\operatorname{tr}(X)\to\max$$

with $\beta_1=\exp(6T_0/t_n)$ and limits (13) for the matrix variables X, Y. Then the matrix is $L=X^{-1}Y$.

For the found matrix L is necessary to provide the stability of the matrix $A-HLC$, by selecting the parameter μ using the best disturbances estimate w_1.

The matrix $\tilde{L}$ is defined similar to the observer (10).

Note that unlike [13], irregular estimation of disturbances is considered here. In particular, the measurement interferences are independent of the disturbance.

If the corrected estimate of the vector $[I_6\ \ 0_{6\times3}]\hat{x}^{(1)}(t_i)$ is not required, then the reduced-order observer can be used instead the observer (10) as in [10]:

$$\xi(t_{i+1}) = P\xi(t_i) + (PG - GA_d)\tilde{y}^{(1)}(t_i) - GD_d\left(\bar{B}(t_i)u_1(t_i) + \gamma\tilde{w}^{(1)}(t_i)\right),$$
$$\hat{\mathrm{w}}_1(t_i) = \xi(t_i) + G\tilde{y}^{(1)}(t_i) + \gamma\tilde{w}^{(1)}(t_i),$$

where $P = I_3 - GD_d$, $G = \mu^{-1}\left(D_d^{+} + L_2L_1^{-1}\right)$, gain of the γ, which can be 0 or 1, in depend of estimation type.

Similarly, the subsystem (3)-(5) can be rewritten in following form:

$$\dot{x}_3 = \tilde{x}_4;$$
$$\dot{\tilde{x}}_4 = \tilde{x}_5;$$
$$\dot{\tilde{x}}_5 = S^{-1}(X_{32})B^{(2)}u_2 + \mathrm{w}_2,$$

where $\tilde{x}_4 = S^{-1}(X_{32})x_4$, $\tilde{x}_5$ is the auxiliary vector, w_2 is the generalized disturbances; $B^{(2)} = A_2B_2 = diag(b_2, b_3, b_4)$, b_i, $i = \overline{2,4}$ is the model parameters. Here, measurement of the vectors is carried x_3, x_4 and vector calculates $\tilde{x}_4$ at discrete time moments without time-delay, but with sampling period significantly less than in the first subsystem. Therefore, to construct estimates of the vectors $\hat{\tilde{x}}_5$ and $\hat{\mathrm{w}}_2$ an observer of the form:

$$\hat{x}^{(2)}(t_{i+1}) = A\hat{x}^{(2)}(t_i) + B(t_i)u_2(t_i) + HL\left(y^{(2)}(t_i) - C\hat{x}^{(2)}(t_i)\right) \tag{14}$$

can be used, where $\hat{x}^{(2)} = \left[\hat{\tilde{x}}_4^T \ \ \hat{\tilde{x}}_5^T \ \ \hat{\mathrm{w}}_2^T\right]^T$, $y^{(2)} = \tilde{x}_4$; $B(t_i) = DS^{-1}(t_i)B^{(2)}$, $C = [I_3 \ \ 0_3 \ \ 0_3]$, the other matrices are the same as mentioned previously.

Taking into account the measurement of the vector $\tilde{x}_4$ instead of the observer (14) the reduced-order observer also can be used.

Controller with delay synthesis

Assume a control signal with a time-delay equal to the sampling period fed to the actuators of main rotor.

For subsystems (1), (2) the discrete model has the form:

$$\bar{x}^{(1)}(t_{i+1}) = A_d\bar{x}^{(1)}(t_i) + B_dB_1^{(1)}(t_i)u_1(t_{i-1}) + D_d\left(\mathrm{w}_1(t_i) + 0.5T_0\dot{\mathrm{w}}_1(t_i)\right) + \Delta\mathrm{w}_1(t_i), \tag{15}$$

where $\bar{x}^{(1)} = \left[x_1^T \ \ \tilde{x}_2^T\right]^T$ is the *n*-order state vector, $\Delta\mathrm{w}_1(t_i)$ is the error of the disturbance approximation, which proportional to the value T_0^3.

Let's write the subsystem (15) in deviations $\Delta\bar{x}^{(1)}(t_i) = \bar{x}^{(1)}(t_i) - \bar{x}_r^{(1)}(t_i)$ from the prescribed helicopter motion $\bar{x}_r(t_i) = \left[x_{1r}^T(t_i) \ \ \dot{x}_{1r}^T(t_i)\right]^T$ in the earth coordinate system:

$$\Delta\overline{x}^{(1)}(t_{i+1}) = A_d\Delta\overline{x}^{(1)}(t_i) + B_d B_1^{(1)}(t_i)u_1(t_{i-1}) + g_r^{(1)}(t_i) + D_d\left(\mathrm{w}_1(t_i) + 0.5T_0\dot{\mathrm{w}}_1(t_i)\right) + \Delta\mathrm{w}_1(t_i), \qquad (16)$$

where $g_r^{(1)}(t_i) = A_d\overline{x}_r^{(1)}(t_i) - \overline{x}_r^{(1)}(t_{i+1})$ is unknown vector.

Using the found estimate of vector $\hat{x}^{(1)}(t_i) = \left[\hat{\overline{x}}^{(1)T}(t_i)\ \hat{\mathrm{w}}_1^T(t_i)\right]^T$ the control law will be presented in the following form:

$$u_1(t_{i-1}) = \left(B_1^{(1)}(t_i)\right)^{-1}\left[K\left(\hat{\overline{x}}^{(1)}(t_i) - \overline{x}_r^{(1)}(t_i)\right) - \hat{\mathrm{w}}_1(t_i) - a(t_i)\right]. \qquad (17)$$

K is the gain matrix, $a(t_i) = D_d^+ g_r^{(1)}(t_i) + 0.5T_0\dot{\hat{\mathrm{w}}}_1(t_i)$ is the known vector using when estimate $\dot{\hat{\mathrm{w}}}_1(t_i) \approx \left(\hat{\mathrm{w}}_1(t_i) - \hat{\mathrm{w}}_1(t_{i-1})\right)/T_0$ is used.

For the current time, the control law will have the form:

$$u_1(t_i) = \left(B_1^{(1)}(t_{i+1})\right)^{-1}\left[K\left(\hat{\overline{x}}^{(1)}(t_{i+1}) - \overline{x}_r^{(1)}(t_{i+1})\right) - \hat{\mathrm{w}}_1(t_{i+1}) - a(t_{i+1})\right], \qquad (18)$$

where the $\hat{\overline{x}}^{(1)}(t_{i+1})$ is defined using equation (10), and also

$$\hat{\mathrm{w}}_1(t_{i+1}) = \tilde{w}^{(1)}(t_i) + \left[0_3\ \ 0_3\ \ I_3\right]\hat{x}^{(1)}(t_{i+1}). \qquad (19)$$

If we substitute the control (17) in (16) we obtain the following close-loop control system:

$$\Delta\overline{x}^{(1)}(t_{i+1}) = (A_d + B_d K)\Delta\overline{x}^{(1)}(t_i) + B_d K\Delta\hat{\overline{x}}^{(1)}(t_i) + D_\Delta\Delta\overline{\mathrm{w}}_1(t_i), \qquad (20)$$

where $\Delta\hat{\overline{x}}^{(1)}(t_i) = \hat{\overline{x}}^{(1)}(t_i) - \overline{x}^{(1)}(t_i)$ is the estimating error, $\Delta\overline{\mathrm{w}}_1(t_i)$ is the specified disturbance, which depends on the $\Delta\mathrm{w}_1(t_i)$, the error of vectors compensation $g_r^{(1)}(t_i)$, $\mathrm{w}_1(t_i) + 0.5T_0\dot{\mathrm{w}}_1(t_i)$, and now we can accept $D_\Delta = I_n$.

Consider the method of determining matrix K taking to account the componentwise limit:

$$|\,u_1(t_i)\,| \le u_{1\max}, \qquad (21)$$

where $u_{1\max} = \left[\sin\theta_{\max}\ \ \sin\phi_{\max}\ \ 1 - |\,\delta_{col}^*\,|\right]^T$, and $\theta_{\max}$, $\phi_{\max}$ are the maximum allowable angles.

For the control law (17) the follow equation can be obtained:

$$K\left(\Delta\overline{x}^{(1)}(t_i) + \Delta\hat{\overline{x}}^{(1)}(t_i)\right) = \Delta u_1,$$

Where the vector $\Delta u_1 = B_1^{(1)}(t_i)u_1(t_{i-1}) - a(t_i)$ is limited above by the vector $\Delta u_{1\max}$ of (21), and $|\,\theta\,| \le \theta_{\max}$, $|\,\phi\,| \le \phi_{\max}$, possible commands and disturbances. Then we have the inequality:

$$K\left(\Delta\bar{x}^{(1)}(t_i)+\Delta\hat{\bar{x}}^{(1)}(t_i)\right)\left(\Delta\bar{x}^{(1)}(t_i)+\Delta\hat{\bar{x}}^{(1)}(t_i)\right)^T K^T \le \Lambda^2, \tag{22}$$

where $\Lambda = diag\{\Delta u_{1\max}\}$ is the diagonal matrix with elements of the vector $\Delta u_{1\max}$ in the main diagonal.

The inequality $\Delta\bar{x}^{(1)}(t_i)\Delta\bar{x}^{(1)T}(t_i) \le X$ is correct for the system (20) if the stability conditions, $\Delta\hat{\bar{x}}^{(1)}(t_i)\Delta\hat{\bar{x}}^{(1)T}(t_i) \le V$ and $\Delta\bar{\mathrm{w}}_1(t_i)\Delta\bar{\mathrm{w}}_1^T(t_i) \le Q_w$ are implemented [12]. The matrix $X>0$ should be suitable for the following matrix inequality:

$$X-\beta_1\left(A_d X A_d^T + AXK^T B_d^T + B_d KXA_d^T\right)-\beta_2 Q_w-\beta_1 B_d KXK^T B_d^T-\beta_3 B_d KVK^T B_d^T \ge 0. \tag{23}$$

where V, Q_w is the defined positive definite matrix, and the gains β_j ($j=\overline{1,3}$) are calculated using equations (12).

We assume that, when the control law (18) begins to work the transients in the observer has ended. The transients are causes by the initial conditions $\hat{\bar{x}}^{(1)}(t_0)=0$. Then the matrix V can be specified in the form $V \le q_v X$, where the parameter $q_v > 0$ characterizes the estimation accuracy of the vector $\Delta\bar{x}^{(1)}(t_i)$ using the observer. In this case from (22) we can obtain the estimate:

$$KXK^T \le \left(1+\sqrt{q_v}\right)^{-2}\Lambda^2. \tag{24}$$

Considering the inequality $V \le q_v X$, designations $Y=KX$ and Shura lemma from the inequality (23) we obtain more the strict inequality:

$$\begin{bmatrix} X-\beta_1\left(A_d X A_d^T + AY^T B_d^T + B_d Y A_d^T\right)-\beta_1\alpha_1^{-1}Q_w & B_d Y \\ Y^T B_d^T & \beta_1\left(1+q_v\left(\beta_1-1-\alpha_1\right)^{-1}\right)X \end{bmatrix} \ge 0,\ X>0, \tag{25}$$

where $0<\alpha_1<\beta_1-1$.

We shall construct helicopter initial deviation area from the reference motion in the form of inequality $\Delta\bar{x}^{(1)}(t_0)\Delta\bar{x}^{(1)T}(t_0) \le q^{-1}X$, $0<q<1$. Also from (24) we can obtain the inequality:

$$YX^{-1}Y^T \le q\left(1+\sqrt{q_v}\right)^{-2}\Lambda^2,\quad 0<q<1. \tag{26}$$

We now need to find the matrix by which the required control time, the minimal sum of the matrix X diagonal elements, and the minimum value of the parameter q are achieved. The minimum value of the parameter q must provide the largest estimate of the initial deviation range for the vector $\Delta\bar{x}^{(1)}(t_0)$. Here the following statement is true.

Statement 2. To determine matrix K in (17) it is sufficient to solve the following task:

$$\operatorname{tr}(X) + q \to \min.$$

Considering the adopted controls limit (22), the adopted control time t_p and value q_v, the adopted parameters $\beta_1 = \exp(6T_0 / t_p)$ и $0 < \alpha_1 < \beta_1 - 1$. For the matrix variables X, Y we must take into account the constraints (25), (26). Then, the matrix $K = YX^{-1}$ and the initial deviation range can be calculated using inequality $\Delta\bar{x}^{(1)}(t_0)\Delta\bar{x}^{(1)T}(t_0) \le q^{-1}X$.

If the noise of measurement is absent, then q_v must be equal to 0 in (25), (26) and parameter $\alpha_1 > 0$ must be include in the variables list of parameterization task.

Unlike [13], here the allowable initial deviation range is developing and the error of the state vector measurement is considering. The error is independently of the disturbances.

Thus, control law $\Delta\delta_{\text{col}}(t_i) = [0\ 0\ 1]u_1(t_i)$ is developed and the required angles change: $\left[\sin\theta_r(t_i)\ \sin\phi_r(t_i)\right]^T = [I_2\ 0_{2\times1}]u_1(t_i)$.

For the subsystem (3)-(5) the adopted control law is of the form:

$$u_1(t_i) = \left(S^{-1}(t_i)B^{(2)}\right)^{-1}\left[K_1\Delta x_3(t_i) + K_2\left(S^{-1}(t_i)x_4(t_i) - \dot{x}_{3r}(t_i)\right) + K_3\hat{\bar{x}}_5(t_i) + \hat{\mathrm{w}}_2(t_i)\right] \tag{27}$$

where $\Delta x_3(t_i) = x_3(t_i) - \left[\theta_r(t_i)\ \phi_r(t_i)\ 0\right]^T - x_{3r}(t_i)$; $x_{3r}(t_i)$ is reference vector of angles, and K_1, K_2, K_3 are diagonal matrices with positive gains.

Thus, using control law (27) the desired angles θ_r, ϕ_r or x_{3r} are tracked. At the same the time vectors $\hat{\bar{x}}_5$ and $\hat{\mathrm{w}}_2$ are estimated using the observer of equation (14).

Helicopter dynamic simulation

The simulation results of the helicopter dynamics with the control laws developed in [10] showed that if time-delays were present in the linear velocity sensors and collective pitch actuator, the required Handling Qualities according to ADS-33E-PRF [14] wer not met. In that case there were large altitude oscillations. The time-delay value for each axis was equal to 0.1 sec.

To validate the performance of the proposed discrete multi-gap observer the pirouette maneuver [14] under wind gusts was considered. The wind acting on the three directions X, Y, Z was of the following form:

$$W_i = 0.5V_{p\max}\left[1-\cos\left(\frac{2\pi}{\Delta t_p}t\right)\right],\ i=\overline{1,3}, \tag{28}$$

where $V_{p\max}=5$ m/sec is the maximum speed of wind gust during the time interval $\Delta t_p = 10$ sec.

Simulation responses of the helicopter landing with two observers (9)-(11) from an initial height equal to 3 m and with the control laws (9)-(11), (17), and $a(t_i)=0$ are shown in figure 1. The wind of (28), was considered without any noise. Estimates of the helicopter states and wind gust using only one observer (9), $\hat{\mathrm{w}}_{13}(t_i)=\tilde{x}_9^{(1)}(t_i)$, are also shown in figure 1 for comparison. It may be concluded that estimates using of two observers are similar with real processes when transient has ended. In this cas,e the estimates using one observer (9) have a phase-delay.

Note that the estimation accuracy of the states and the disturbances is essential in solving the problem of fault detection.

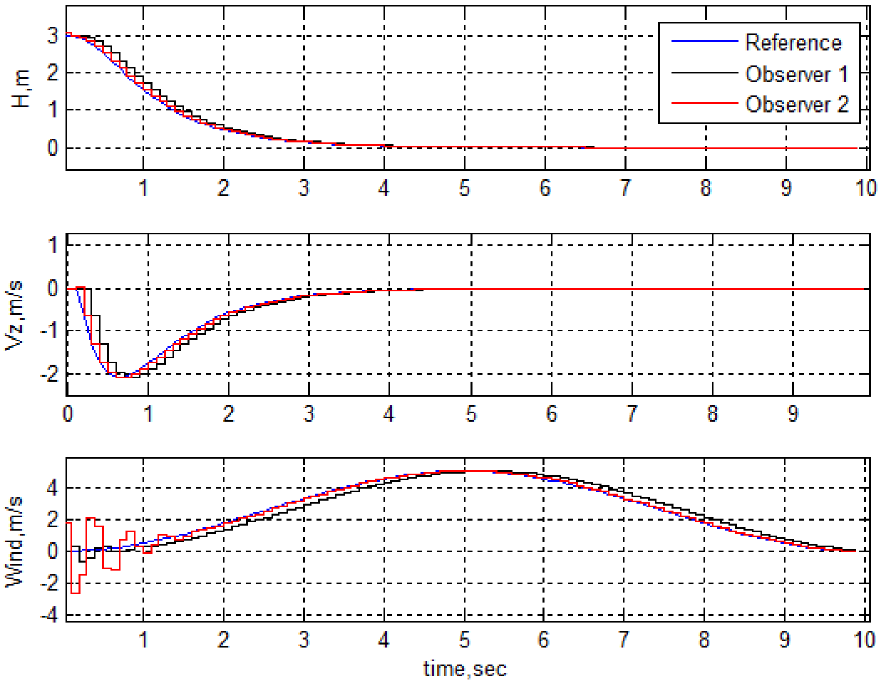

Figure 1. Simulation responses of the helicopter landing under action of wind (28)

To validate the performance of the discrete controller (17), (27), (14) for the full model (1)-(5) with the one observer (9) the simulation responses of the landing under with the wind gusts of (28) and white noise in linear velocities sensors and height sensor are considered. A dispersion of the white noise is equal to 0.01. The results are shown in figure 2. In this case, the longitudinal and lateral movement of the helicopter and its velocity were not measured.

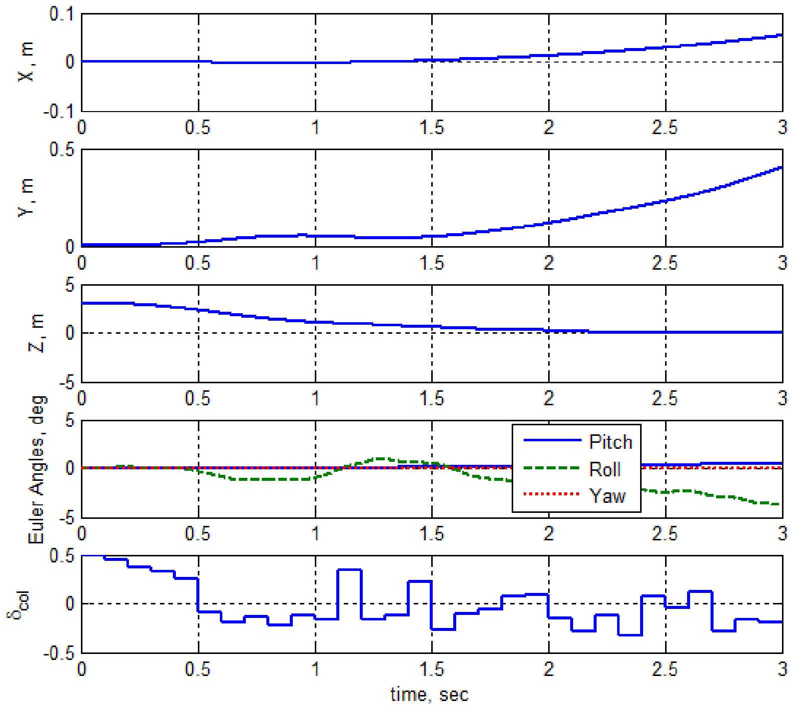

Figure 2. Simulation responses of the landing under action of wind gusts (28) and white noise

Figure 2 shows that the helicopter landed 2.4 sec after the start of the maneuver. And in this case it achieved linear and angular deviations within the allowable limits according to [14].

Thus, the developed discrete controller with observer provides the required control quality for cases with time-delays of the measured and control signals, and under wind disturbances. The main advantage of the developed controller is its easy gain tuning for the real helicopters. The controller gains can be changed for different flight modes using analytical dependences.

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