Iterative Slepian-Wolf Decoding and FEC Decoding for Compress-and-forward Systems

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Abstract: While many studies have concentrated on providing theoretical analysis for the relay assisted compress-and-forward systems little effort has yet been made to the construction and evaluation of a practical system. In this paper a practical CF system incorporating an error-resilient multilevel Slepian-Wolf decoder is introduced and a novel iterative processing structure which allows information exchanging between the Slepian-Wolf decoder and the forward error correction decoder of the main source message is proposed. In addition, a new quantization scheme is incorporated as well to avoid the complexity of the reconstruction of the relay signal at the final decoder of the destination. The results demonstrate that the iterative structure not only reduces the decoding loss of the Slepian-Wolf decoder, it also improves the decoding performance of the main message from the source.

Keywords: Wyner-Ziv coding, multilevel Slepian-Wolf coding/decoding, quantization, compress-and-forward

I. INTRODUCTION

With recent increases in worldwide demand for wireless data services, the incorporation of various relay techniques is becoming more and more attractive. Among those relay techniques, compress-and-forward (CF) has drawn considerable attention recently [1]-[4]. In CF, the source broadcasts a message to the relay and the destination, and the relay compresses and forwards its observation to the destination. At the destination, this observation is reconstructed and combined with the signal directly received from the source to provide the diversity gain. The compression is actually a distributed source coding problem and can be solved by Wyner-Ziv (WZ) coding [5]. A feasible WZ encoder/decoder which consists of a quantizer/dequantizer and a multilevel Slepian-Wolf (SW) encoder/decoder is provided in [6] and proved to perform close to the theoretical bound in [7]. However, this structure cannot be applied in the scenario of CF directly because a different correlation model should be considered rather than the quadratic Gaussian one, which is more applicable in the sensor network scenario [11]. In addition, the compressed information is not protected in [6], which causes considerable amount of decompression errors. Joint source-channel coding/decoding schemes at the relay are introduced in [11] and [13], where iterative structure is used at the destination to improve the performance.

Based on the same line of argument, the iterative decoding structure can be introduced for the main message from the source and the compressed message from the relay as well. In this paper, we use the novel quantization scheme in [8] combined with the error-resilient multilevel SW encoder/decoder in [9] and propose a structure incorporating the concept of soft processing, and establish a fruitful interaction between the SW decoder and the forward error correction (FEC) decoder of the main source message in an iterative manner. Our work is focused on the half-duplex relay channel with additive Gaussian noises. The rest of the paper is organized as follows. The signal model is introduced in the next section. In section III, the iterative structure is proposed. Simulation results are given in section IV and the final section concludes.

II. SIGNAL MODEL

Let's assume a simple cooperative communication system composed of three nodes: source node S, relay node R, and destination node D. Let's further assume that nodes transmission-reception is based on a simple protocol composed of two phases. In the first phase of this protocol S broadcasts its signal to R and D, and in the second phase only R transmits to D. Even though more efficient approach is to allow S and R jointly transmit in the second phase, for the convenience of introduction of the proposed approach we adhere to this simple protocol. We assume that the channels associated to S-D, S-R, and R-D links to be memoryless and defined by probability transitions $p(y_{1,n}|x_n, \theta_{1,n})$, $p(y_{r,n}|x_n, \theta_{r,n})$, and $p(y_{2,n}|x_{r,n}, \theta_{2,n})$, respectively. Here $x_n \in \Xi$ is the symbol transmitted by S at the n-th use of the S-D, and S-R channels, and $y_{1,n} \in \Psi_d$ and $y_{r,n} \in \Psi_r$ are the corresponding outputs, respectively. Ψ_d and Ψ_r are received alphabets of D and R, respectively. In practice they are either P (set of real numbers) or X (set of complex numbers), depending on the employed modulation. Similarly, $x_{r,n} \in \Xi_r$ is the symbol transmitted by R at the n-th use of R-D channel and $y_{2,n} \in \Psi_d$ is the corresponding output. Ξ and Ξ_r are modulation alphabet used by S and R transmitters and for the simplicity of exposition we assume both to be {-1, +1}. $\theta_{1,n}$, $\theta_{r,n}$, and $\theta_{2,n}$ are representing the channel states of the respective channels at their n-th use. Later on at section IV where we provide simulation results we will assume Gaussian model for all the channels. This will imply $\theta_{1,n}$, $\theta_{r,n}$, and $\theta_{2,n}$ are fixed for all the channel uses n.

Now let's describe the details of the processings carried out at S and R. During phase 1, the information sequence $\boldsymbol{d} \in \{0,1\}^K$ at S is encoded to $\boldsymbol{c}_s=[c_{s,1},\ldots,c_{s,N}]$, using FEC code X_S with coding rate K/N. Then $\boldsymbol{c}_s$ is modulated to $\boldsymbol{x}= [x_1, x_2,\ldots, x_N]$. The received signals at the destination and the relay are $\boldsymbol{y}_1= [y_{1,1}, y_{1,2},\ldots, y_{1,N}]$ and $\boldsymbol{y}_r= [y_{r,1}, y_{r,2},\ldots, y_{r,N}]$, respectively. At R, $y_{r,n}$ is quantized to bin index $\boldsymbol{v}^{(n)} \in \{0,1\}^{M\times 1}$:

$$\boldsymbol{v}^{(n)}=[v_{1,n}, v_{2,n},\ldots, v_{M,n}]^T,$$

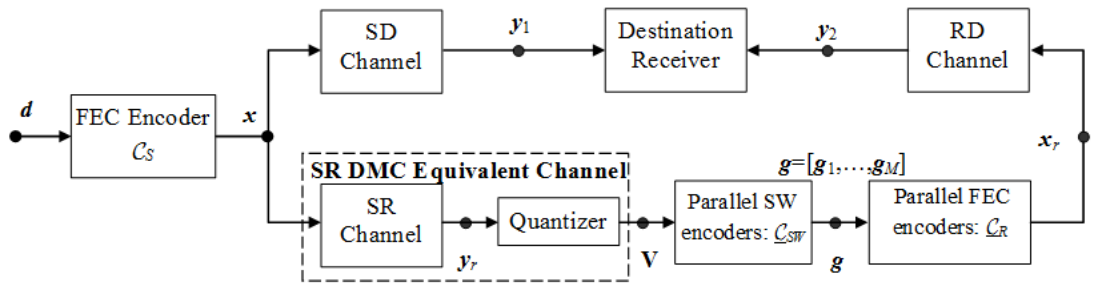

Fig.1 System Model ($g=[g_1,\ldots,g_M]$)

where M is the quantization level. Thus a binary matrix $\mathbf{V}$ will be obtained as

$$\mathbf{V}=[\boldsymbol{v}^{(1)},\boldsymbol{v}^{(2)},\ldots,\boldsymbol{v}^{(N)}]=\left[v_{m,n}\right]_{\substack{m=1,\ldots,M\\ n=1,\ldots,N}}=[\boldsymbol{v}_1,\boldsymbol{v}_2,\ldots,\boldsymbol{v}_M]^t$$

The multilevel SW encoding is performed in the following way. The m-th row of $\mathbf{V}$, denoted by $\boldsymbol{v}_m$, is compressed by the m-th level SW encoder. Here we use LDPC codes with compression rate R_m due to their better performance [6], [10]-[11]. Let $X_{SW,m}$ be the code to compress $\boldsymbol{v}_m$ to

$$\boldsymbol{g}_m=\boldsymbol{v}_m\mathbf{H}_m=[g_{m,1}, g_{m,2},\ldots, g_{m,K_m}],$$

where $\mathbf{H}_m$ is the $N\times K_m$ parity check matrix of $X_{SW,m}$, and $K_m=R_mN$. The compressed $\boldsymbol{g}_m$ is then encoded by a FEC code X_m into the sequence $\boldsymbol{c}_m\in\{0,1\}^{1\times N_m}$ to help the system combat the errors introduced in the R-D link. The codewords $\boldsymbol{c}_m$ $m=1,\ldots,M$ are multiplexed to $\boldsymbol{c}_r=[\boldsymbol{c}_1,\ldots,\boldsymbol{c}_M]$ and then modulated to $\boldsymbol{x}_r=[x_{r,1}, x_{r,2},\ldots, x_{r,\tilde{N}}]$ where $\tilde{N}=N_1+\ldots+N_M$. The received signal at D in the second phase is $\boldsymbol{y}_2=[y_{2,1}, y_{2,2},\ldots, y_{2,\tilde{N}}]$. The received signals $\boldsymbol{y}_1$ and $\boldsymbol{y}_2$ are fed into the final receiver to be used for decoding of the main information sequence $\boldsymbol{d}$. The overall system is depicted in Fig.1, where $\underline{X}_{SW}=\{X_{SW,1},\ldots, X_{SW,M}\}$ and $\underline{X}_R=\{X_{R,1},\ldots, X_{R,M}\}$ denote the multi-level SW codes, and the relay FEC codes, respectively.

In Fig. 1, the S-R link and the quantizer are combined and regarded as an equivalent discrete memoryless channel (DMC). This will help us to understand the information exchange process described in the next section more clearly. Also, it should be emphasized that in above descriptions the encoding of $\boldsymbol{g}_m$ to $\boldsymbol{c}_m$ is before the multiplexing. Since the length of $\boldsymbol{g}_m$ varies according to the compression rate, the length of $\boldsymbol{c}_m$ varies as well to keep the encoding rate constant. However, it can be conducted in a more flexible manner by placing the encoder after multiplexing. After $\boldsymbol{g}$ is obtained, we can either encode $\boldsymbol{g}$ with a longer FEC code or conduct similar parallel coding scheme after dividing $\boldsymbol{g}$ into arbitrary number of parts. By doing this, the FEC coding is not necessarily constrained within each SW encoding level. Some simulations are presented later to demonstrate this point.

III. ITERATIVE DECODING AND DECOMPRESSIING

The final receiver at the destination node can use an iterative architecture to perform close to the optimal joint decoding and decompressing. Considering the system structure composed of FEC codes and SW compressions, applying message passing principle will lead us to the iterative receiver architecture depicted in Fig. 2. As illustrated in this figure the receiver contains an iterative decoder called SW-FEC$_R$. While this decoder benefits from some iterations inside, it also iteratively interacts with soft-input soft-output (SISO) decoder of X_S. We call these two types of iterations as inner and outer iterations, respectively. First we describe the outer iteration loop and then briefly go through the inside structure of the SW-FEC$_R$ decoder.

Upon reception of the $\boldsymbol{y}_1$ and $\boldsymbol{y}_2$ sequences, the log-likelihood-ratio (LLR) vectors $\boldsymbol{l}_1=[l_{1,1}, l_{1,2},\ldots, l_{1,N}]$ and $\boldsymbol{l}_2=[l_{2,1}, l_{2,2},\ldots, l_{1,\tilde{N}}]$ will be calculated. These two vectors are expressing soft values of the coded signals $\boldsymbol{x}$ and $\boldsymbol{x}_r$ received from the corresponding SD and RD channels, respectively. $\boldsymbol{l}_1$ will be propagated through SR-DMC block to provide soft information $\boldsymbol{\Lambda}^{(V)}$ about variable $\mathbf{V}$. This information along with $\boldsymbol{l}_2$ is used by iterative SW-FEC$_R$ decoder to provide improved soft information $\mathbf{L}^{(V)}$ about variable $\mathbf{V}$. Then $\mathbf{L}^{(V)}$ is send through SR-DMC^{-1} block to produce soft information $\boldsymbol{l}^{(x)}=[l_1^{(x)},l_2^{(x)},\ldots,l_N^{(x)}]$ about $\boldsymbol{x}$. The SISO decoder of X_S will use $\boldsymbol{l}^{(x)}+\boldsymbol{l}_1$ and will provide soft extrinsic values $\boldsymbol{\lambda}^{(x)}=[\lambda_1^{(x)},\lambda_2^{(x)},\ldots,\lambda_N^{(x)}]$ about $\boldsymbol{x}$ and soft or hard output values $\boldsymbol{l}^{(d)}$ about information $\boldsymbol{d}$. In the next outer iteration $\boldsymbol{\lambda}^{(x)}+\boldsymbol{l}_1$ will propagate through SR-DMC block towards the SW-FEC$_R$ decoder.

As SISO decoding for most of FEC codes are well described and understood in the literature, we avoid describing the details of the SISO decoder of X_S. The new ingredients in the outer iteration of the receiver that needs detailed descriptions are the SR-DMC and SR-DMC^{-1} blocks. These two blocks apply message passing principle to the equivalent DMC channel of the source to relay link of Fig. 1. This DMC channel can be described by the probability transitions $P_{DMC}(\boldsymbol{b}|u)$ with $\boldsymbol{b}\in\{0,1\}^{M\times 1}$ and $u\in\{-1,+1\}$. The two variables u and $\boldsymbol{b}$ are dummy variables representing x_n and $\boldsymbol{v}^{(n)}$ for any given time index n. The probability transitions for any given $\boldsymbol{b}$ and x can be calculated using the characteristics of the SR original channel and the relay quantizer. Assuming SR channel to be memoryless, for example due to use of an interleaver at the source node, the transition probability of $p(\mathbf{V}|\boldsymbol{x})$ will be expressed as $\prod_n P_{DMC}(\boldsymbol{v}^{(n)}|x_n)$.

Fig. 3 presents a simplified Bayesian graph of the system. Based on the causal relation between the variables the local function for each variable node is identified in this figure. Therefore, the overall joint probability function of the system will be

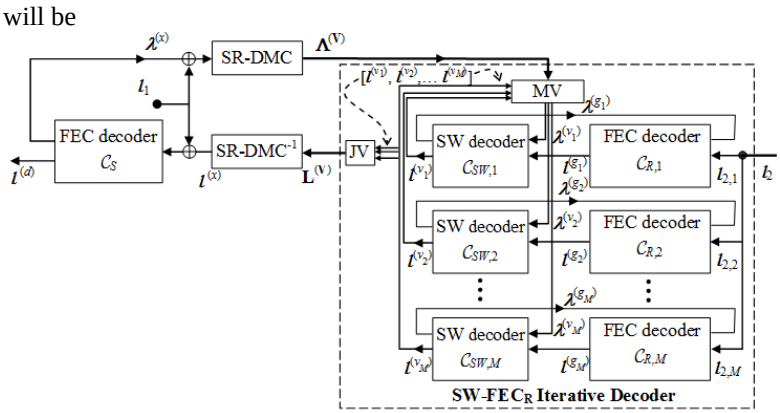

Fig.2 Iterative structure

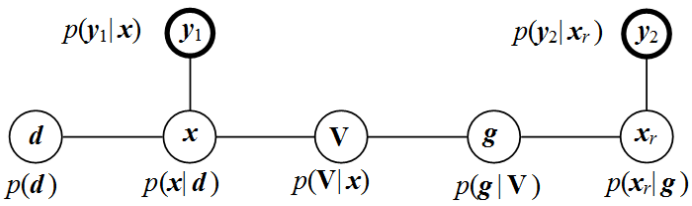

Fig. 3 A simplified Bayesian graph of the system

$$p(\boldsymbol{d}, \boldsymbol{x}, \mathbf{V}, \boldsymbol{g}, \boldsymbol{x}_r, \boldsymbol{y}_1, \boldsymbol{y}_2) = p(\boldsymbol{d}).p(\boldsymbol{x}|\boldsymbol{d}).p(\mathbf{V}|\boldsymbol{x}).p(\boldsymbol{g}|\mathbf{V}).p(\boldsymbol{x}_r|\boldsymbol{g}).p(\boldsymbol{y}_1|\boldsymbol{x}).p(\boldsymbol{y}_2|\boldsymbol{x}_r).$$

Essentially upon reception of $\boldsymbol{y}_1$ and $\boldsymbol{y}_2$ the receiver should perform a marginalization over the graph to calculate the posterior distribution of $\boldsymbol{d}$. Apparently, the direct marginalization over this graph with regard to size of its variable nodes will be prohibitive. The iterative receiver of Fig. 2 in fact attempts to approach this marginalization with a reasonable complexity. Based on the presented graph, now we can work out what kind of messages should be sent from $\boldsymbol{x}$ to $\mathbf{V}$ and vice versa. As the SR channel is assumed to be memoryless and thus the SR-DMC channel, the messages related to $\boldsymbol{x}$ and $\mathbf{V}$ can be decomposed to their component levels, i.e. x_n and $\boldsymbol{v}^{(n)}$, respectively. As x_n is binary its corresponding soft value can be expressed as the logarithmic ratio of its two possible message values, while $\boldsymbol{v}^{(n)}$ is 2^M-aray and its soft value will be expressed as the logarithm of its every possible message value. In this regard the soft values $\mathbf{\Lambda}^{(\mathbf{V})}$ and $\mathbf{L}^{(\mathbf{V})}$ in Fig. 2 are 2^M-aray symbol reliabilities. The blocks SR-DMC and SR-DMC^{-1} perform the corresponding per component message updates for the two directions of $\boldsymbol{x}$ to $\mathbf{V}$ and vice versa, respectively:

SR-DMC message update: This block using the new incoming message $\{ f_{in,n}^{(x)}(u) \}_{u\in\{-1,+1\}}$ about variable x_n provides an output message $\{ f_{out,n}^{(v)}(\boldsymbol{b}) \}_{\boldsymbol{b}\in\{0,1\}^{M\times 1}}$ about variable $\boldsymbol{v}^{(n)}$:

$$f_{out,n}^{(v)}(\boldsymbol{b}) = \sum_{u\in\{-1,+1\}} f_{in,n}^{(x)}(u) P_{DMC}(\boldsymbol{b}|u) \;\; \forall\, \boldsymbol{b}\in\{0,1\}^{M\times 1}$$

SR-DMC^{-1} message update: This block using the new incoming message $\{ f_{in,n}^{(v)}(\boldsymbol{b}) \}_{\boldsymbol{b}\in\{0,1\}^{M\times 1}}$ about variable $\boldsymbol{v}^{(n)}$ provides the output message $\{ f_{out,n}^{(x)}(u) \}_{u\in\{-1,+1\}}$ about variable x_n:

$$f_{out,n}^{(x)}(u) = \sum_{\boldsymbol{b}\in\{0,1\}^{M\times 1}} f_{in,n}^{(v)}(\boldsymbol{b}) P_{DMC}(\boldsymbol{b}|u) \;\; \forall\, u\in\{-1,+1\}$$

It is straightforward to see that these two message updates are just multiplication of the local function $P_{DMC}(\boldsymbol{b}|u)$ to the incoming message and then performing a proper marginalization depending on the direction of the outgoing message. For a given time index n, the incoming message $\{ f_{in,n}^{(x)}(u) \}_{u\in\{-1,+1\}}$ is calculated from $\lambda_n^{(x)} + l_{1,n}$ (the n-th component of the $\boldsymbol{\lambda}^{(x)} + \boldsymbol{l}_1$) and $\{ f_{in,n}^{(v)}(\boldsymbol{b}) \}_{\boldsymbol{b}\in\{0,1\}^{M\times 1}}$ is calculated from the n-th column of $\mathbf{L}^{(\mathbf{V})}$:

$$f_{in,n}^{(x)}(u) = \kappa\left(u; \lambda_n^{(x)} + l_{1,n}\right) \;\; \forall\, u\in\{-1,+1\},$$

$$f_{in,n}^{(v)}(\boldsymbol{b}) = \exp\left(l_{val(\boldsymbol{b})+1,n}^{(v)}\right) \;\; \forall\, \boldsymbol{b}\in\{0,1\}^{M\times 1},$$

where $\kappa(u;l) := e^{ul/2}/(e^{-l/2}+e^{+l/2})$ and $val(\boldsymbol{b})$ is the decimal value of the binary label $\boldsymbol{b}$, and $l_{m,n}^{(v)}$ is the (m,n)-th component of the matrix $\mathbf{L}^{(\mathbf{V})}$. Again for a given n, $l_n^{(x)}$ (the n-th component of $\boldsymbol{l}^{(x)}$) and $\Lambda_{m,n}^{(v)}$ (the (m,n)-th component of $\mathbf{\Lambda}^{(\mathbf{V})}$) are calculated as follows:

$$l_n^{(x)} = \log\left(f_{out,n}^{(x)}(+1) \big/ f_{out,n}^{(x)}(-1) \right) \text{ and}$$

$$\Lambda_{val(\boldsymbol{b})+1,n}^{(v)} = \log\left(f_{out,n}^{(v)}(\boldsymbol{b}) \right) \;\; \forall\, \boldsymbol{b}\in\{0,1\}^{M\times 1}.$$

Iterative SW-FEC$_R$ Decoder: This decoder uses an iterative architecture to exploit the coupling between different levels of the multilevel SW code and at the same time also to exploit the relation between the SW code $X_{SW,m}$ and FEC code $X_{R,m}$ at each level m=1,…,M. For each level m the SISO decoders of $X_{SW,m}$ and $X_{R,m}$ iteratively exchange the extrinsic information $\boldsymbol{l}^{(g_m)}$ and $\boldsymbol{\lambda}^{(g_m)}$ to approach joint decoding and decompressing. On the other hand the levels of SW decoders are coupled through iterative exchange of the soft extrinsic values $\{\boldsymbol{l}^{(v_m)}\}_{m=1,\ldots,M}$ and $\{\boldsymbol{\lambda}^{(v_m)}\}_{m=1,\ldots,M}$.

The block MV performs the marginalization of the variable $\mathbf{V}$ onto its composing levels m=1,…,M. It uses the input information $\mathbf{\Lambda}^{(\mathbf{V})}$ and $\{\boldsymbol{l}^{(v_m)}\}_{m=1,\ldots,M}$ and produces the marginalized information $\{\boldsymbol{\lambda}^{(v_m)}\}_{m=1,\ldots,M}$:

$$f_{out,n}^{(v_m)}(u) = \sum_{\boldsymbol{b}\in\{0,1\}^{M\times 1},\text{s.t: } b_m = u} \prod_{j=1,\, j\neq m}^{M} f_{in,n}^{(v_j)}(b_j) f_{in,n}^{(v)}(\boldsymbol{b}) \;\forall\, u\in\{0,1\},$$

where $f_{in,n}^{(v)}(\boldsymbol{b}) = \exp\left(\Lambda_{val(\boldsymbol{b})+1,n}^{(v)}\right)$, $f_{in,n}^{(v_m)}(b_m) = \kappa\left(2b_m - 1; l_n^{(v_m)}\right)$, and $\lambda_n^{(v_m)}$ will be calculated as $\lambda_n^{(v_m)} = \log\left(f_{out,n}^{(v_m)}(1) \big/ f_{out,n}^{(v_m)}(0) \right)$ for m=1,…,M. b_m in the above expressions denotes the m-th component of the binary vector $\boldsymbol{b}$.

The block JV combines the marginal information $\{\boldsymbol{l}^{(v_m)}\}_{m=1,\ldots,M}$ of the levels to calculate soft output value $\mathbf{L}^{(\mathbf{V})}$ for variable $\mathbf{V}$. $L_{m,n}^{(v)}$ (the (m,n)-th component of $\mathbf{L}^{(\mathbf{V})}$) is simply calculated by:

$$L_{val(\boldsymbol{b})+1,n}^{(v)} = \log\left(\prod_{m=1}^{M} \kappa\left(2b_m - 1; l_n^{(v_m)}\right) \right) \;\; \forall\, \boldsymbol{b}\in\{0,1\}^{M\times 1}.$$

The order of iterations within SW-FEC$_R$ Decoder depends on the dependency of the levels to each other. For a quantizer with set-partitioning based labeling, the decoding starts from the first level and goes to the next level in an increasing order. At level m of decoding the decoder of $X_{R,m}$ uses input $\boldsymbol{l}_{2,m}$ as part of $\boldsymbol{l}_2$ related to the coded sequence $\boldsymbol{c}_m$ and produces the soft output values $\boldsymbol{l}^{(g_m)}$. Then the SISO decoder of $X_{SW,m}$ performs its decompression using $\boldsymbol{l}^{(g_m)}$ and the marginalized information $\boldsymbol{\lambda}^{(v_m)}$ and produces soft output values $\boldsymbol{l}^{(v_m)}$ and extrinsic soft values $\boldsymbol{\lambda}^{(g_m)}$. The input information $\boldsymbol{\lambda}^{(v_m)}$ benefits from newly calculated $\{\boldsymbol{l}^{(v_j)}\}_{j=1,\ldots,m-1}$, of the earlier levels, the current values of $\{\boldsymbol{l}^{(v_j)}\}_{j=m+1,\ldots,M}$ of the next levels, and the soft values $\mathbf{\Lambda}^{(\mathbf{V})}$ coming from the outer iteration. In this regard the block MV is executed M times, once per each level. After sequentially going through all the levels new extrinsic values $\{\boldsymbol{\lambda}^{(g_m)}\}_{m=1,\ldots,M}$ will be ready to be used in the next iteration of the SW-FEC$_R$ Decoder. After a number of iterations within this decoder the updated and improved soft information $\{\boldsymbol{l}^{(v_m)}\}_{m=1,\ldots,M}$ of the levels will be combined by JV block to produce new $\mathbf{L}^{(\mathbf{V})}$) that subsequently will be used in the outer iteration of the receiver. In contrast to conventional schemes [5], the presented iterative receiver avoids reconstructing the relay received signal. While distributed source coding concept is useful in allowing us to perform compression at the relay node, apparently for our application there is no need for signal reconstruction. The proposed receiver architecture efficiently utilize this aspect by considering the equivalent SR-DMC channel and the respected derived message passing updates

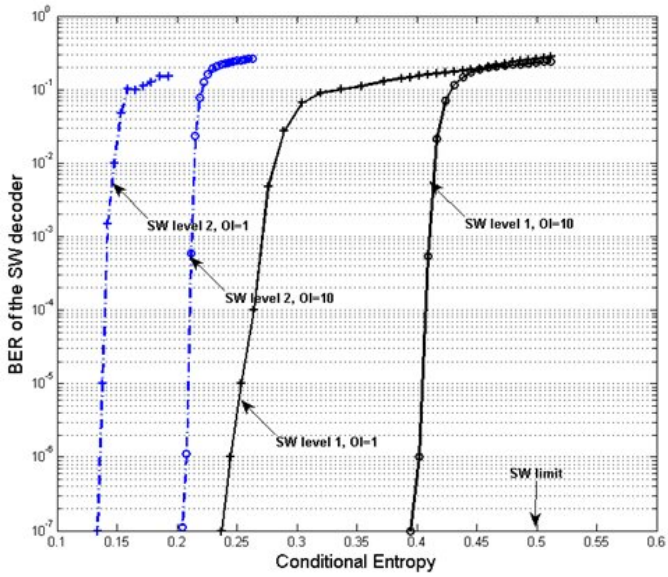

Fig. 4 BER of the SW decoder (SNR_{s2r}=4dB, SNR_{r2d}=2dB, II: inner iteration =3, OI: outer iteration, *Scenario A*)

through SR-DMC and SR-DMC^{-1} blocks. This approach also avoids the loss of the soft contents of $\mathbf{L}^{(V)}$ produced by iterative SW-FEC$_R$ Decoder.

IV. SIMULATION RESULTS

We evaluate the performance of the new joint structure in fixed compression rate and flexible compression rate, respectively. Two simulation scenarios for fixed compression rate are set as follows:

-Scenario A

- 2 levels SW encoding.
- Main FEC code X_s is a LDPC code with length 10^2 and coding rate 1/2.
- The ideal compression rates for two levels can be calculated through the conditional entropy $H(v_1|Y_d^1)$ and $H(v_2|\ v_1, Y_d^1)$, respectively. However, a LDPC code with length $8*10^3$, coding rate 1/2 is used as $X_{SW,1}$ and $X_{SW,2}$, i.e. the compression rate is fixed for both levels at 0.5. This setting does not influence the performance evaluation of the new structure.
- A LDPC code with length $8*10^3$, coding rate 1/2 is used as X_1 and X_2 to protect $\boldsymbol{g}_1$ and $\boldsymbol{g}_2$, respectively, in the relay to the destination link.

-Scenario B

- 2 levels SW encoding.
- Main FEC code X_s is a LDPC code with length 10^2 and coding rate 1/2.
- A LDPC code with length 13298, coding rate 1/4 is used as $X_{SW,1}$ and $X_{SW,2}$, i.e. the compression rate is fixed for both levels at 0.75.
- The compressed signal of two levels $\boldsymbol{g}_1$ and $\boldsymbol{g}_2$ are concatenated as $\boldsymbol{g}$ which is divided into short pieces and encoded by a LDPC code with length 10^2, coding rate ½, where the zero padding is used for the last code.

For comparison purpose, we also simulate two scenarios as:

- No cooperation (NC) where a direct link is applied without relay.

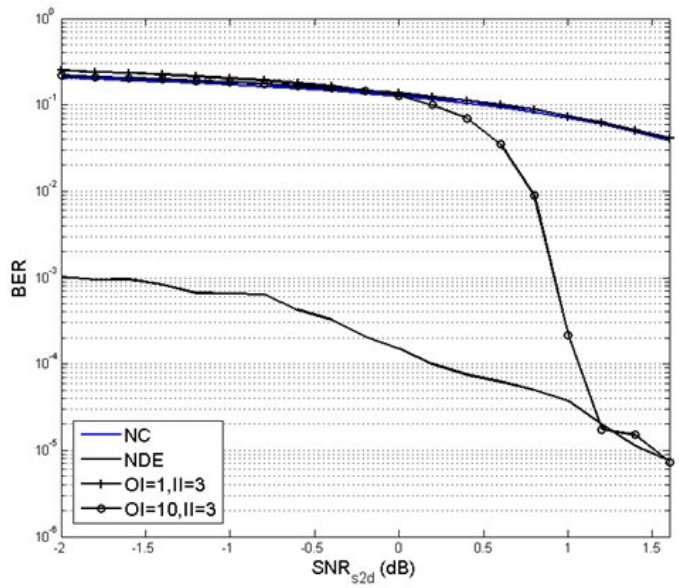

Fig. 5 BER of the main FEC SISO decoder at the destination (*Scenario A*)

- NDE (no decompression error) where we assume the SW decoding is error-free.
- IRD (ideal receive diversity) where the observation of the relay node is perfectly known to the destination.

Fig 4 shows the error performance of the multilevel SW decoder in the first level and the second level, respectively. With SW coding rate 1/2, the SW limit is (1-1/2). Ideally speaking, once the conditional entropy is smaller than the SW limit, the SW decoding error should be zero. However, this figure indicates two facts: firstly, the SW decoding error rate decreases with the decreasing of the conditional entropy; secondly, there is a gap, i.e. decoding loss, between the SW limit and the practical results. This loss can be reduced with longer codes but cannot be eliminated [11]. It is shown that in the first level, with 10 outer iterations, the error rate is greatly improved and even better than the results in [10] and [11], where the LDPC used for SW coding is longer and carefully designed. This result clearly indicates that the SW decoder is able to benefit from the priori information of $\Pr\left(X_s^1(n)\right)$ from the main FEC decoder. The error performance of the second level has the similar shape as level 1 because the multilevel SW decoding is performed level by level and the errors from the upper level will propagate to lower levels and dominate the error performance in lower levels.

Fig 5 depicts the BER performance of the main FEC SISO decoder. For the case of no decompression error (NDE), with error free SW decoding, its performance is much better than the NC case and can act as a lower limit. When the iterative structure is used, the BER performance is high at the beginning because in that SNR region the large amount of errors due to SW decoding will propagate to the final FEC decoder and hence degrade its performance. With improved SNR_{s2d}, i.e. decreased conditional entropy, where the SW decoding error does not dominate the final BER anymore, the iterative structure attain much better performance and is able to approach the performance of NDE. However, note that even without the iterative processing, the BER curve is able to approach the NDE case as well but with a much higher SNR of S-D link.

The benefit of this iterative structure is clearly demonstrated by Fig. 6, where the SNRs of SR and SD links are 0dB. Since both SR and SD links are weak, the decompression error with only one outer iteration is quite high, which leads to high BER

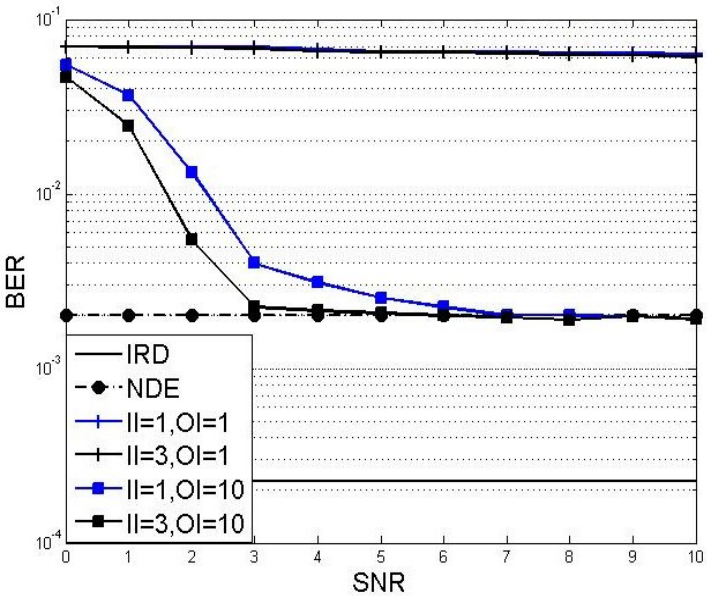

Fig. 6 BER of the main message at the destination (*Scenario B*, IRD: ideal receive diversity, NDE: no decompression error, II: inner iteration, OI: outer iteration)

no matter how many inter iterations are performed. When more outer iterations are performed, the performance of the SW decompressor is greatly improved such that the BER for the main message is reduced with increased SNR_{r2d} and finally converge to the performance of the case without any decompression error. It also shows that inner extrinsic information exchange can improve the BER as well by speeding up this convergence process.

Finally, we present the results for continuous compression rates, where the FEC codes for the main message and the compressed information are the same as in *scenario B*. However, the compression rates of two levels are no longer fixed at 0.5 as in *scenarios A* and *B* but adjusted gradually from the maximum value until the BER of the main message is below 10^{-3}. To evaluate the overall transmission efficiency, i.e. bits/symbol, we have

$$e_{tx} = \frac{\text{number of total information bits of the main message}}{\text{number of total transmitted symbols during two phases}} = \frac{N_{in}}{2N_{in} + 4N_{in}\left(c_{r1} + c_{r2}\right)},$$

where N_{in} is the number of information bits of the main message from the source, c_{ri} is the compression rate for level i. Here the first and second terms in the denominator are the number of transmitted symbols during phase 1 and 2, respectively. For idealistic compression, we have $c_{ri} = H(v_i | Y_d^1)$ for i=1,2. The results are shown in Fig. 7. As we can see, the iterative structure improves the decoding performance. It in turn improves the transmission efficiency by approximately 27.5% and 23.5% when SNR is equal to 1dB and 2.5dB, respectively.

Here we must point out that with delicately designed LDPC codes, the SW decompression error rate could be further improved as well as the BER of the main message. However, this is not in the scope of this paper. Our main objective is to show the benefit from this iterative structure which is independent of the code design.

V. CONCLUSION

There is always a decoding loss when SW coding is used. In this paper, we propose a practical CF system with novel iterative

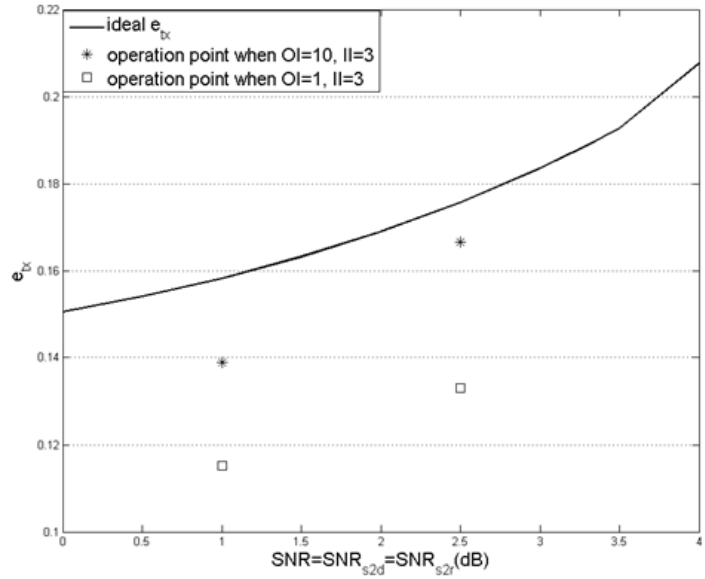

Fig. 7 e_{tx} (SNR=SNRs2d=SNRs2r, SNRr2d=4dB)

structure which targets on reducing this loss. This wisely placed structure makes use of the interaction and information exchange between the main decoder and the SW decoder and incorporates the concept of soft processing. The simulation results show that this structure is able to greatly improve the decompression performance as well as the overall system performance in terms of BER.

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