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A SON Solution for Sleeping Cell Detection using Low-Dimensional Embedding of MDT Measurements

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Abstract—Automatic detection of cells which are in outage has been identified as one of the key use cases for Self Organizing Networks (SON) for emerging and future generations of cellular systems. A special case of cell outage, referred to as Sleeping Cell (SC) remains particularly challenging to detect in state of the art SON because in this case cell goes into outage or may perform poorly without triggering an alarm for Operation and Maintenance (O&M) entity. Consequently, no SON compensation function can be launched unless SC situation is detected via drive tests or through complaints registered by the affected customers. In this paper, we present a novel solution to address this problem that makes use of minimization of drive test (MDT) measurements recently standardized by 3GPP and NGMN. To overcome the processing complexity challenge, the MDT measurements are projected to a low-dimensional space using multidimensional scaling method. Then we apply state of the art k -nearest neighbor and local outlier factor based anomaly detection models together with pre-processed MDT measurements to profile the network behaviour and to detect SC. Our numerical results show that our proposed solution can automate the SC detection process with 93% accuracy.

Keywords—Anomaly Detection, Cell Outages, Low-Dimensional Embedding, LTE, Self-Organizing Networks, Sleeping Cell

I. INTRODUCTION

The increased demands of high throughput, coverage and end user quality of service (QoS) requirements, driven by ever increasing mobile usage incur additional challenges for the network operators. One such challenge is the optimization and maintenance of network performance in a cost-efficient manner which can be addressed through high degree of automation in cellular networks. Automation of the network management process through SON concepts [1] as specified in 3GPP Release 10 standards, is aimed at increasing the robustness and efficiency of LTE network, while minimizing the capital investment and operational expenditures (CAPEX and OPEX). One of the highly desirable functionality in SON is to automate the detection of cells in outage i.e., cells which are not providing normal service level either due to software (SW) or hardware (HW) failure. A special case of cell outage, referred to as *Sleeping Cell* [1], is particularly tricky to deal with even with SON because in this case cell goes into outage or may perform poorly without triggering an alarm for Operation and Maintenance (O&M) system, consequently no

SON compensation function can be launched. Thus a sleeping cell can remain undetected and uncompensated for hours or even days, unless site visit or drive tests are performed or complaints are received by affected customers.

To overcome this problem, in this paper we present novel solution to automatically detect SC using a machine learning approach. A special case of SC has been examined in which a cell becomes *catatonic* (i.e., no service is available) due to bidirectional antenna gain failure, which may occur due to the malfunctioning of transmitting and receiving modules in Evolved Universal Terrestrial Radio Access (E-UTRA) NodeB (eNB). The reported studies in literature that addressed the problem of cell outage detection are either based on quantitative models [2] which requires domain expert knowledge, or simply rely on performance deviation metrics for detection [3]. Just recently interest has emerged in applying methods from the machine learning domain such as clustering algorithms [4] as well as Bayesian Networks [5] to automate the detection of faulty cell behavior. Coluccia *et al.* [6] analyzed the variations in the traffic profiles for 3G cellular systems to detect real-world traffic anomalies. In particular, the problem of detecting catatonic sleeping cells has been addressed by leveraging the *Neighbor Cell List (NCL)* reports [7] to construct a visibility graph, whose topology changes are used to detect cells that are experiencing outage.

Compared to aforementioned approaches, the solution proposed in this paper differs in various aspects. This study adopts a model-driven approach that makes use of mobile terminal assisted data gathering solution based on minimize drive testing (MDT) functionality [1] as specified by 3GPP. The main idea of MDT is that the network can request the user equipment to report the key performance indicators (KPIs) including radio specific measurements from the serving and neighboring cells along with the location information. Our proposed method first maps these KPIs to a low-dimensional embedding space and then further uses them in conjunction with global and local anomaly detection models to build a “normal” network profile. This is in contrast to state of the art techniques that analyze one or two KPIs to learn the threshold levels and use it as a reference for detecting network anomalies. The models once learned leverage the intrinsic characteristics of embedded network measurements to automatically recognize SC situation. Moreover, the geo-locations associated to the measurements are used to localize the position of SC, so that

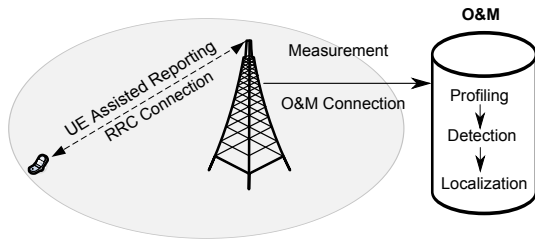


Fig. 1. An overview of Sleeping Cell Detection and Localization Framework

self-healing functionality can be triggered. To the best of our knowledge, no prior study examines the use of global and local anomaly detection methods from the machine learning domain and subsequently applies it for SC detection. Secondly, the framework provided paves a way towards implementing self-healing functionality in emerging (LTE) as well as future (5G) self organizing networks. Therefore, the proposed solution is validated with simulations that are setup in accordance with 3GPP LTE standard in order to construct MDT database for further analysis. The remainder of this paper is structured as follows: Section II presents the proposed framework for SC detection. It also includes a brief discussion on two state of the art anomaly detection models namely k -nearest neighbor and Local Outlier Factor based Anomaly detector. The details of our simulation setup and evaluation methodology are provided in Section III. Finally, Section IV and V report the results and the conclusion, respectively.

II. SLEEPING CELL DETECTION FRAMEWORK

The main idea is to use the MDT reports acquired from a fault-free operating scenario to profile the behaviour of the network. The subsequent step is to use the learned profile to identify the SC situation. The proposed SC detection framework adopts a four step approach including measurements, profiling, detection and localization as shown in Figure (1). The steps involved are further elaborated in the following subsections.

A. Measurement

The MDT reporting schemes have been defined in LTE Release 10 during 2011 [1]. The release proposes to construct a data base of MDT reports from the network using *Immediate* or *Logged* MDT reporting configuration. In this study, the UE's are configured based on immediate MDT configuration to report the cell identification and radio-measurement data to eNB, as specified in Table I, periodically as well as whenever an A2 event (i.e., Serving cell becomes worse than a *threshold*) occurs. The eNB after retrieving these measurements further appends time and wide-band channel quality information (CQI) and forward it to the O&M system to construct the MDT database. The reports obtained from the reference scenario (i.e., fault-free) acts as a benchmark data and used by the target anomaly detection models to learn the network profile. After the completion of network profiling, the models compare the test measurements against the learned profile to detect possible coverage problems as discussed in the following subsection.

B. Profiling and Detection

In the profiling phase, initially data cleaning and normalization operations are performed to pre-process the collected

Features	Description
Location	longitude and latitude information
Serving Cell info	Cell Global Identification (CGI)
RSRP	Reference Signal Received Power in dBm
RSRQ	Reference Signal Received Quality in dB
Neighboring Cell Information	Three Strongest intra-LTE RSRP, RSRQ information

TABLE I. MDT REPORTED MEASUREMENTS

measurements within the database. The KPIs including reference signal received power and quality of the serving as well as of the three strongest neighbouring cells along with the CQI are augmented into one feature vector as shown in Equation 1

$$V = \{RSRP_S, RSRP_{N1}, RSRP_{N2}, RSRP_{N3}, RSRQ_S, RSRQ_{N1}, RSRQ_{N2}, RSRQ_{N3}, CQI\} \quad (1)$$

where S and N stands for serving and neighboring cells, respectively. The 9-dimensional feature vector V corresponds to one measurement sample which is further embedded to only three dimensions in the Euclidean space using Multi-Dimensional Scaling (MDS) method [8]. MDS provides a low-dimensional embedding of the target KPI vectors V while preserving the pairwise distances amongst them. Given, a $t \times t$ dissimilarity matrix Δ^X of the MDT dataset, MDS attempts to find t data points $\psi_1 \dots \psi_t$ in m dimensions, such that Δ^Ψ is similar to Δ^X . Classical MDS (CMDS) operates in Euclidean space and minimizes the following objective function

$$\min_{\psi} \sum_{i=1}^t \sum_{j=1}^t (\delta_{ij}^{(X)} - \delta_{ij}^{(\Psi)})^2 \quad (2)$$

where $\delta_{ij}^{(X)} = \|x_i - x_j\|^2$ and $\delta_{ij}^{(\Psi)} = \|\psi_i - \psi_j\|^2$. Equation 2 can be reduced to a simplified form by representing Δ^X in terms of a kernel matrix using Equation 3

$$X^T X = -\frac{1}{2} H \Delta^X H \quad (3)$$

where $H = I - \frac{1}{t} e e^T$ and e is a column vector of all 1's. This allows us to rewrite Equation 2 as

$$\min_{\Psi} \sum_{i=1}^t \sum_{j=1}^t (x_i^T x_j - \psi_i^T \psi_j)^2 \quad (4)$$

As shown in [8], that the Ψ can be obtained by solving $\Psi = \sqrt{\Lambda} V^T$, where V and Λ are the matrices of top m eigenvectors and their corresponding eigenvalues of $X^T X$ respectively. The m dimensional embedding of the data points are the rows of $\sqrt{\Lambda} V^T$, whereas the value of m is chosen to be 3 in our case. The embedding of KPI's into a lower dimension has several advantages. First, it makes the framework generic allowing it to incorporate new KPI's and network-centric features such as call drop ratios, data traffic etc without imposing higher computational requirements. Subsequently, the interrelationships of high-dimensional databases can be explored in a lower-dimension space. Second, given the growing complexity of the networks, particularly in case of SON, it is challenging to identify few KPIs that accurately capture the behavior of the system. Thus, the embedded representation of KPI's not only addresses this challenge but also aids in the cell profiling process by increasing the separation amongst the dissimilar measurements and vice versa. Consequently, the target algorithms obtain a better estimation of data density and

can identify abnormal measurements as anomalies with higher accuracy, as discussed below.

The embedded KPI representation is then used together with state of the art anomaly detection algorithms to learn the "normal" network profile. This involves defining a SC detection rule to differentiate between normal and abnormal MDT measurements by computing a threshold ' θ ' based on a dissimilarity measure ' \mathcal{D} '. Thus, the SC detection translates to a binary classification problem which can formally be expressed as follows:

$$f(x_i) = \begin{cases} Normal, & \text{if } \mathcal{D}(x_i, D_M) \leq \theta \\ SC, & \text{if } \mathcal{D}(x_i, D_M) \geq \theta \end{cases} \quad (5)$$

where D_M is an embedded MDT dataset that contains 70% samples from the reference scenario. The rest of the samples in the dataset are obtained from the SC scenario to optimize the θ of two state-of-the-art algorithms: k -Nearest Neighbor based anomaly detector (k -NNAD) and Local Outlier Factor based anomaly detector (LOFAD). On the other hand, x_i is the observation obtained from the test dataset D_{test} . The k -NNAD and LOFAD calculate a global and local dissimilarity measure to rank the observed measurements which is used to categorize them as belonging to the normal or a SC scenario, as briefly summarized in the following discussion.

1) *k -Nearest Neighbor based Anomaly Detector (k -NNAD)*: Let x_i be the test instance, and k be the k^{th} neighbor in the D_M . To label x_i as normal or abnormal, the k -NNAD computes a \mathcal{D}_{k-NNAD} based on Equation 6

$$\mathcal{D}_{k-NNAD}(x_i, k, D_M) = \frac{1}{N_{tr}} \sum_{i=1}^{N_{tr}} \mathcal{I}(d_t \leq d_i) \quad (6)$$

The $N_{tr} = |D_M|$, and d_t is the distance of x_i from its k^{th} nearest neighbor and d_i is the distance between i and its k^{th} nearest training object in D_M , whereas $\mathcal{I}(\cdot)$ is an indicator function. Equation 6 represents a global density-based anomaly detection score as proposed in [9]. The test measurement is marked as anomalous if it receives a score greater than the θ value.

2) *Local Outlier Factor based Anomaly Detector (LOFAD)*: The LOFAD [10] tries to compare the local density ρ of the object to that of its k neighbors. It constructs a local neighborhood of an instance x_i and defines its distance to k^{th} nearest neighbor $NN(x_i, k)$:

$$d_b(x_i, k) = d(x_i, NN(x_i, k)) \quad (7)$$

The $d_b(x_i, k)$ is used to construct a neighborhood $\mathcal{N}(x_i, k)$ by including all those points which fulfills the following criteria: $d(x_i, x_j) \leq d_b(x_i, k)$. Formally, reachability distance d_r is defined to estimate the $\rho(x_i, k)$ as follows:

$$d_r(x_i, k) = \max\{d_b(x_j, k), d(x_j, x_i)\} \quad (8)$$

and ρ can be defined as

$$\rho(x_i, k) = \frac{|\mathcal{N}(x_i, k)|}{\sum_{x_j \in \mathcal{N}(x_i, k)} d_r(x_i, x_j, k)} \quad (9)$$

The $d_r(x_i, x_j, k)$ ensures that instances that lie farther away from x_i have lesser impact on $\rho(x_i, k)$. Finally the \mathcal{D} can be

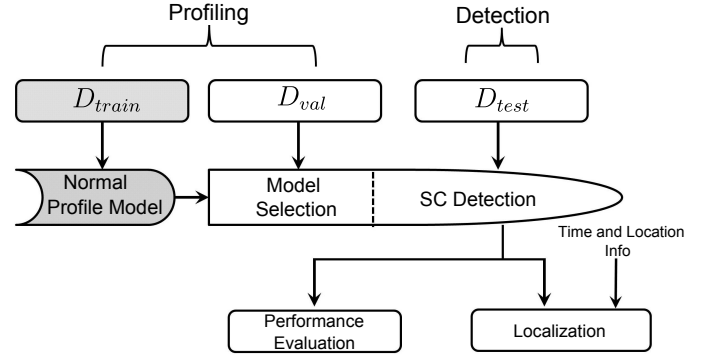


Fig. 2. An overview of Profiling and SC Detection Framework

calculated by comparing the ρ of x_i to its $\mathcal{N}(x_i, k)$, formally defined as:

$$\mathcal{D}_{LOFAD}(x_i, k, D_M) = \frac{\sum_{x_j \in \mathcal{N}(x_i, k)} \frac{\rho(x_j, k)}{\rho(x_i, k)}}{|\mathcal{N}(x_i, k)|} \quad (10)$$

\mathcal{D}_{LOFAD} represents a local density-estimation score whereas value close to 1 mean x_i has same density relative to its neighbours. On the other hand, a significantly high \mathcal{D}_{LOFAD} score is an indication of anomaly.

The parameter selection for k -NNAD and LOFAD is performed using cross-validation (CV) method as listed in Algorithm 1. The D_M is divided into training D_{train} and validation dataset D_{val} using K -folds approach, whereas the value of K is chosen to be 10 in our framework. To select the optimal model, each target detector is trained for different values of k and the model achieving the average highest detection score is selected. The detection performance of the selected models are then compared by evaluating them against D_{test} as shown in Figure 2.

Algorithm 1 Model Selection using CV Method

- 1: Split the target dataset D_M into K chunks.
 - 2: **for** $l = 1, 2, \dots, K$: **do**
 - 3: Set D_{val} to be the l^{th} chunk of data
 - 4: Set D_{train} to be the other $K - 1$ chunks.
 - 5: Fit each model to D_{train} and evaluate its performance on D_{val} .
 - 6: **end for**
 - 7: **Model Selection**: Select the model with a average highest detection score
 - 8: **Performance Estimation**: Evaluate the performance of the selected model on D_{test}
-

C. Localization of SC

During the profiling phase, the location information in the MDT measurements is used to estimate the coverage area of the best serving cell which we refer to as dominance area of the eNB. As soon as the SC situation triggers in the network, the malfunctioning eNB becomes no longer available. Consequently, the dominance area of the neighboring cells increases to serve the affected area. Therefore, if only CGI information is utilized to localize SC, the anomalous MDT reports within the target area, would erroneously be associated to its neighboring cells. However, the location of SC can be established by

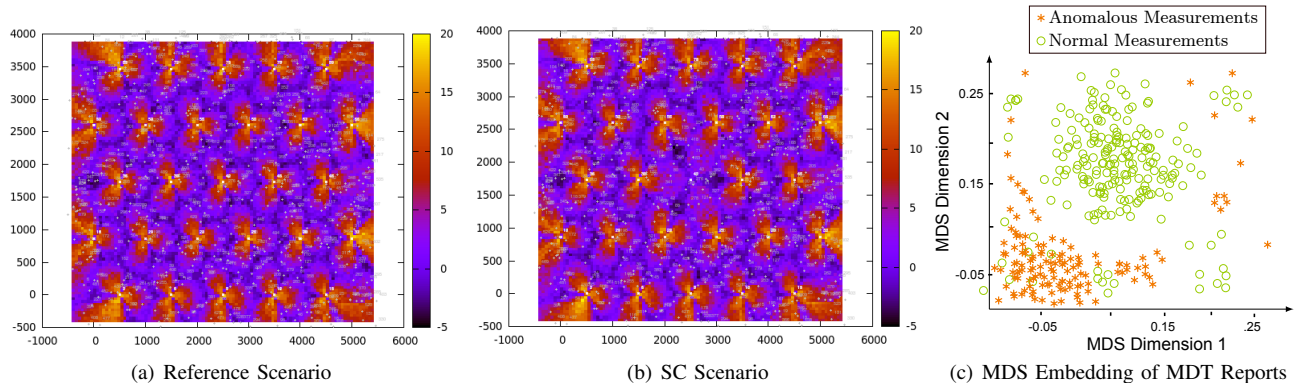


Fig. 3. (a) SINR plot of reference scenario (b) SINR plot of SC scenario where antenna gain of cell 11 is attenuated to -50 dBi (c) MDT measurements in the embedded space classified into normal and anomalous categories by k -NNAD

Parameter	Values
Cellular Layout	27 Macrocell sites
Sectors	3 Sectors per cell
User Distribution	Uniform Random Distribution
Path Loss	$L[dB] = 128.1 + 37.6 \log_{10}(R)$
Antenna Gain (Normal Scenario)	15 dBi
Antenna Gain (SC Scenario)	-50 dBi
Slow Fading Std	8 dB
Simulation Length	420s (1 time step = 1ms/TTI)
BS Tx Power	46 dBm
Network Synchronization	Asynchronous
HARQ	Asynchronous, 8 SAW channels, Maximum Retransmission = 3
Cell Selection Criteria	Strongest RSRP defines the target cell
Load	20 users/cell
MDT Reporting Interval	240 ms
Traffic Model	Infinite Buffer
HO Margin	3dB

TABLE II. SIMULATION PARAMETERS

correlating the geo-location of the measurements labelled as anomalous with the dominance areas estimated during the profiling stage. To establish such a correlation, we calculate a standard z -score for each eNB corresponding to its estimated dominance area. The z -score is calculated as $z_b = \frac{|n_b - \mu_n|}{\sigma_n}$ where n_b is the number of MDT reports labeled as anomalies for the eNB b , and variables μ_n and σ_n are the mean and standard deviation scores of the neighbouring cells. The change in the z -score for each eNB in the SC scenario compared to reference scenario is used to localize the position of SC.

III. SIMULATION SETUP AND EVALUATION METRIC

A. Simulation Setup

A full dynamic system tool is employed to simulate the LTE network based on 3GPP specifications. The simulation run time was 7 minutes for each scenario with parameter configurations as listed in Table II. A reference scenario is used to profile the normal network operating behaviour by collecting the UE reported MDT measurements. In the SC scenario, the antenna gain of cell 11 is attenuated to -50 dBi for a duration of three minutes. The SINR plots of the reference and SC scenario obtained during the simulation has been shown in Figure 3. The collected measurements are used by the global and local anomaly detection models to profile the network which allows them to detect anomalous situations as discussed in Section II.

B. Detection Performance

The Receiver Operating Characteristic (ROC) curves [11] plots the true positive rate or also detection rate (DR) (i.e., a percentage of anomalous measurements correctly classified as anomalies) against the false positive rates (FPR) (i.e., a percentage of normal cell measurements classified as anomalies). In this study, a standard performance metric named as Area under ROC curve (AUC) is used to access the performance of the target algorithms for detecting anomalous measurements from the SC scenario. The ROC curves are generated by plotting the DR against FPR by varying the θ for each model until a DR value reaches 100%. To select the optimal model for each anomaly detector a parameter search (i.e. $k = 1, 2, \dots, 30$) is performed using Algorithm 1. The final values are found to be 20 and 8 for for k -NNAD and LOFAD, respectively.

IV. RESULTS AND DISCUSSION

The employed anomaly detection algorithms profile the network behaviour using local and global approaches. The target models are then used to classify test measurements into normal or anomalous categories. It has been observed that the most of the KPIs from the reference scenario, grouped themselves into a large cluster when projected to an embedded space. Conversely, the measurements belonging to SC lie far from the samples that conforms to normal operations as depicted in Figure 3(c). MDS tries to maximize the variance between the data points and consequently dissimilar points are projected far from each other allowing the models to compute a robust dissimilarity measure for anomaly detection. k -NNAD based global profiling technique that relies on global density estimation procedure outperformed local density estimation method LOFAD, since the anomalous measurements obtained from the outage scenario largely act as global anomalies. Moreover, some of the normal measurement also form small micro clusters. This is due to exceptionally good RSRP values reported by the mobile terminals while they were in close proximity to the serving eNB. However, LOFAD treats them as local anomalies. Additionally, the measurements obtained from the cell edges show similarity with data samples that corresponds to outage scenario. Hence, in the embedded space they are projected close to the samples that corresponds to abnormal measurements. From a classification perspective, the target models wrongly classify such measurements as belonging to a SC scenario. But from a SON perspective,

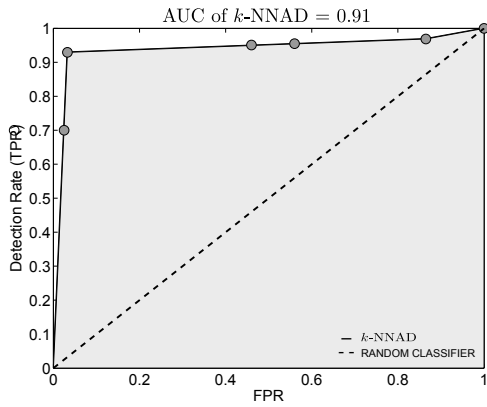


Fig. 4. ROC Curve of k -NNAD based profiling technique

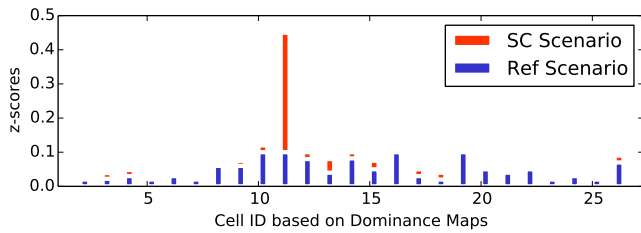


Fig. 5. Cell Dominance areas versus z -scores for Localization of SC

identification of such abnormality indicates a weak coverage problem and can be used to trigger automated actions for coverage optimization. Similarly, some of the UE generated measurements, as a result of radio link failure are also treated as anomalies. Figure 4 shows that k -NNAD achieved a 93% detection rate which is 5% higher than LOFAD at a false alarm rate of 10%. As shown in Table III, the AUC value achieved by k -NNAD and LOFAD are 0.91 and 0.85, respectively, that shows the superiority of global anomaly detection methods over local approaches for profiling the network behaviour.

We use k -NNAD as our target model to calculate the z -score for each eNB separately for reference and SC scenarios as shown in Figure 5. It can be seen that even the cells which are not in outage receives a z -score, since a fraction of the UE reported measurements belonging to their dominance areas are identified as anomalies due to several reasons as already discussed above. Therefore, to classify a cell as a SC, each eNB must collect a minimum number of anomalous reports (i.e., n_b) to achieve a significantly higher z -score compared to rest of the cells. In our case, we found out that a minimum number of 4800 MDT measurements are required to observe a significant change in the z -score. The value of n_b further determines the detection delay since the measurement count is dependent on the number of active users in the eNB dominance area. For example, in our case 20 uniformly distributed UE's are sending reports with a periodicity of 240 ms, and therefore the system would take approximately one minute to detect an outage situation. Likewise, the delay value can increase or decrease depending on the user density in the target cell.

V. CONCLUSION

This paper has presented a machine learning framework for automating the sleeping cell detection process in an LTE

Model	Approach	AUC score
k -NNAD	Global	0.91
LOFAD	Local	0.85

TABLE III. PERFORMANCE OF TARGET ANOMALY DETECTION MODELS FOR SC DETECTION

network using minimization of drive testing functionality. The proposed approach builds a normal profile of the network behaviour in a low-dimensional embedding space. The measurements are labeled as anomalous if they deviate from the learned profile. For this purpose, multi-dimensional scaling method in conjunction with global and local anomaly detection models were examined. It was found that k -NNAD, a global anomaly detection model achieved a higher detection accuracy compared to LOFAD which adopts a local approach for classifying abnormal measurements. Finally the UE reported coordinate information is employed to establish the dominance areas of target cells which are subsequently used to localize the position of sleeping cell. The proposed SC detection framework can act as a foundation for next generation network monitoring tool, since it allow easy inclusion of other key performance indicators from the network and can be extended to detect other issues including coverage holes, weak coverage as well as performance degradation problems.

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