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On Receiver Design for Uplink Low Density Signature OFDM (LDS-OFDM)

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Abstract—Low density signature orthogonal frequency division multiplexing (LDS-OFDM) is an uplink multi-carrier multiple access scheme that uses low density signatures (LDS) for spreading the symbols in the frequency domain. In this paper, we introduce an effective receiver for the LDS-OFDM scheme. We propose a framework to analyze and design this iterative receiver using extrinsic information transfer (EXIT) charts. Furthermore, a turbo multi-user detector/decoder (MUDD) is proposed for the LDS-OFDM receiver. We show how the turbo MUDD is tuned using EXIT charts analysis. By tuning the turbo-style processing, the turbo MUDD can approach the performance of optimum MUDD with a smaller number of inner iterations. Using the suggested design guidelines in this paper, we show that the proposed structure brings about 2.3 dB performance improvement at a bit error rate (BER) equal to $10^{-5}$ over conventional LDS-OFDM while keeping the complexity affordable. Simulations for different scenarios also show that the LDS-OFDM outperforms similar well-known multiple access techniques such as multi-carrier code division multiple access (MC-CDMA) and group-orthogonal MC-CDMA.

Index Terms—Multi access communication, Iterative methods, Code division multiple access, Multi-user channels.

I. INTRODUCTION

MULTI-carrier code division multiple access (MC-CDMA) is considered to be a suitable approach to cope with challenging service demands due to its ability of exploiting both time and frequency resources [1]–[4]. In the uplink channel using MC-CDMA, non-orthogonality of received effective signatures causes multi-user interference (MUI). Multi-user detection (MUD) can be used to moderate the detrimental effects of MUI. However, implementation of optimum MUDs is not practical due to their prohibitively high computational complexity. Even for a moderate number of interfering users, the complexity grows exponentially with the number of users [5]. In order to reduce the complexity of MUD for MC-CDMA systems, group-orthogonal MC-CDMA (GO-MC-CDMA) [6] has been proposed. It has been shown that by dividing users into subgroups, GO-MC-CDMA is able to achieve a performance very close to the single-user bound [6] while keeping the complexity affordable.

In order to achieve a further reduction in the complexity of MUD for uplink MC-CDMA, we have recently proposed low density signature (LDS) structures for MC-CDMA systems known as LDS-OFDM [7]. In LDS-OFDM systems, because of their low density signature structures, each data symbol is only spread over a limited number of chips (effective processing gain). Then, each user’s generated chip is transmitted over an orthogonal sub-carrier. Furthermore, each sub-carrier is only used by a limited number of data symbols that may possibly belong to different users. Therefore each user, transmitting on a given sub-carrier, will experience interference from only a small number of other data symbols. In other words, LDS-OFDM is a special case of the MC-CDMA system where its spreading sequences have low density. In theory, low density signatures were introduced in [8] for downlink multi-carrier systems. Since the LDS-OFDM system is proposed for uplink multi-carrier systems, the design of the receiver is different from the one proposed in [8], because in uplink we have a single centralised receiver. Designing this receiver to have minimal or affordable complexity while being able to achieve the single user performance bound is a challenge. This paper aims to address this challenge by proposing a suitable design and analysis tool to obtain an efficient detection/decoding algorithm. The contributions of this paper are as follows:

1) An efficient implementation of the message-passing algorithm (MPA) [9] to approximate the maximum a posteriori (MAP) detection for the LDS structure is presented. This algorithm is able to reduce the complexity of the receiver compared to the existing state-of-the-art multiple access systems. This technique calculates the extrinsic information for each sub-carrier by using the MAP-based detections: Log-MAP with brute-force searching among possible combinations of symbols.

2) The iterative detector is analysed using extrinsic information transfer (EXIT) charts. Furthermore, design guidelines are provided for improving the LDS structure.

3) A tuned turbo multi-user detector/decoder (MUDD) for LDS-OFDM is proposed. The proposed turbo receiver comprises two basic components: an LDS multi-user symbol detector and a collection of users’ decoders. Using the EXIT chart analysis we show how to tune the receiver in a way that the turbo MUDD can approach its final performance using a smaller number of inner iterations. In [10], we have already shown a primary study on LDS-OFDM with turbo iterations to demonstrate the effect of various loads, and here we extend this work further by showing how the turbo receiver is tuned using a novel technique.

4) Finally, using design guidelines extracted from the EXIT...
different users have identical number of data symbols, indices $k$
obreakdashes{} to zero). In this section, we first explain the system model of density (a large number of chips in the sequence are equated with those of other multiple access techniques. Finally, section VII is devoted to concluding remarks.

II. LDS-OFDM SYSTEM MODEL

Consider a multiple access channel (MAC), corresponding to the uplink communications from multiple users to a single base station in a practical system. The block diagram of LDS-OFDM for an uplink system is shown in Fig. 1. It can be noted that the main blocks are similar to an MC-CDMA system. Similar to an MC-CDMA spreading process, we multiply the modulated symbol with a spreading signature (a random sequence of chips). However, in the LDS-OFDM case, the main difference is that the spreading signature has a low density (a large number of chips in the sequence are equated to zero). In this section, we first explain the system model of LDS-OFDM, which is similar to the one published in [10]. Then the iterative receiver for this system is described. It is assumed that the LDS-OFDM system has $K$ users with user indices $k = 1, \ldots, K$. Let $X_k$ be the constellation alphabet, from which the transmitted symbol for user $k$ will take its value. Thus, the modulated symbol for user $k$ is formed by mapping a sequence of independent information bits $b_k \in \mathbb{F}_q^d$ to $X_k$, and can be represented by the function $\phi_k : \mathbb{F}_q^d \rightarrow X_k$, where $q_k$ is the number of information bits per symbol for user $k$. Without loss of generality, all users are assumed to take their symbols from the same constellation alphabet, i.e., $X = X_k, \forall k = 1, \ldots, K$. In addition, we assume that the different users have identical number of data symbols, $M$.

Introducing $N_c$ as the number of total chips, the spreading matrix for user $k$ will be $S_k = [s_{k,1}, \ldots, s_{k,M}] \in \mathbb{C}^{N_c \times M}$, where matrix $S_k$ has only $d_v$ (effective spreading factor) non-zero values on each column. Let us define $S = [S_1, \ldots, S_K] \in \mathbb{C}^{N_c \times MK}$ as the low density signature matrix of the multi-user system. The norm of each signature vector is unity. Furthermore, let variable $d_v$ be the number of symbols that interfere with each other at each chip. We define $A = \text{diag}(A_1, \ldots, A_K)$ as the transmit power gain of users and $G_k = \text{diag}(g_{k,1}, \ldots, g_{k,N_c})$ as the corresponding channel gain for the $k^{th}$ user.

In LDS-OFDM, each user’s generated chip will be transmitted over an orthogonal sub-carrier. Therefore, the received spreading sequence for data symbol $m \in \{1, \ldots, M\}$ of user $k$ can be represented by $h_{k,m} = A_kG_kS_{k,m}$. Let $\mathcal{J}_n = \{(k,m) : s_{k,m}^n \neq 0\}$ be the set of data symbols (which may belong to different users) that interfere on chip $n$. Also, let $\mathcal{E}_{k,m} = \{n : s_{k,m}^n \neq 0\}$ be the set of different sub-carriers that the $m^{th}$ symbol of user $k$ is spread on.

For an uplink MC-CDMA system, the received signal at sub-carrier index $n$ is written as

$$y_n = \sum_{k=1}^K \sum_{m=1}^M h_{k,m}^n x_{k,m} + v_n, \quad (1)$$

where $v_n$ is the AWGN for sub-carrier $n$ and $x_{k,m}$ is the $m^{th}$ data symbol of user $k$. Thus, considering that in LDS-OFDM system, the signature has a limited number of non-zero positions, we can express the received signal at the $n^{th}$ chip (sub-carrier) as follows:

$$y_n = \sum_{(k,m) \in \mathcal{J}_n} h_{k,m}^n x_{k,m} + v_n. \quad (2)$$

At the receiver side, after OFDM demodulation, the signal is sent to a near-optimum MUD based on MPA. An LDS system, with $K$ users and $N_c$ sub-carriers, is represented by the factor graph $G(\mathcal{U}, \mathcal{C})$ where $\mathcal{C}$ and $\mathcal{U}$ are the sets of function nodes and variable nodes, respectively. Each chip is represented by a function node $c \in \mathcal{C}$ and each user-symbol is represented by a variable node $u \in \mathcal{U}$. The connections between the received sub-carrier and its related users are shown by edges. Let $c_n(x_{k,m})$ be the edge that connects function node $n$ to variable node $m$ of user $k$. Because this receiver is iterative and operates at the chip-level, the resultant technique is termed the chip-level iterated (CLi) MUD. Because of small number of interferers in each sub-carrier, applying maximum a posteriori based chip-level iterated (MAP-CLi) MUD is feasible for LDS-OFDM. The design of this receiver is based on MPA, which requires iterative exchange of messages between the function and the variable nodes.

The CLi MUD technique can be explained as follows. Let $L_{h_{n,-}}(x_{k,m})$ be the message sent along the edge $c_n(x_{k,m})$, at $j^{th}$ iteration, from variable node $u_{k,m}$ to function node $c_n$. Similarly, the message sent from the function node to the variable node is given by $L_{h_{n,\rightarrow}}(x_{k,m})$. Assuming there are no a priori probabilities available, the initial messages ($j = 0$) are set to zeros: $L_{h_{n,\rightarrow}}(x_{k,m}) = 0, \forall k,m, \forall n$. The messages...
are updated using the following rules
\[ L^j_{n,\rightarrow}(x_{k,m}) = \sum_{l \in E_{k,m} \setminus n} L^{j-1}_{l,\rightarrow}(x_{k,m}), \tag{3} \]

\[ L^j_{n,\rightarrow}(x_{k,m}) \propto f(x_{k,m},y_n, L^j_{n,\rightarrow}(x_{k',m'}), \forall (k',m') \in J_n \setminus (k,m)). \tag{4} \]

It can easily be seen from (3) and (4), that all messages are updated by the extrinsic information. In order to approximate the optimum MAP detector, the right hand side of (4) represents marginalization function, which is based on (2), and can be written as
\[ f(x_{k,m},y_n, L^j_{n,\rightarrow}(x_{k',m'}), \forall (k',m') \in J_n \setminus (k,m)) = \]
\[ \log \left( \sum_{x^{[n]} \in \mathbb{X}^{[n]}} p^j(y_n|x^{[n]}) P_n^{f}(x^{[n]} \setminus x_{k,m}) \right) \]
\[ \log \left( \prod_{x^{[n]} \in \mathbb{X}^{[n]}} p^j(y_n|x^{[n]}) P_n^{f}(x^{[k',m']}) \right), \tag{5} \]

Where the conditional probability density function (PDF) \( p^j(y_n|x^{[n]}) \) and a priori probability \( P_n^{f}(x^{[k',m']}) \) are given as
\[ p^j(y_n|x^{[n]}) \propto \exp \left( -\frac{1}{2\sigma^2} \|y_n - h_{n}^{T}x^{[n]}\|^2 \right), \tag{6} \]
\[ P_n^{f}(x^{[k',m']}) = \exp \left( L^j_{n,\rightarrow}(x_{k',m'}) \right), \tag{7} \]

where \( x^{[n]} \) and \( h_{n} \) denote the vector containing the symbols transmitted by every user that spread its data on chip \( n \) and their corresponding effective received signature values, respectively. As can be seen in (5) based on received chip \( y_n \) and a priori input information \( P_n^{f}(x^{[k',m']}) \), extrinsic values are calculated for all the constituent bits involved in (2). Combining (6) and (7) into (5), the message update will be
\[ L^j_{n,\rightarrow}(x_{k,m}) = \kappa_{n,k,m} \max_{x^{[n]} \in \mathbb{X}^{[n]}} \left\{ \sum_{(k',m') \in J_n \setminus (k,m)} \frac{1}{2\sigma^2} \|y_n - h_{n}^{T}x^{[n]}\|^2 \right\}, \tag{8} \]

where \( \kappa_{n,k,m} \) denotes the normalization coefficient and
\[ \max(a,b) \triangleq \log \left( e^a + e^b \right). \tag{9} \]

This technique is termed log-MAP CLI detection. After the message-passing has converged or has reached the maximum number of iterations \( J \), a posteriori probability of the transmitted symbol \( x_{k,m} \) is estimated as
\[ L_{k,m}(x_{k,m}) = \sum_{l \in E_{k,m}} L^j_{l,\rightarrow}(x_{k,m}). \tag{10} \]

By using the hard decision, the estimated value of \( x_{k,m} \) is
\[ \hat{x}_{k,m} = \arg \max_{x_{k,m}} L_{k,m}(x_{k,m}). \tag{11} \]
by the other detector. In this section, EXIT charts are used to characterize the operation of these two iterative detectors.

EXIT charts are applied to the MUD of LDS using the property of the factor graph of LDS receiver that both variable and function nodes exchange the same information. The information exchanged is the soft value that represents the reliability of the symbol related to each edge. In other words, the information on each edge is about the adjusted variable node, whether it is sent from the variable node itself or the connected function node. Therefore, knowing the transmitted signal we could find the mutual information between the actual value for the variable nodes and the a priori or extrinsic values on the edges. In this paper, $I_A$ refers to the average mutual information between the bits on the detector graph edges and the a priori L-values. Also, $I_E$ is the average mutual information between the bits on the graph edges and the extrinsic L-values. Therefore, in order to find the EXIT curve for each detector constituent we must represent $I_E$ as a function of $I_A$ for VND and FND separately.

1) EXIT Curve for the Variable Node Detector: In a regular LDS each variable node has $d_v$ incoming messages from the edge interleaver, which is equal to the variable node degree in the factor graph. The variable node produces its messages by calculating (3) for $d_v$ connected function nodes. In order to compute an EXIT curve for variable nodes, $L_{d_v}^{-1}(x_{k,m})$ in (3) is modelled as the soft output of an AWGN channel when the input is the $i^{th}$ interleaver bit transmitted using BPSK (binary phase shift keying). Then the mutual information between the variable node’s extrinsic messages and actual values of symbols on the edges is calculated. Thus, a priori L-value can be calculated by

$$ A = \mu_A x + n_A, \tag{12} $$

where $n_A$ is an independent Gaussian random variable with variance $\sigma_A^2$ and mean zero and $x \in \pm 1$ are original bits on the detector graph edges. Then we will have

$$ \mu_A = \frac{\sigma_A^2}{2}, \tag{13} $$

The mutual information $I_A = I(X; A)$ can be calculated by [11]

$$ I_A = \frac{1}{2} \sum_{x=-1,1} \int_{-\infty}^{\infty} p_A(\xi|X = x) \log_2 \left( \frac{2p_A(\xi|X = x)}{p_A(\xi|X = -1) + p_A(\xi|X = 1)} \right) d\xi. \tag{14} $$

Knowing that the conditional probability density function $p_A(\xi|X = x)$ depends on $L$-value $A$, with Gaussian distribution and with properties mentioned in (14), we can write

$$ I_A(\sigma_A) = 1 - \int_{-\infty}^{\infty} e^{-\left(\left(\xi - \frac{\sigma_A^2}{2}\right)^2 / 2\sigma_A^2\right)} \frac{1}{\sqrt{2\pi\sigma_A}} \log_2(1 + e^{-\xi}) d\xi. \tag{15} $$

For abbreviation we define

$$ J(\sigma) := I_A(\sigma_A = \sigma), \tag{16} $$

with

$$ \lim_{\sigma \to 0} J(\sigma) = 0, \quad \lim_{\sigma \to \infty} J(\sigma) = 1, \quad \sigma \geq 0. \tag{17} $$

Considering (3) together with the fact that the sum of two normally distributed random variables is also normally distributed with the mean and variance equal to the sum of theirs, the EXIT function of a degree $d_v$ variable node is

$$ I_{E,VND}(I_A, d_v) = J \left( \sqrt{(d_v - 1) \left(J^{-1}(I_A)\right)^2} \right). \tag{18} $$

Fig. 3 plots variable node EXIT curves for different $d_v$ using (18).

2) EXIT Curve for the Function Node Detector: A function node of degree $d_f$ has $d_f + 1$ incoming messages, $d_f$ from the edge interleaver and one message from the channel. The output L-values are calculated in (8) in details. We model a priori L-values, $L_{d_f}^{-1}(x_{k,m'})$, as the output of an AWGN channel that its input is the corresponding transmitted bit using BPSK and then calculate the mutual information of output with regards to the actual value on the edges. Due to the complexity of the calculation in function nodes, the EXIT curve for them is computed by simulations over AWGN channel. Therefore, first the distribution $p_E$ for extrinsic information is determined by Monte Carlo simulation (histogram measurements). Then using (14) the mutual information between the extrinsic information and the bits on the detector graph edges, is calculated.

Fig. 4 illustrates the EXIT charts for the MUD with 252 chips and different number of users (overloading conditions) which lead to different $d_v$s for a fixed $d_v = 2$. Fig. 4 shows that for a load of 400% the performance degrades noticeably compared to a load of 200% and 300%. As we can see in this figure, the iterative process starts with $I_{A,FND} = 0$ since no prior information is given to the function nodes in the beginning. In the next steps, the output L-values are exchanged between VND and FND curves. Fig. 4 also shows that the FND curves for different loading conditions intersect at $I_A = 1$, which is because for complete a priori information, the function nodes are able to eliminate the multiple access.
interference (MAI) fully and the effect of overloading will be
cancelled. Fig. 5 reveals the EXIT charts and the trajectories of
the system for $E_b/N_0$ equal to 5 dB and 8 dB. It shows that an
increase in signal to noise ratio (SNR) would shift the FND
curve vertically towards higher extrinsic output. This figure
also shows the trajectories of the iterative process obtained by
simulation. Except for a slight difference in the early iterations,
which occurs because of the finite size of edge interleaver in
the LDS structure, the system trajectories closely follow the
transfer curves of the MUD components, which indicates that
the EXIT charts analysis is valid for LDS-OFDM’s MUD.

B. EXIT Chart Analysis Aided Signature Design for LDS

There are different parameters in the design of the LDS ma-
trix that affect the performance of the system. Two important
parameters are the number of chips that each symbol is spread
on, $d_v$, and the number of symbols that share the same chip,
$d_c$. The relation between these two parameters for overloading
equal to $\beta$ is as follows:

$$d_c = \beta d_v.$$  \hspace{1cm} (19)

To design a robust LDS, the EXIT charts are drawn for
different $d_v$ s to evaluate the effect of processing gain on the
convergence behavior of MUD. Fig. 6 shows the EXIT charts
for a load of 200% and $E_b/N_0 = 8$ for an LDS design with
different processing gains. As it can be noted, using $d_v = 3$
 improves the intersection point compared to $d_v = 2$. This
is due to the larger frequency diversity offered at $d_v = 3$.
Also the performance at $d_v = 4$ is close to $d_v = 3$, therefore
considering that complexity increases exponentially with $d_v$
(for fixed overloading conditions), $d_v = 3$ can be regarded as
a suitable value in the design of LDS systems with a load of
200%. Thus, as an example, we have found that $d_v = 3$ is a
proper value for a load of 200%, this can be extended to other
scenarios to find suitable parameters for them.

IV. EXIT CHART ANALYSIS OF TURBO MULTI-USER
DETECTOR/DECODER

In this section we propose a new receiver for LDS-OFDM
based on iterative exchange of extrinsic information between

LDS multi-user detector and FEC decoders of different users.
There exist two iterative processes in the turbo MUDD: inner
processing for MUD and outer processing for turbo MUDD.
Having two iterative processes involved, the message update
algorithm must be modified accordingly. Defining $t$ as the
turbo iteration index for outer iteration, the message updates
in (3) and (4) can be adapted

$$L_{i\rightarrow k,m}^j(x_{k,m}) = \sum_{l\in\mathcal{E}_{k,m}} L_{l\rightarrow k,m}^{j-1,t}(x_{k,m}) + L_{dec,\text{out}}^{t-1}(x_{k,m}),$$ \hspace{1cm} (20)

$$L_{h\rightarrow k,m}^j(x_{k,m}) \propto f(x_{k,m} | y_n, L_{h\rightarrow k,m}^{j-1,t}(x_{k,m}), \gamma(k',m') \in \mathcal{J}_n \setminus (k,m)).$$ \hspace{1cm} (21)

Where $L_{dec,\text{out}}^{t-1}$ is the extrinsic information that an FEC
decoder sent to its corresponding variable node in the prior
turbo iteration. Therefore, considering the message-passing
algorithm the messages sent by variable nodes to the decoder
are as follows:

$$L_{i\rightarrow k,m}^{j,t} = \sum_{l\in\mathcal{E}_{k,m}} L_{l\rightarrow k,m}^{j-1,t}(x_{k,m}).$$ \hspace{1cm} (22)
BER can be calculated as follows:

\[ \text{BER} = \frac{1}{2} \text{erfc} \left( \frac{\mu_{E,MUD}^2}{\sigma_{E,MUD}^2} \right) = \frac{1}{2} \text{erfc} \left( \frac{\sigma_{I}^2}{2} \right). \]  

(24)

Application of the message-passing rule to the new graph, the message sent by the variable node to a function node must be based on all the messages it has received from the remainder of the connected function nodes in addition to the message that has been received from the corresponding FEC decoder. For function nodes the message update follows the same procedure as for the conventional LDS-OFDM MUD.

In this section, EXIT charts are used to investigate and reduce the complexity of the turbo MUD. To calculate the EXIT charts, both the MUD and FEC decoder are assumed to be a device that produce a new sequence of extrinsic information using a sequence of observations and the input \( a \) priori information. Therefore, in order to calculate the mutual information using (15), it is necessary to estimate the PDF from the histogram of the soft output of a constituent decoder. Furthermore, EXIT chart is a useful tool for predicting the BER performance [12]. The BER can be predicted by estimating the soft output of the coded bits by summing up the extrinsic information and the \( a \) priori information

\[ \Lambda = L_{MUD} + L_{DEC}, \]  

(23)

\[ \sigma_{\Lambda}^2 = \sigma_{E,MUD}^2 + \sigma_{L,DEC}^2, \]  

(24)

where (24) assumes \( a \) priori information and the extrinsic information are independent. Both variances can be calculated from their related mutual information using (17). Thus, the BER can be calculated as follows:

\[ p_b \approx \frac{1}{2} \text{erfc} \left( \frac{\mu_{E,MUD}^2}{\sigma_{E,MUD}^2} \right) = \frac{1}{2} \text{erfc} \left( \frac{\sigma_{I}^2}{2} \right). \]  

(25)

Fig. 7 illustrates the EXIT chart for turbo MUD with different number of inner iterations for a system load of 200% over AWGN channel. Here we implemented the system with 60 chips and a convolutional code with generator \( G = (133, 171) \) in octal notation (FEC decoder). Additionally, the BER scaling according to (25) is given as a contour plot.

Fig. 7 shows that the curves for different numbers of inner iterations intersect at the same point at input mutual information equal to one. This is due to full interference cancellation at FNDs when perfect \( a \) priori information is provided to them. In this case just one inner iteration will suffice to detect the data symbols. However, as it is observed from Fig. 7 larger number of turbo iterations will be required if only one inner iteration is used. In contrast, the cases with 3 and 6 inner iterations exhibits a marginally open tunnel between the EXIT curves for the FEC decoders and the MUD, hence for these cases turbo MUD can approach its final performance by less number of turbo iterations.

To efficiently utilize both inner and outer iterations we suggest using 6 inner iterations at first turbo iteration and then reducing the inner iterations to 1 from the second turbo iteration and onward. This will considerably reduce the overall complexity while maintaining the same performance. As the simulated trajectory in Fig. 7 shows, the simulated results follow the suggested iteration schedule very closely and the turbo MUDD converges by only 4 turbo iterations.

Monte Carlo based simulation results for LDS-OFDM’s BER with a load of 200% , \( N_c = 60 \) over AWGN channel is shown in Fig. 8. This result proves the prediction concluded using EXIT charts. As it is observed, the receiver brings about a significant performance improvement over conventional LDS-OFDM receivers. 2.3 dB gain at BER=10\(^{-5}\) is achieved by turbo MUD with 3 turbo iterations over conventional LDS-OFDM receiver. As expected from EXIT chart analysis, the performance of the turbo MUDD with (6,1,1) inner iterations is close to the one with (6,6,6) inner iterations. Consequently, the complexity was reduced while keeping the performance of the two systems close to each other. This result is important because in a real system, turbo iteration requires a huge processing overhead.

Finally, to show the validity of using previous analysis on a fading channel, the EXIT chart is calculated for the turbo LDS-OFDM under multi-path fading channel (ITU Pedestrian Channel B). Fig.9 shows the convergence behavior of LDS-OFDM with effective spreading size of 3 and 60 chips and 120 data symbols which belong to 10 users. The graphs for different SNRs show that an increase in the SNR only result in a vertical shift of MUD curve, which is a complete match.
to the behaviour of the system for AWGN channel shown in
Fig. 5. It is also shown that the simulated decoding trajectory
travel within the open tunnel between the EXIT curves for the
MUD and the FEC decoder. Therefore, the validity of EXIT
chart analysis for multi-path fading channels can be verified.
It is necessary to mention that EXIT chart analysis assumes
that the PDF of the exchanged messages approaches Gaussian-
like distributions with increasing number of iterations, thus, it
can be applied under multi-path fading channel as long as the
trajectory follows the curves of the receiver components.
The receiver that its parameters are selected according to
our proposed design guidelines can be called tailor-designed
LDS receiver. In next section, the performance of a tailor-
designed LDS-OFDM system is shown and compared with
well-known multiple access techniques with similar principles.

V. PERFORMANCE COMPARISON WITH MC-C DMA AND
GO-MC-C DMA
In this section, the performance of tailor-designed LDS-OFDM is evaluated and compared with MC-C DMA and GO-
MC-C DMA in terms of BER performance and computational
complexity. Considering the prohibitive complexity of the
optimum MUD, LDS-OFDM is compared with a MC-C DMA
system that has linear minimum mean-square error (MMSE)
detector. The linear MMSE detector is the optimal linear
detector that maximizes the output signal-to-interference and
noise ratio (SINR) [13]. On the other hand, it has been shown
that the performance of GO-MC-C DMA is very close to the
single-user bound [6], thus it is necessary to show that the
proposed receiver design for LDS-OFDM is able to keep the
performance the same while reducing the complexity.
The performances are evaluated using Monte Carlo based
simulations over multi-path fading channel. The single-user
BER bound with the same channel profile is also considered
in the comparison. Throughout the simulations, the overall
number of information bits transmitted by all the systems is
kept equal to ensure a fair comparison between the systems.
The simulation parameters are listed in Table I. The number of
sub-carriers used is the same as in [6] to keep the complexity
of the optimum MUD feasible. For GO-MC-C DMA, the
number of sub-carriers per group is set to 4 and maximum
likelihood (ML) detection is employed per group. For MC-
C DMA the processing gain is equal to the number of data
sub-channels. For MC-C DMA and GO-MC-C DMA systems
orthogonal codes for a load of 100% and Welch bound equality
(WBE) for a load of 200% are used. The spreading codes
are constructed using the algorithm developed in [14]. A
regular graph structure is maintained for LDS-OFDM while its
signatures are generated randomly. Using the design guidelines
achieved in section III, the effective spreading factor for LDS-
OFDM is equal to 3. To keep the complexity of LDS-OFDM’s
receiver at a similar level with the receiver for MC-C DMA
and GO-MC-C DMA, its receiver is not turbo MUD (the
performance of turbo MUD is shown in section IV).

Fig. 10 shows the BER results for systems with loads of
100% and 200%. As it can be observed from the figure, LDS-
OFDM system achieves performance close to the GO-MC-
C DMA system, which employs optimum MUD. However, the
complexity of the LDS detector is less than the complexity
of GO-MC-C DMA. As in LDS-OFDM the complexity increases

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
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<tbody>
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<td>Number of Users</td>
<td>10</td>
</tr>
<tr>
<td>Number of data sub-channels</td>
<td>60</td>
</tr>
<tr>
<td>FFT size</td>
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<tr>
<td>Sub-channel bandwidth</td>
<td>15KHz</td>
</tr>
<tr>
<td>LDS-OFDM MUD technique</td>
<td>Max-Log-MAP</td>
</tr>
<tr>
<td>Channel coding</td>
<td>Half-rate convolutional code</td>
</tr>
<tr>
<td>Modulation</td>
<td>BPSK</td>
</tr>
<tr>
<td>Data streams per user</td>
<td>100% Loading 6</td>
</tr>
<tr>
<td></td>
<td>200% Loading 12</td>
</tr>
<tr>
<td>Effective spreading factor (LDS)</td>
<td>( d_e = 3 )</td>
</tr>
<tr>
<td>Symbols per sub-carrier (LDS)</td>
<td>100% Loading 3</td>
</tr>
<tr>
<td></td>
<td>200% Loading 6</td>
</tr>
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</table>

Fig. 9. EXIT chart for turbo MUDD for 200% load under multi-path fading channel.

Fig. 10. Performance results for LDS-OFDM, MC-C DMA and GO-MC-C DMA.
Therefore, at each received chip a user’s symbol will have only a complexity order of $O(|X|^K)$ and $O(|X|^K)$ is required for a load of 100% and 200%, respectively. However, in GO-MC-CDMA the complexity increases exponentially by the number of transmitted symbols in each group, which results in complexity order of $O(|X|^K)$ and $O(|X|^K)$ for loads of 100% and 200%, respectively. Furthermore, the results show that, LDS-OFDM outperforms the performance of MC-CDMA with MMSE detector. This is because the LDS detector is more efficient than the MMSE detector in eliminating the MUI. For a load of 100%, LDS-OFDM outperforms MC-CDMA by $8.7 \text{ dB}$ at BER $= 10^{-3}$. The MMSE detector fails to attain a satisfactory BER performance under the overloaded conditions. Considering all the results, it can be concluded that the tailor-designed LDS-OFDM system can achieve performance close to the GO-MC-CDMA system with less complexity.

Moreover, the LDS-OFDM system outperforms the system that employs MC-CDMA with linear MMSE detector. At the end, to highlight the importance of LDS-OFDM receiver we will compare this newly proposed technique with the existing multiple access techniques in next section.

### VI. LDS-OFDM in Comparison with Existing Multiple Access Techniques

In this section, the properties of LDS-OFDM are discussed and the scheme is compared with different MAC techniques.

#### A. Frequency Diversity

For OFDMA systems, it is not possible to exploit the frequency domain diversity at modulation symbol level because the symbols of users are assigned directly to sub-channels in such systems. Therefore, in order to exploit the frequency diversity for OFDMA system at a later stage, it is crucial to incorporate properly the designed error correction coding and interleaving schemes [15].

For LDS-OFDM, the performance of the system was improved by introducing a larger degree of frequency diversity proportional to $d_c$, the effective spreading factor. Furthermore, considering that the detection of user symbols in LDS structure is done completely in the sub-carrier level, frequency diversity can be achieved by assigning distributed and spaced-sufficient sub-carriers for spreading of a given data symbol. Also, the created LDS structure will produce diversity on the interference experienced by each data symbol. Although LDS-OFDM better exploits frequency diversity, as compared to OFDMA, its diversity gain is less than the one for MC-CDMA and GO-MC-CDMA, as they offer full frequency diversity [6], [16].

#### B. Computational Complexity of MUD

Applying the close-to-optimum MUD based on MPA became feasible for LDS structure by reducing the number of interferers in each chip. As mentioned earlier, $d_c$ is the number of symbols that interfere with each other on each chip. Therefore, at each received chip a user’s symbol will have only $d_c - 1$ interferers, where $d_c \ll K$.

The complexity of the proposed receiver for LDS-OFDM will be in order of $O(|X|^{d_c})$ which is a considerable reduction compared to $O(|X|^K)$ for optimal MUD. The complexity of LDS MUD will be

\[
\text{Complexity} = |X|^{d_c} \times \text{(No. of MUD iterations)} \times N_c. \quad (26)
\]

Consequently, although the complexity is increased compared to OFDMA, its growth is much less than the optimum MUD used in MC-CDMA. Comparing the factor graph of GO-MC-CDMA and LDS-OFDM we can see that in contrast to the LDS structure for GO-MC-CDMA in each group there is full connection between the function nodes and variable nodes and there is no connection between the groups. Therefore, applying message-passing algorithm is inefficient due to the full connectivity of the graph for GO-MC-CDMA. For GO-MC-CDMA the complexity of the system depends on the size of orthogonal groups because the complexity for maximum likelihood detection in GO-MC-CDMA increases exponentially with the number of active users per group [6]. As mentioned earlier the complexity of LDS-OFDM is less than the complexity of GO-MC-CDMA.

#### C. Overloaded Conditions

In wireless systems the number of users naturally exceeds the available dimensions as the demand for the spectrum is increasing while the bandwidth is limited. Conventional signature design for multiple access systems such as MC-CDMA and GO-MC-CDMA is based on orthogonality between the sequences to avoid interference. Therefore, these systems can perform reliably when the number of users is equal to the number of chips [17]. However, overloaded conditions can be supported by LDS-OFDM as the design of this system is not based on orthogonality. The LDS system is able to approach near single-user performance for a CDMA system with up to 200% loading [18].

#### D. Near-Far Effect

The near-far problem (i.e., the effect of unequal received energies) is the principal shortcoming of direct sequence CDMA systems. On the other hand, by jointly detecting all users’ symbols, optimum multi-user detection for CDMA systems is near-far resistant [19]. Considering that in the MUD of LDS-OFDM an iterative joint process has been employed for detecting different users’ symbols and the fact that the performance of LDS MUD is close to optimum MUD, we can conclude that LDS-OFDM is robust against unequal received powers. Fig. 11 shows the performance of near-far resistance for LDS-OFDM MUD with different numbers of chips. The simulation is done for the case where $E_b/N_0$ is equal to $15 \text{ dB}$ for the first user, and $E_b/N_0$ of other users is different. The BER performance of the first user is shown according to $\Delta E_b/N_0$, which represents the difference in $E_b/N_0$ of user of interest from the $E_b/N_0$ of other users (all other users have equal $E_b/N_0$). The results are shown for an LDS-OFDM system with different numbers of chips and loads. It can be seen that unequal received power has a minor effect on the performance of user of interest for all the scenarios. Furthermore, the performance of the LDS-CDMA system against the near-far effect problem has been analysed in [18].
These results match the results shown for LDS-OFDM, since both techniques are based on low density signatures. In order to achieve an even better near-far resistance, the system can resort to power control mechanisms. Several algorithms are introduced in the literature for the optimum power control of MC-CDMA systems [20]–[22], which can be applied to LDS-OFDM if necessary.

VII. CONCLUSIONS

In this paper, a close-to-optimum MUD for LDS-OFDM systems was introduced and analysed. The LDS-OFDM receiver was evaluated using EXIT charts. By analyzing the inner iterations for MUD under AWGN channel, it was shown that the signature design with effective processing gain equal to 3 was a suitable value for a load of 200%, considering the complexity and performance. This could be extended to other scenarios in order to find suitable values for other conditions. In addition, we showed how the performance is affected by loading as the curves intersected in lower mutual information points for higher loading values. Furthermore, the proposed turbo MUD was investigated and by deriving and analysing its EXIT charts we could reduce the complexity by properly tuning the number of inner iterations along the outer turbo iterations. It was shown that 2.3 dB performance improvement at BER equal to $10^{-5}$ could be obtained when turbo MUD was employed over an AWGN channel.

Applying the achieved guidelines, the performance of LDS-OFDM was evaluated over a typical multi-path fading channel under different spectral efficiency conditions. The simulation results showed a noticeable performance improvement compared to MC-CDMA systems with MMSE MUD. This large performance gain could be achieved at the cost of slightly increased computational complexity. This improvement is mainly due to capability of LDS-OFDM to exploit frequency domain diversity in addition to avoiding a strong interference to corrupt all the sub-carriers. In terms of complexity it was shown that the performance of LDS-OFDM was close to GO-MC-CDMA while reducing the computational complexity of the MUD. Interference handling property of LDS-OFDM makes it a suitable candidate for use in heterogeneous wireless networks. However, their use in these specific scenarios needs further detailed analysis.

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