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Fiscal consolidation in an open economy with sovereign premia and without monetary policy independence*

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Abstract

We welfare rank various tax-spending-debt policies in a New Keynesian model of a small open economy featuring sovereign interest-rate premia and loss of monetary policy independence. When we compute optimized state-contingent policy rules, our results are: (a) Debt consolidation comes at a short-term pain but the medium- and long-term gains can be substantial. (b) In the early phase of pain, the best fiscal policy mix is to cut public consumption spending to address the debt problem, and, at the same time, to cut income tax rates to mitigate the recessionary effects of debt consolidation. (c) In the long run, the best way of using the fiscal space created is to reduce capital taxes.

Keywords: Feedback policy, New Keynesian, Sovereign premia, Debt consolidation.

JEL classification: E6, F3, H6

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1 Introduction

Since the global crisis in 2008, and after years of deficits and rising debt levels, public finances have been at the center of attention in most eurozone periphery countries. Although several policy proposals are under discussion, a particularly debated one is public debt consolidation. Proponents claim this is for good reason: as a result of high and rising public debt, borrowing costs have increased, causing crowding out problems and undermining government solvency. Opponents, on the other hand, claim that debt consolidation worsens the economic downturn and leads to a vicious cycle at least in the short term. At the same time, as members of the single currency, these countries cannot use an independent monetary policy.

What is the best use of fiscal policy under these circumstances? Is debt consolidation beneficial? Should the debt ratio be stabilized at its currently historically high level or should it be brought down? If brought down, how quickly? Do the answers to these questions depend on which tax-spending policy instruments are used over time?

This paper welfare ranks various fiscal policies in light of the above. The setup is a rather conventional New Keynesian model of a small open economy, where the interest rate, at which the country borrows from the world capital market, increases with the public debt-to-GDP ratio. We focus on a monetary policy regime in which the small open economy fixes the exchange rate and loses monetary policy independence; this mimics membership in a currency union. Hence, the key national macroeconomic tool left is fiscal policy.

Then, following a rule-like approach to policy, we assume that fiscal policy is conducted via simple and implementable feedback policy rules. In particular, we assume that public spending and the tax rates on consumption, capital and labor are all allowed to respond to the inherited public debt-to-GDP ratio, as well as to contemporaneous output, as deviations from policy targets. We experiment with various policy target values depending on whether policymakers

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1 We will use the terms debt consolidation, fiscal adjustment and fiscal austerity interchangeably. For a discussion of the tradeoffs faced by policymakers in the case of fiscal adjustment, see e.g. the EEAG Report on the European Economy (2014).

2 For empirical support of this assumption, see e.g. European Commission (2012). For the small open economy model and various deviations from it, see Schmitt-Grohé and Uribe (2003). Further details and extensions are below.

3 For empirical support of such simple rules, see e.g. European Commission (2011). There is a rich literature on monetary and fiscal feedback policy rules that includes e.g. Schmitt-Grohé and Uribe (2005 and 2007), Pappa and Vassilatos (2007), Kirsanova et al. (2007), Leith and Wren-Lewis (2008), Batini et al. (2008), Kirsanova et al. (2009), Leeper et al. (2009), Bi (2010), Bi and Kumhof (2011), Kirsanova and Wren-Lewis (2012), Cantore
aim just to stabilize the economy around its status quo, or whether they also want to move the economy to a new reformed steady state. The status quo is naturally defined as the solution consistent with the euro period data. The new reformed steady state, on the other hand, is defined as the case in which the fiscal authorities adjust their policies as much as needed so as to end up with lower debt and zero sovereign interest-rate premia; we also consider the case in which the new reformed steady state is the associated Ramsey steady state. In addition, since we do not want our results to be driven by ad hoc differences in feedback policy coefficients across different policy rules, we focus on optimized ones. In other words, we compute simple and implementable policy rules that also maximize households’ welfare. In particular, adopting the methodology of Schmitt-Grohé and Uribe (2004a, 2005 and 2007), we compute welfare-maximizing rules by taking a second-order approximation to both the equilibrium conditions and the welfare criterion around the new reformed steady state(s).

The model is solved numerically using common parameter values and fiscal-public finance data from the Italian economy during 2001-2013. We choose Italy simply because it exhibits most of the features discussed in the opening paragraph above and, at the same time, it continues to participate in the world capital market without receiving foreign aid like other eurozone periphery countries. It thus looks as a natural choice to quantify our model.

Before presenting our results, it is worth pointing out that there is no such a thing like "the" debt consolidation: the implications of debt consolidation depend heavily on which policy instrument bears the cost in the early phase of austerity and on which policy instrument is anticipated to reap the benefit in the late phase, once the debt burden has been reduced and fiscal space has been created.\(^4\) The costs in the early phase are due to spending cuts and/or tax increases, while the opposite holds once fiscal space has been created. Our results (see below) confirm all this. Hence, the choice of fiscal policy instruments matters for lifetime utility and output. This choice also matters for how quickly public debt should be brought down: the more distorting are the fiscal policy instruments used during the early costly phase, the slower the speed of fiscal adjustment should be. Naturally, there is more choice when we allow for

\(^4\)In other words, the debate about the benefits and costs of each instrument used for debt consolidation is essentially a debate about the size of the multiplier of each instrument (see the discussion in the EEAG Report on the European Economy, 2014). See also e.g. Coenen et al. (2008), Leeper et al. (2009) and Davig and Leeper (2011) on how the impact of current policy depends on expectations of possible future policy regimes.
policy mixes (for instance, when the policy instrument(s) used in the early costly phase can be different from those used in the late phase of fiscal space) than when we are restricted to use a single instrument all the time.

Our main results are as follows. First, in most cases, debt consolidation is beneficial only if we are relatively far-sighted. For instance, in our baseline computations, debt consolidation is welfare-improving only after the first ten years. In other words, debt consolidation comes at a short-term loss (this loss is bigger if one uses one fiscal instrument only, instead of a fiscal policy mix). Nevertheless, once the short-term pain is over, the gains from debt consolidation get substantial over time. All this means that the argument for, or against, debt consolidation involves a value judgment. On the other hand, we find that debt consolidation is welfare-improving all the time, even in the short term, when we travel to the Ramsey steady state; but, in that steady state, the (optimal) values of the tax rates are far away from their values in the actual data.

Second, under debt consolidation, a general result is that the fiscal authorities should use all available tax-spending instruments during the early costly phase of fiscal austerity and reduce capital tax rates - which are particularly distorting - during the late phase of fiscal space. Actually, the anticipation of a reduction in capital taxes plays a key role in the recovery from fiscal austerity. During the early costly phase, the assignment of instruments to intermediate targets (or economic indicators) should be as follows: cut public consumption spending to address the public debt problem, while, at the same time, reduce income (capital and labor) tax rates in order to mitigate the recessionary effects of austerity. Sometimes, consumption tax rates should be also used, if changes in other taxes are restricted. The bottom line is that the choice of the fiscal policy mix is important (as also argued by Wren-Lewis, 2010) and that the short-term cost becomes too big if the fiscal authorities pay attention to debt imbalances only.

Third, when we solve the model in the fictional case in which Italy would have followed an independent monetary policy (meaning that now there is also a feedback Taylor-type rule for the nominal interest rate), the main results do not change. Also, to the extent that the feedback policy coefficients, both in fiscal and monetary policy rules, are selected optimally, the welfare gain from switching to flexible exchange rates appears to be negligible, at least in
this class of New Keynesian models.

What is the value added of our paper? Papers on fiscal consolidation in an open economy, which are close to ours, include Coenen et al. (2008), Forni et al. (2010a, 2010b), Cogan et al. (2013), Erceg and Lindé (2013), Almeida et al. (2013) and Roeger and in’ t Veld (2013). But these papers a priori set the fiscal instruments through which debt consolidation is implemented or the speed/pace of this adjustment. Our work differs mainly because: (i) Following an optimized feedback rule-like approach to policy, we search for the best mix of fiscal action in an open economy facing sovereign interest-rate premia and loss of monetary policy independence. In doing this, we put special emphasis on which instruments should bear the cost of consolidation in the early phase and which instruments should reap the benefits in the later phase. (iii) We study transition results depending on whether the government simply stabilizes the economy from exogenous shocks, or it also leads the economy to a new reformed steady state with lower debt. (iii) We study what would have happened with monetary policy independence other things equal.

The rest of the paper is organized as follows. Section 2 presents the model. Section 3 presents the data, parameterization and the status quo solution. Section 4 discusses how we work. The main results are in Section 5. A rich sensitivity analysis is in Section 6. Section 7 studies the case with independent monetary policy. Section 8 closes the paper. Algebraic details and additional robustness results are in an online Appendix.

2 Model

Consider a small open economy where the interest-rate premium is debt-elastic (see e.g. Schmitt-Grohé and Uribe, 2003). On other dimensions, our setup is the standard New Keynesian model of an open economy with domestic and imported goods featuring imperfect competition and Calvo-type price rigidities (see e.g. Galí and Monacelli, 2005 and 2008, and Benigno and Thoenissen, 2008).

The economy is composed of \( N \) identical households indexed by \( i = 1, 2, ..., N \), of \( N \) firms

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5Papers on debt consolidation in a closed economy include Cantore et al. (2012), Bi et al. (2013), Pappa et al. (2014) and Philippopoulos et al. (2015). Econometric studies on the effects of debt consolidation include e.g. Perotti (1996), Alesina et al. (2012) and Batini et al. (2012).

indexed by $h = 1, 2, ..., N$, each one of them producing a differentiated domestically produced tradable good, as well as of monetary and fiscal authorities. Similarly, there are $f = 1, 2, ..., N$ differentiated imported goods produced abroad. Domestic firms and owned by domestic households and any profits are equally divided to these households. Population, $N$, is constant over time.

### 2.1 Aggregation and prices

#### 2.1.1 Consumption bundles

The quantity of variety $h$ produced by domestic firm $h$ and consumed by domestic household $i$ is denoted as $c^H_{i,t}(h)$. Using a Dixit-Stiglitz aggregator, the composite domestic good consumed by household $i$, $c^H_{i,t}$, consists of $h$ varieties and is given by the function:

$$
\left( \frac{\phi}{\phi - 1} \right) \left( \sum_{h=1}^{N} [c^H_{i,t}(h)] \right)^{\frac{\phi - 1}{\phi}}
$$

where $\phi > 0$ is the elasticity of substitution across goods.

Similarly, the quantity of imported variety $f$ produced abroad by foreign firm $f$ and consumed by domestic household $i$ is denoted as $c^F_{i,t}(f)$. Using a Dixit-Stiglitz aggregator, the composite imported good consumed by household $i$, $c^F_{i,t}$, consists of $f$ varieties and is given by the function:

$$
\left( \frac{\phi}{\phi - 1} \right) \left( \sum_{f=1}^{N} [c^F_{i,t}(f)] \right)^{\frac{\phi - 1}{\phi}}
$$

In turn, household $i$'s consumption bundle, $c_{i,t}$, is defined as:

$$
c_{i,t} = \nu^{\nu} \left( c^H_{i,t} \right)^{1-\nu} \left( c^F_{i,t} \right)^{1-\nu}
$$

where $\nu$ is the degree of preference for domestic goods (if $\nu > 1/2$, there is a home bias).

#### 2.1.2 Consumption expenditure, prices and terms of trade

Household $i$’s total consumption expenditure is:

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As in e.g. Blanchard and Giavazzi (2003), we work with summations rather than with integrals. This does not affect the results.
\[ P_t c_{i,t} = P_t^H c_{i,t}^H + P_t^F c_{i,t}^F \]  \hspace{1cm} (4)

where \( P_t \) is the consumer price index (CPI), \( P_t^H \) is the price index of home tradables, and \( P_t^F \) is the price index of foreign tradables (expressed in domestic currency).

In turn, \( i \)'s expenditures on home and foreign goods are respectively:

\[ P_t^H c_{i,t}^H = \sum_{h=1}^{N} P_t^H(h) c_{i,t}^H(h) \]  \hspace{1cm} (5)

\[ P_t^F c_{i,t}^F = \sum_{f=1}^{N} P_t^F(f) c_{i,t}^F(f) \]  \hspace{1cm} (6)

where \( P_t^H(h) \) is the price of variety \( h \) produced at home and \( P_t^F(f) \) is the price of variety \( f \) produced abroad, both denominated in domestic currency.

We assume that the law of one price holds meaning that each tradable good sells at the same price at home and abroad. Thus, \( P_t^F(f) = S_t P_t^H*(f) \), where \( S_t \) is the nominal exchange rate (where an increase in \( S_t \) implies a depreciation) and \( P_t^H*(f) \) is the price of variety \( f \) produced abroad denominated in foreign currency. A star denotes the counterpart of a variable or a parameter in the rest-of-the-world. Note that the terms of trade are defined as \( \frac{P_t^F}{P_t^H} = \frac{S_t P_t^H *}{P_t^H} \), while the real exchange rate is defined as \( \frac{S_t}{P_t} \).

### 2.2 Households

Each household \( i \) acts competitively to maximize expected discounted lifetime utility:

\[ E_0 \sum_{t=0}^{\infty} \beta^t U(c_{i,t}, n_{i,t}, g_t) \]  \hspace{1cm} (7)

where \( c_{i,t} \) is \( i \)'s consumption bundle as defined above, \( n_{i,t} \) is \( i \)'s hours of work, \( g_t \) is per capita public spending, \( 0 < \beta < 1 \) is the time preference rate, and \( E_0 \) is the rational expectations operator.

The period utility function is assumed to be of the form\(^8\)

\(^8\)See also Galí (2008), Galí and Monacelli (2008) and many others in this literature.
\[ u_{i,t}(c_{i,t}, n_{i,t}, g_t) = \frac{c_{i,t}^{1-\sigma}}{1-\sigma} - \chi_n \frac{n_{i,t}^{1+\eta}}{1+\eta} + \chi_g \frac{g_t^{1-\zeta}}{1-\zeta} \]  

(8)

where \( \chi_n, \chi_g, \sigma, \eta, \zeta \) are standard preference parameters. That is, \( 1/\sigma \) is the elasticity of intertemporal substitution and \( \eta \) is the inverse of Frisch labour elasticity.

The period budget constraint of each household \( i \) written in real terms is (notice that, for simplicity, we assume a cashless economy; we report that our results do not depend on this):

\[
\begin{align*}
(1 + \tau^k_t) \left[ \frac{P_t^H}{P_t} c^H_{i,t} + \frac{P_t^F}{P_t} c^F_{i,t} \right] + \frac{P_t^H}{P_t} x_{i,t} + b_{i,t} + \frac{S_t P_t^s}{P_t} f_{i,t}^h &+ \frac{\phi^h}{2} \left( \frac{S_t P_t^s}{P_t} f_{i,t}^h - SP_t^s \right)^2 = \\
(1 - \tau^k_t) \left[ \tau^k_t \frac{P_t^H}{P_t} k_{i,t-1} + \bar{\omega}_{i,t} \right] + (1 - \tau^n_t) w_t n_{i,t} + R_{t-1} \frac{P_{t-1}}{P_t} b_{i,t-1} + \\
+ Q_{t-1} \frac{S_t P_t^s}{P_t} \frac{P_{t-1}^s}{P_t^s} f_{i,t-1}^h - \tau^l_{i,t}
\end{align*}
\]

(9)

where \( x_{i,t} \) is \( i \)'s domestic investment, \( b_{i,t} \) is the real value of \( i \)'s end-of-period domestic government bonds, \( f_{i,t}^h \) is the real value of \( i \)'s end-of-period internationally traded assets denominated in foreign currency (if negative, it denotes foreign private debt), \( \tau^k_t \) denotes the real return to the beginning-of-period domestic capital, \( k_{i,t-1}, \bar{\omega}_{i,t} \) is \( i \)'s real dividends received by domestic firms, \( w_t \) is the real wage rate, \( R_{t-1} \geq 1 \) denotes the gross nominal return to domestic government bonds between \( t - 1 \) and \( t \), \( Q_{t-1} \geq 1 \) denotes the gross nominal return to international assets between \( t - 1 \) and \( t \), \( \tau^l_{i,t} \) denotes real lump-sum taxes to each household (if negative, it denotes transfers), and \( 0 \leq \tau^c_r, \tau^k_r, \tau^n_r \leq 1 \) are tax rates on consumption, capital income and labour income respectively. Letters without time subscripts denote steady state values. The parameter \( \phi^h \geq 0 \) measures adjustment costs related to private foreign assets as a deviation from their steady state value, \( f_{i,t}^h \); these adjustment costs help us to avoid excess volatility and get plausible (in line with the data) short-term dynamics for private foreign assets following a policy reform; further details are in subsection 3.1 below.

The law of motion of physical capital for each household \( i \) is:

\[
k_{i,t} = (1 - \delta) k_{i,t-1} + x_{i,t} - \frac{\xi}{2} \left( \frac{k_{i,t}}{k_{i,t-1}} - 1 \right)^2 k_{i,t-1}
\]

(10)
where $0 < \delta < 1$ is the depreciation rate of capital and $\xi \geq 0$ is a parameter capturing adjustment costs related to physical capital.

Details on the household’s problem and the first-order conditions are in Appendix A.

### 2.3 Firms

Each firm $h$ produces a differentiated good of variety $h$ enjoying market power on its own good and facing Calvo-type price fixities.

The profit of each firm $h$ in real terms is (see also e.g. Benigno and Thoenissen, 2008):

$$\widehat{\omega}_t(h) = \frac{P^H_t(h)}{P_t}y_t^H(h) - \frac{P^H_t}{P_t}k_{t-1}(h) - w_t n_t(h)$$ (11)

All firms use the same technology as represented by the production function:

$$y_t^H(h) = A_t[k_{t-1}(h)]^\alpha [n_t(h)]^{1-\alpha}$$ (12)

where $A_t$ is an exogenous stochastic TFP process whose motion is defined below and $0 < \alpha < 1$.

Profit maximization by firm $h$ is subject to the demand for its product (see Appendix B):

$$y_t^H(h) = C_t^H(h) + X_t(h) + G_t(h) + C_t^{F*}(h) = \left[ \frac{P^H_t(h)}{P^H_t} \right]^{-\phi} Y_t^H$$ (13)

so that, demand for firm $h$’s product, $y_t^H(h)$, comes from domestic households’ consumption and investment, $C_t^H(h)$ and $X(h)$ respectively, where $C_t^H(h) = \sum_{i=1}^N c^H_{i,t}(h)$ and $X_t(h) = \sum_{i=1}^N x_{i,t}(h)$, from the domestic government, denoted as $G_t(h)$, and from foreign households’ consumption, $C_t^{F*}(h) = \sum_{i=1}^{N^*} c^{F*}_{i,t}(h)$, and where $Y_t^H$ denotes aggregate demand.

In addition, firms are subject to a Calvo-type pricing mechanism. In particular, in each period, each firm $h$ faces an exogenous probability $\theta$ of not being able to reset its price. A firm $h$, which is able to reset its price at time $t$, chooses its price $P^h_t(h)$ to maximize the sum of discounted expected nominal profits for the next $k$ periods in which it may have to keep its price fixed. This objective is:

$$E_t \sum_{k=0}^{\infty} \theta^k \Xi_{t,t+k} \left\{ P^h_t(h) y_{t+k}^H(h) - \Psi_{t+k} (y_{t+k}^H(h)) \right\}$$
where $\Xi_{t, t+k}$ is a discount factor taken as given by the firm (but it equals the household’s intertemporal marginal rate of substitution in consumption, in equilibrium), $y^H_{t+k}(h) = \left[ \frac{p^\#(h)}{p^H_{t+k}} \right]^{-\phi} y^H_{t+k}$ as said above, and $\Psi_t(h)$ is the minimum nominal cost function for producing $y^H_t(h)$ at $t$ so that $\Psi_t(h)$ is the associated nominal marginal cost.

Details on the firm’s problem and the first-order conditions are in Appendix B.

### 2.4 Government budget constraint

The period budget constraint of the government in real and per capita terms is (details are in Appendix C):

$$d_t = R_{t-1} \frac{P_{t-1}}{P_t} \lambda_{t-1} d_{t-1} + Q_{t-1} \frac{S_t P^*_t}{P_t} \frac{P^*_t}{P_{t-1}} \frac{P_{t-1}}{P_t} \frac{P_{t-1}}{P_t} (1 - \lambda_{t-1}) d_{t-1} + \frac{P^H_t}{P_t} g_t - \tau_t^o \left( \frac{P^H_t}{P_t} c^H_t + \frac{P^F_t}{P_t} c^F_t \right) - \tau_t^k (r^k - \frac{P^H_t}{P_t} k_{t-1} + \bar{\omega}_t) - \tau_t^n w_t n_t - \tau_t^l + \frac{\phi^g}{2} (1 - \lambda_t) d_t - (1 - \lambda) d_t^2$$

where $d_t$ is the real and per capita value of end-of-period total public debt. Thus, total nominal public debt, $D_t$, can be held by domestic private agents, $\lambda_t D_t$, as well as by foreign private agents, $(1 - \lambda_t) D_t$, where the fraction $0 \leq \lambda_t \leq 1$ is exogenously given.\(^9\) The parameter $\phi^g \geq 0$ measures adjustment costs related to public foreign debt and are similar to those of the household in equation (9) above (further details are in subsection 3.1 below).

In each period, one of $(\tau_t^o, \tau_t^k, \tau_t^n, g_t, \tau_t^l, \lambda_t, d_t)$ needs to adjust to satisfy the government budget constraint (see subsection 2.7 below).

### 2.5 Closing the model: debt-elastic interest rate premium

As is well known, to avoid nonstationarity and convergence to a well-defined steady state, we have to depart from the benchmark small open economy model (see Schmitt-Grohé and Uribe, 2003, for alternative ways). Here, we do so by endogenizing the interest rate faced by the domestic country when it borrows from the world capital market, $Q_t$.\(^10\) In particular, we start

\(^9\)Public debt differs from foreign debt. The end-of-period public debt, written in total nominal terms, is $D_t = B_t + S_t F^F_t$, where $B_t = \lambda_t D_t = \sum_{i=1}^N B_{i, t}$, is domestic government bonds held by domestic agents and $S_t F^F_t = (1 - \lambda_t) D_t$ denotes domestic government bonds held by foreign investors. On the other hand, the country’s end-of-period net foreign debt, written in total nominal terms, is $S_t (F^F_t - F^F_t) = (1 - \lambda_t) D_t - S_t F^F_t$, where $F^F_t = \sum_{i=1}^N F^F_{i, t}$ is nominal foreign assets held by domestic agents (if negative, it denotes liabilities). As said, further details are in Appendix C. Note that, by focusing on a single open economy, we do not model the behavior of foreign investors, so that we treat $0 \leq \lambda_t \leq 1$ as exogenous.

\(^10\)See also Christiano et al. (2011), García-Cicco et al. (2010) and many others for endogenous country premia. Note that, although endogeneity of the country premium is also used by the literature on sovereign
by assuming that the country premium between \( t \) and \( t + 1 \), namely \( Q_t - Q_t^* \), is an increasing function of the end-of-period total nominal public debt as share of nominal GDP, \( \frac{D_t}{P_t Y_t} \), when this share exceeds a certain threshold. In the robustness section below, we will also study the case where the country premium is increasing in the country’s net foreign liabilities or debt\(^{11}\).

In particular, following e.g. Schmitt-Grohé and Uribe (2003) and García-Cicco et al. (2010), we use:

\[
Q_t = Q_t^* + \psi \left( e^{\left( \frac{D_t}{P_t Y_t} - \bar{d} \right)} - 1 \right)
\]

where the world interest rate, \( Q_t^* \), is exogenously given, \( \bar{d} \) is an exogenous threshold value above which the interest rate on government debt starts rising above \( Q_t^* \), and the parameter \( \psi \) measures the elasticity of the interest rate with respect to deviations of total public debt from its threshold value (see subsection 3.1 below for these parameter values)\(^{12}\).

### 2.6 Exchange rate and fiscal policy regimes

To solve the model, we need to specify the exchange rate and the fiscal policy regimes. Concerning exchange rate policy, since the model is applied to Italy over the last decade, we solve it for a case without monetary policy independence. In particular, we assume that the nominal exchange rate, \( S_t \), is exogenously set and, at the same time, the domestic nominal interest rate on domestic government bonds, \( R_t \), becomes an endogenous variable\(^{13}\). Concerning fiscal policy, we assume that, along the transition, the residually determined public financing policy instrument is the end-of-period total public debt, \( D_t \) (see below for other public financing cases at the steady state).

Before we turn to fiscal policy rules in the next subsection, it is worth clarifying that, along default, the study of the latter is beyond the scope of our paper. As Corsetti et al. (2013) point out, there are two approaches to sovereign default. The first models it as a strategic choice of the government (Eaton and Gersovitz, 1981, Arellano, 2008, and many others). The second assumes that default occurs when debt exceeds its endogenous limit (Bi, 2012, and many others).

\(^{11}\) As said above, this rather common assumption (namely, that the interest rate, at which the country borrows from the rest of the world, is increasing in public and/or foreign debt) is supported by a number of empirical studies (see e.g. European Commission, 2012).

\(^{12}\) The value of \( \bar{d} \) can be thought of as any value of debt above which sustainability concerns start arising. As we discuss below in subsections 3.1 and 6.1, our qualitative results are robust to the exact value of \( \bar{d} \) used, to the extent that each time the value of \( \psi \) is recalibrated.

\(^{13}\) This is similar to the modeling of e.g. Erceg and Linde (2012). Recall that in the case of flexible or managed floating exchange rates, \( S_t \) and \( R_t \) switch positions, in the sense that \( S_t \) becomes an endogenous variable, while \( R_t \) is used as a policy instrument usually assumed to follow a Taylor-type rule for the nominal interest rate (see Section 7 below).
the transition path, nominal rigidities imply that money is not neutral so that monetary policy, and the exchange rate regime in particular, matter to the real economy.

2.7 Fiscal policy rules

Without room for monetary policy independence, only fiscal policy can be used for policy action. In this paper, we follow a rule-like approach to policy. We focus on simple rules, meaning that the fiscal authorities react to a small number of easily observable macroeconomic indicators capturing the current state of the economy. The magnitude of reaction coefficients (namely, how fiscal policy instruments respond to macroeconomic indicators) will be chosen optimally.

Specifically, we allow the main spending-tax policy instruments (namely, government spending as share of output, \( s^g_t \), and the tax rates on consumption, capital income and labor income, \( \tau^c_t, \tau^k_t \) and \( \tau^n_t \)) to react to the public liabilities to GDP ratio as deviation from a target value, \((l_{t-1} - l)\), as well as to the contemporaneous output gap, \((y^H_t - y^H)\), according to the simple linear rules:

\[
\begin{align*}
\Delta s^g_t &= -\gamma^g_t (l_{t-1} - l) - \gamma^g_y (y^H_t - y^H) \\
\Delta \tau^c_t &= \gamma^c_t (l_{t-1} - l) + \gamma^c_y (y^H_t - y^H) \\
\Delta \tau^k_t &= \gamma^k_t (l_{t-1} - l) + \gamma^k_y (y^H_t - y^H) \\
\Delta \tau^n_t &= \gamma^n_t (l_{t-1} - l) + \gamma^n_y (y^H_t - y^H)
\end{align*}
\]

where, from the government budget constraint in subsection 2.4, \( l_{t-1} \) is defined as:

\[
l_{t-1} = \frac{R_{t-1} \lambda_{t-1} d_{t-1} + Q_{t-1} \frac{s^g_t}{S_{t-1}} (1 - \lambda_{t-1}) d_{t-1}}{P^H_{t-1} y^H_{t-1}}
\]

and where, in the above rules, \( l_t, y^H_t \) and \( d_t \) are written in real and per capita terms, variables without time subscripts denote policy target values and \( \gamma^q_t, \gamma^q_y \geq 0 \) for \( q \equiv (g, c, k, n) \) are

---

14 We focus on “distorting” policy instruments, because using lump-sum instruments (like \( \tau^l_t \) in our model) to bring public debt down would be like a free lunch.

15 For similar rules, see e.g Schmitt-Grohé and Uribe (2007), Bi (2010) and Cantore et al. (2012). As said above, see European Commission (2011) for similar fiscal reaction functions used in practice. On the other hand, see Kliem and Kriwoluzky (2014) for a critical approach.
feedback policy coefficients on public debt to GDP and output targets respectively. The rest of fiscal policy instruments (namely, lump-sum government transfers as share of output, denoted as $s^l_t$, and the share of domestic public debt in total public debt, $\lambda_t$) are assumed to be constant over time and equal to their average values in the data (see subsection 2.8 below).

Notice that, in the above rules, a policy target value (like $s^g, \tau^e, \tau^k, \tau^n, l, y^H$) will be the steady state value of the corresponding variable. This value will depend on whether we are in the status quo economy, or in a reformed economy. For example, as further discussed in section 4 below, the debt policy target, $l$, can be either the average public debt-to-GDP ratio in the data (this will be the benchmark case without reforms where fiscal policy adjusts so as to keep the public debt ratio at its average value) or it can be set to a value less than in the data (this will be the case of debt consolidation where fiscal policy systematically brings public debt down over time).

Also, keep in mind that below we will also allow for persistence in policy instruments, as well as for the case in which the debt policy target is not fixed but it follows an $AR(1)$ rule (see section 6 below).

2.8 Exogenous variables and shocks

In this subsection, we define the exogenous variables and, among them, the exogenous stochastic processes that drive extrinsic fluctuations in our model.

We assume that foreign imports or equivalently domestic exports, $c_F^*$, are a function of terms of trade, $TT_t \equiv \frac{P^F_t}{P^H_t}$, where both variables are expressed as deviations from their steady state values:

$$\frac{c^*_t}{c^*_s} = \left(\frac{TT_t}{TT}\right)^\gamma$$  \hspace{1cm} (20)

where $0 < \gamma < 1$ is a parameter. The idea is that foreign imports rise as the domestic economy becomes more competitive. The value of the exogenous $c^*_s$ is specified in the calibration subsection 3 below).

Regarding the other rest-of-the-world variables, namely, the exogenous part of the foreign interest rate, $Q^*_t$, and the gross rate of domestic inflation in the foreign country, $\Pi^H_t = \frac{P^H_t}{P^H_{t-1}}$, we assume that they are constant over time and equal to $Q^*_t = 1.0115$ (which is the data
average value - see below) and $\Pi_t^{H^*} \equiv 1$ at all $t$.

Regarding the exogenously set policy instruments, we set the nominal exchange rate $S_t$ at 1 (under fixed exchange rates), while, the output share of government transfers, $s_t^I$, and the fraction of domestic public debt in total public debt, $\lambda_t$, are set at their data averages values at all $t$ (see subsection 3.1 below).

Finally, in the main part of the paper, stochasticity comes from shocks to TFP, which follows:

$$\log (A_t) = (1 - \rho^a) \log (A) + \rho^a \log (A_{t-1}) + \varepsilon_t^a$$

(21)

where $0 < \rho^a < 1$ is a parameter, $\varepsilon_t^a \sim N (0, \sigma^2_a)$ and, as said above, variables without time subscript denote steady state values. As we report below in section 6 our main results do not change when we add extra shocks, like shocks to the world interest rate.

### 2.9 Decentralized equilibrium (for any feasible policy)

We now combine all the above equations to present the Decentralized Equilibrium (DE) which is for any feasible policy. The DE is defined to be a sequence of allocations, prices and policies such that: (i) households maximize utility; (ii) a fraction $(1 - \theta)$ of firms maximize profits by choosing an identical price defined as $P_t^\#$, while a fraction $\theta$ just set prices at their previous period level; (iii) all constraints, including the government budget constraint and the balance of payments, are satisfied; (iv) markets clear; (v) policymakers follow the feedback rules assumed in subsection 2.7. This DE is given the values of feedback policy coefficients in the policy rules (16)-(19), the exogenous variables, $\{c_t^{F^*}, Q_t^*, \Pi_t^{H^*}, \epsilon_t, s_t^I, \lambda_t, A_t\}_{t=0}^\infty$, which have been defined in subsection 2.8 and initial conditions for the state variables.

We thus end up with a first-order non-linear dynamic system of 32 equations. Algebraic details, the final equilibrium system and the associated steady state (also used for calibration) are in Appendix D.
2.10 Solution methodology

We will work as follows. In the next section (section 3), we solve the above model numerically employing parameter values and data in accordance with the Italian economy over 2001-2013. As we shall see, the steady state solution of this model economy will do well at mimicking the output shares of key macroeconomic variables observed in the Italian data since 2001; hence, we will call it the "status quo steady state". In turn, the next sections will study the transition dynamics, when we depart from this status quo steady state and travel to a new reformed steady state (the latter is defined in section 4.1 below). To compute the transition dynamics, driven by policy reforms (like debt consolidation policies) and/or exogenous stochastic processes (like TFP shocks), we will take a second-order approximation of the stochastic problem around the deterministic new reformed steady state.\footnote{A potential criticism might be that an approximate solution might not be reliable because we approximate the equilibrium equations around a new steady state different from the initial one. To address this issue, we have also solved a deterministic version of the model using non-linear, or non-approximate, numerical methods. This means that we repeat the equilibrium equations in each time period until the economy converges to a steady state position where variable changes are negligible. We use Dynare for these computations. The key results, namely the optimal policy mix during the transition, do not change.} In all policy experiments, the feedback coefficients in the policy rules are computed optimally to maximize the household’s welfare.

Further details on the reformed economy and the computational methodology are in section 4 below. The quantitative implementation is in sections 5 and 6. But we first need to solve for the status quo steady state. This is in the next section.

3 Data, parameterization and the status quo steady state

This section parameterizes the above model economy using average data from Italy over 2001-2013 (the exact end period for each variable may vary depending on data availability) and then presents the resulting steady state solution. As said, the latter will then serve as a point of departure to study policy reforms.

3.1 Data and parameter values

The data are from Eurostat over 2001-2013 (in the case of TFP, the data source is Ameco over 1980-2014). The time unit is meant to be a year. In calibrating the model, we assume that the economy is in the deterministic steady state of the decentralized equilibrium presented above.
with zero inflation (see Appendix D for the steady state system). Recall that, since policy instruments react to deviations of macroeconomic indicators from their steady state values, feedback policy coefficients do not play any role at the steady state.

The baseline parameter values, as well as the values of the fiscal policy variables, are listed in Table 1. As reported in detail in section 6, our results are robust to changes in these parameter values.

[Table 1 here]

The value of the time preference rate, $\beta$, follows from setting the gross interest rate at $R = Q = 1.0225$ (the latter is consistent with an interest-rate premium of 1.1% over the German 10-year bond rate, which is the average value in the data, and with a gross inflation rate equal to one at the steady state).

The value of $a$ implies a labor share, $(1 - a)$, equal to 0.62, which is the average value in the Italian data over 2001-2013. We employ conventional parameter values, as used by the literature, for the elasticity of intertemporal substitution, $1/\sigma$, the inverse of Frisch labour elasticity, $\eta$, and the price elasticity of demand, $\phi$, which are as in e.g. Andrés and Doménech (2006) and Galí (2008). Regarding the preference parameters in the utility function, $\chi_n$ is calibrated from the household’s labour supply condition, while $\chi_g$ is set at 0.1. The price rigidity parameter, $\theta$, is set at 0.5. The value of $\gamma$, in equation (20) for foreign imports, is set at 0.9.

In our baseline parameterization, the threshold parameter value of the public debt to GDP ratio above which sovereign interest-rate premia emerge, $\overline{d}$, is set at 0.9 (see equation (15)). This value is consistent with evidence provided by e.g. Reinhart and Rogoff (2010) and Checherita-Westphal and Rother (2012) that, in most advanced economies, the adverse effects of public debt arise when it is around 90-100% of GDP. It is also within the range of thresholds for sustainable public debt estimated by the European Commission (2011). In turn, the associated premium parameter, $\psi$, is calibrated by using equation (15). In other words, assuming a value for the parameter $\overline{d}$, and using data averages over the sample period for the interest-rate premium and the public debt to GDP ratio, the value of $\psi$ follows from equation (15). In our baseline parameterization, the resulting value of $\psi$ is 0.0505, which means that
a one percentage point increase in the debt-to-GDP ratio leads to an increase in the interest rate premium by 5.05 basis points. Such values are in line with empirical findings for OECD countries (see e.g. Ardagna et al., 2008). As reported in subsection 6.1 below, our results are robust to changes in $\tilde{\theta}$, to the extent that each time we recalibrate $\psi$.

The parameters $\phi^h$ and $\phi^g$, measuring adjustment costs associated with changes in private and public foreign assets respectively (see equations (9) and (14)) are both set to 0.3. As already said above, this value gives plausible short-run dynamics for private foreign assets and, in turn, for the country’s net foreign debt following a policy reform. Robustness checks for both $\phi^h$ and $\phi^g$ are reported in subsection 6.1 below. We also report that a positive value for $\phi^h$ is needed to give bounded solutions for the welfare when we compute optimized feedback policy rules below. Similarly, the value of $\xi$ measuring capital adjustment costs is set equal to 0.3.

Concerning the exogenous variables, the persistence and standard deviation parameters in the TFP process (21) are set at $\rho^a = 0.9479$ and $\sigma_a = 0.007636$. These values have been estimated by running simple regressions using Ameco data for total factor productivity. As reported below, our results are robust to changes in these parameter values. Regarding the rest-of-the-world variables, $\Pi^H_t$, $Q^*_t$ and $c^F_t$, we set their steady state values equal to $\Pi^H = 1$, $Q^* = 1.0115$ (which is the data average) and $\frac{c^F}{x} = 1.01$ (which is the ratio of exports to imports in the Italian data).

The steady state values of fiscal and public finance policy instruments, $\tau^c_t$, $\tau^k_t$, $\tau^n_t$, $s^g_t$, $s^l_t$, $\lambda_t$, are set at their data averages. In particular, $\tau^c$, $\tau^k$, $\tau^n$ are the effective tax rates on consumption, capital and labor in the data over 2001-2013. Moreover, $s^g$ and $s^l$, namely, government spending on goods/services and on transfer payments as shares of output, are set at their average values in the data, 0.2222 and 0.2326 respectively. Finally, $\lambda$, the fraction of total public debt held by domestic private agents is set at 0.64, which is again its average value in the data during the same period.

### 3.2 Status quo steady state

The steady state system is in Appendix D. Table 2 presents the numerical solution of this system when we use the parameter values and policy instruments in Table 1.
we treat public debt, $d$, as the residually determined public financing instrument. In Table 2, we also present some key ratios in the Italian data. Notice that most of the solved ratios, produced endogenously by the model, are meaningful and close to their actual values. For instance, the solution for the country’s net foreign debt as share of GDP (denoted as $\tilde{f}$\textsuperscript{17}) is 0.2134; its average value in the data is 0.2109. Also, the solution for total public debt as share of GDP\textsuperscript{18} is 1.0965; its average value in the data is 1.098.

In what follows, this steady state solution will serve as a point of departure to study various policy experiments. Before we move on to reforms, it is worth pointing out that, in the above model, a lower public debt implies a lower sovereign premium and this leads to higher capital, higher output and higher welfare; this can rationalize the debt consolidation policies studied in what follows.

![Table 2 here]

4 Description of policy experiments

In this section, we define the policy reform studied, how we model debt consolidation and how we compute optimized feedback policy rules.

4.1 Definition of the reformed economy

The main experiment we want to consider in this paper is the case in which the economy departs from the status quo and travels over time to a new reformed state with lower debt. Regarding the status quo, this is provided by the steady state solution of the model economy calibrated and solved numerically in the previous section (see Tables 1 and 2). Regarding the new reformed steady state, this is defined as the case in which the public debt-to-GDP ratio is permanently reduced, so as there are no sovereign interest-rate premia in the new steady state. In other words, in the new reformed steady state, we set $Q = Q^*$ so that $\frac{TT_{1-d}}{y^h} = \tilde{d}$. Specifically, the government reduces the output share of public debt from 1.0965 (which is the status quo solution) to the threshold value of $\tilde{d} = 0.9$ corresponding with zero premia. To put

\textsuperscript{17}Thus, $\tilde{f} \equiv \frac{S_t(f^g - f^h)}{p^f Y^h} = \frac{(1-\lambda)D_t - S_t F^h}{p^f Y^h}$, where $TT_t \equiv p^h_{TT}Y^h$ is the terms of trade. Details are in Appendix D.

\textsuperscript{18}This is $\frac{D}{p^f Y^h} \equiv \frac{TT_{1-d}}{y^h}$, where $TT_t \equiv p^h_{TT}Y^h$ is the terms of trade. Details are in Appendix D.
it differently, since, in our model, sovereign premia arise whenever the public debt-to-output ratio happens to be above the 0.9 threshold, premia are eliminated \( Q = Q^* \) once such debt reduction has been achieved. As said above, we will conduct robustness exercises for the values of the exogenously set threshold, \( \bar{d} \). Also, again as a robustness check, we will consider the case in which the new reformed steady state is the Ramsey steady state, meaning that the policy targets are the Ramsey steady state values.

In addition, we assume that, in the new reformed steady state, the country’s net foreign debt position is zero or, equivalently, that the country ends up with a balanced trade.\(^{19}\) In other words, in the new reformed steady state, we set the country’s net foreign debt as share of output to zero, \( \bar{f} = 0 \). This means that the country’s net foreign debt as share of output is permanently reduced from 0.2134 (which is the status quo solution) to zero. Note that this assumption is not important to our qualitative results (its robustness is checked in subsection 6.1 below) but, if the foreign position of the country is left free in the long run with relatively low interest rates \( Q = Q^* \), private agents have an incentive to overborrow and this leads to unrealistically high values of private foreign debt, \( f^h < 0 \).

Details of the equilibrium conditions of the reformed economy, the steady state implied by these conditions, as well as numerical solutions of the reformed steady state under unrestricted or restricted \( \bar{f} \), and under alternative public financing scenarios, are provided in Appendix E.

4.2 Public debt consolidation and the intertemporal tradeoff

It is widely recognized that debt consolidation implies a tradeoff between short-term fiscal pain and medium-term fiscal gain. In our model, during the early phase of the transition, debt consolidation comes at the cost of higher taxes and/or lower public spending. On the other hand, in the medium- and long-run, a reduction in the debt burden allows, other things equal, a cut in tax rates, and/or a rise in public spending. Thus, one has to value the early costs of stabilization vis-a-vis the medium- and long-term benefits from the fiscal space created.

This intertemporal tradeoff also implies that the implications of consolidation depend heavily on the public financing policy instruments used, namely, which policy instrument adjusts endogenously to accommodate the exogenous change in fiscal policy (see also e.g. Leeper et

\(^{19}\)For a similar practice (namely, to assume a zero net foreign debt position in the steady state and then check its robustness), see e.g. Mendoza and Tesar (2005).
Specifically, these implications depend both on which policy instrument bears the cost of adjustment in the early period of adjustment and on which policy instrument is anticipated to reap the benefit, once consolidation has been achieved. In the policy experiments we consider below, we experiment with fiscal policy mixes, which means that the fiscal authority can use various instruments in the transition and in the steady state. Details are in subsection 5.3 below.

4.3 The reformed economy vs a reference regime

To evaluate the implications of debt consolidation as defined in subsection 4.1 above, we need to compare them to a reference regime. Here, we find it natural to use as reference regime the case without debt consolidation other things equal. In other words, we will study two scenarios regarding policy action. The first, used as a reference, is the scenario without debt consolidation. Here, the role of policy is only to stabilize the economy against shocks. For instance, say that the economy is hit by an adverse temporary TFP shock, which, as the impulse response functions reveal, produces a contraction in output, a rise in the public debt to output ratio and a rise in the sovereign premium. Then, the policy questions are which policy instrument to use, and how strong the reaction of policy instruments to deviations from targets should be, in order to maximize household’s welfare criterion. Note that, in this case, the values of the policy targets in the feedback rules (16)-(19) are given by the status quo steady state solution. In other words, in this policy scenario, we depart from, and end up, at the status quo steady state solution of subsection 3.2 above, so that transition dynamics are driven by exogenous shocks only.

The scenario with debt consolidation is richer. Now the role of policy is twofold: to stabilize the economy against the same shocks as above and, at the same time, to improve resource allocation by gradually reducing the debt to GDP ratio over time as defined in subsection 4.1. The policy questions are as above in the reference regime, except that now the policy targets in the feedback rules (16)-(19) are given by the steady state solution of the new reformed economy. In other words, in this case, we depart from the status quo solution with sovereign premia, but we end up at a new reformed steady state with lower (public and foreign) debt. Thus, now there are two sources of transition dynamics: temporary shocks and the deterministic
difference between the initial and the new reformed long run (see also Cantore et al., 2012).

4.4 Computational methodology

Irrespectively of the policy experiments studied, to make the comparison of different policy regimes meaningful, we compute optimized policy rules, so that our results do not depend on ad hoc differences in feedback policy coefficients across different policy rules. The welfare criterion is the household’s expected discounted lifetime utility (see equations (7)-(8) above).

We work as follows: First, we take a second-order approximation of the equilibrium conditions, as well as of household’s expected discounted lifetime utility, around the deterministic reformed steady state (see Appendix E for the latter).\(^{20}\) This is a function of feedback policy coefficients in the policy rules and initial values for the state variables. Secondly, we select the feedback policy coefficients so as to maximize the conditional mean of household’s expected discounted lifetime utility, where conditionality refers to the initial conditions chosen; the latter are given by the status quo steady state solution (see subsection 3.2 above). Thus, the initial values of the endogenous and exogenous predetermined variables are set equal to their status quo values. If necessary, the ranges of the feedback policy coefficients or, equivalently, the values of the policy instruments themselves, will be restricted to give determinate solutions and/or meaningful values for policy instruments (e.g. tax rates less than one and non-negative nominal interest rates). All this is similar to Schmitt-Grohé and Uribe (2004a, 2005, 2007).\(^{21}\)

As said above, we work similarly in the case without debt consolidation (where we depart from, and return to, the same status quo steady state) which serves as a reference regime.

\(^{20}\)Thus, we take a second-order approximation to both the equilibrium conditions and the welfare criterion. As is known, this is consistent with risk-averse behavior on the part of economic agents and can also help us to avoid possible spurious welfare results that may arise when one takes a second-order approximation to the welfare criterion combined with a first-order approximation to the equilibrium conditions (see e.g. Gali, 2008, and Benigno and Woodford, 2012).

\(^{21}\)Specifically, to compute a second-order accurate approximation of both the conditional welfare and the decentralized equilibrium, as functions of feedback policy coefficients, we use the perturbation method of Schmitt-Grohé and Uribe (2004a). In turn, we use a matlab function (such as fminsearch.m) to compute the values of the feedback policy coefficients that maximize this approximation.
5 Macroeconomic implications of fiscal consolidation

In this section, we present the main results. The emphasis will be on the case of the reformed economy as defined in subsection 4.1 above, but, for reasons of comparison, we will also present results for the reference case without debt consolidation. Recall that, in the case of debt consolidation, transition dynamics are driven both by a temporary supply shock and by deterministic changes in fiscal policy instruments aiming at debt reduction and elimination of the sovereign premium over time.

5.1 Steady state utility and output in the reformed economy

The steady state of the reformed economy with debt consolidation is as defined in the previous section, while details are in Appendix E. Thanks to the fiscal space created by debt reduction, public spending can rise or a tax rate can be reduced residually.

Table 3 summarizes steady-state utility and output under alternative public financing scenarios in this reformed economy (for the full numerical solutions, see Appendix E). In the first row of Table 3, the assumption is that it is public spending that takes advantage of debt reduction, in the sense that, once the debt burden has been reduced, public spending can increase relative to its value in the status quo solution. In the next three rows, the fiscal space is used to finance cuts in one of the three tax rates. The last row reports, for comparison, the case in which it is lump-sum tranfers, \( s^l \), that rise. The best outcome (both in terms of utility and output) is achieved when the fiscal space is used to finance a cut in capital tax rates. This is as expected and is consistent with the Chamley-Judd normative result. Therefore, in what follows, we will use the capital tax rate, \( \tau^k \), as the residually determined fiscal policy instrument in the steady state of the reformed economy with debt consolidation.

[Table 3 here]

5.2 Determinacy and bounds on policy coefficients

As is known, local determinacy, or implementability, depends crucially on the values of feedback policy coefficients in the rules (16)-(19) above. This is also the case in our model. Our experiments show that economic policy can ensure determinacy when at least one of the fiscal
policy instruments \((s^q_t, \tau_c^k, \tau_n^k)\) reacts to public liabilities between critical minimum and maximum values, where these bounds vary depending on which fiscal policy instrument is used. In other words, determinacy requires restrictions on the magnitude of \(\gamma_t^q\), where \(q \equiv (g,c,k,n)\). By contrast, the values of \(\gamma_t^q\), measuring the reaction of fiscal policy instruments to the output gap, are not found to be critical to determinacy.

Nevertheless, although not important for determinacy, we set bounds on the feedback reaction of the capital tax rate to the output gap, \(\gamma_t^k\), so as the implied capital tax rate is within \(0.11 < \tau_t^k < 0.51\) in the transition. In other words, since the average value of \(\tau_t^k\) in the data is 0.31, we limit attention to changes in \(\tau_t^k\) that are less than minus/plus 20 percentage points than in the data. We set these bounds because, if the capital tax rate were left unrestricted, the policymaker would find it optimal to set it at a very low value as one would expect given the Chamley-Judd logic. Our bounds exclude this possibility. Note that such practice is usual in the related literature; for instance, when they compute optimized rules, Schmitt-Grohé and Uribe (2007) and Kliem and Kriwoluzky (2014) impose similar intervals for monetary and fiscal policy respectively.

5.3 Optimal fiscal policy mix

We can now study optimal policy mixes when we depart from the status quo steady state and travel towards the reformed steady state as defined in subsection 4.1 above. Policymakers are allowed to use different instruments in the transition and in the steady state. In particular, in the reformed steady state, given the evidence in Table 3, we assume that it is the capital tax rate that takes advantage of the fiscal space created once the debt burden has been reduced and sovereign premia have been eliminated; as said, a cut in the capital tax rate is the most efficient way of using this fiscal space. On the other hand, in the transition to this reformed steady state, all available fiscal instruments, and at the same time, are allowed to be used, as in the policy rules (16)-(19). Since feedback policy coefficients are chosen optimally, this will also tell us how to assign different policy instruments to different macroeconomic indicators.

\(^{22}\)For example, when we use one fiscal instrument at a time, the ranges of fiscal reaction to public liabilities are \(0.027 < \gamma_t^q < 2.72, 0.048 < \gamma_t^c < 4.34, 0.064 < \gamma_t^k < 2.96, 0.063 < \gamma_t^n < 1.56\) for \(s^q_t, \tau_c^k, \tau_n^k\) and \(\tau_t^q\) respectively. When we switch on all fiscal instruments, these ranges become narrower.

\(^{23}\)Results in the special case in which the fiscal authority is restricted to use one fiscal instrument at a time are in Appendix E. These results can help to understand the working of the model. Here, we only present the optimal mix, where are instruments can be used simultaneously, to save on space.
Results for the optimal mix during the transition to the reformed steady state are reported in Table 4.

[Table 4 here]

The values of the optimized feedback policy coefficients in Table 4 imply a clear-cut assignment of instruments to targets. Government spending should be used to address the public debt problem, while income (capital and labor) taxes should be used to address the output problem. On the other hand, it is better to avoid changes in consumption tax rates (the optimized feedback coefficients in the rule for the consumption tax rate are practically zero in this case). These signs and magnitudes of the feedback policy coefficients mean that government spending should be reduced in order to bring public debt down, while, at the same time, the capital and labor tax rates should be also cut so as to stimulate the real economy in an attempt to increase the denominator in the debt-to-output ratio (impulse response functions are shown below).

Table 5 also reports some associated statistics (like elasticities and min/max values). As can be seen, in the short term, public spending should fall by a lot vis-a-vis its data value so as to bring public debt down and, at the same time, capital and labor tax rates should also fall by a lot vis-a-vis the data to stimulate the real economy. Then, over time, they return to their data average values (this is also shown by impulse response functions below). In subsection 5.6 below, we also report results when the use of policy instruments is restricted.

[Table 5 here]

5.4 Welfare over time with, and without, debt consolidation

Setting the feedback policy coefficients as in Table 4, the associated expected discounted utility over various time horizons is reported in the first row of Table 6. Studying what happens to welfare over various time horizons can be useful because, for several (e.g. political-economy) reasons, economic agents can be short-sighted. It can also help us to understand the possible conflicts between short-, medium- and long-term effects from debt consolidation. The second

\[24\] The elasticities report the percentage change in the fiscal instrument with respect to a 1% change in the macroeconomic indicator, other things equal.
row in Table 6 reports results without debt consolidation, other things equal. Thus, we again compute the best policy mix meaning that all fiscal policy instruments at the same time are allowed to react to debt and output gaps but now the debt and output targets in the policy rules remain as in the status quo solution (see subsection 4.3 above). Finally, the last row in Table 6 gives the welfare gain, or loss, of debt consolidation expressed in permanent consumption equivalent units. A positive number means that welfare would increase with debt consolidation. And vice versa: a negative number means that welfare would decrease with debt consolidation.

The results in Table 6 reveal that, other things equal, debt consolidation improves welfare only if we are relatively far-sighted. In particular, expected discounted utility is higher with debt consolidation, only when we care beyond the first ten years. Reversing the argument, debt consolidation comes at a short-term cost. Once the short-term pain is over, the welfare gain in consumption equivalents is substantial.

### 5.5 Impulse response functions with debt consolidation

To get a clearer picture of the above results, we also present the implied impulse response functions illustrating the time paths of the fiscal policy instruments used for consolidation as well as some key macroeconomic variables. They are shown in Figure 1. As can be seen, public spending should fall, while, at the same time, capital and labor tax rates should be cut, for the reasons discussed above. This optimal mix allows a gradual reduction in the public debt to GDP ratio, in the foreign debt to GDP ratio and in the interest-rate premium. Private consumption falls in the short term, as a consequence of debt consolidation, but recovers soon. Hours of work need to rise (or leisure to fall) for some time. All variables converge to their new, reformed values over time.

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25It should be pointed out that the rise in welfare is partly driven by the fact that debt consolidation and elimination of sovereign premia in the reformed long-run equilibrium allow a higher value of the time preference rate than in the pre-reformed long-run solution in section 3 (in particular, the calibrated value of $\beta$ was 0.978 in the status quo steady state in section 3, while it is 0.9886 without premia). We report that the main results do not change when we allow for persistence in the change of the time preference rate as it rises from 0.978 to 0.9886. Actually, when we allow the related autoregressive parameter to be optimally chosen, along the feedback policy coefficients, its optimal value is close to zero meaning that it is better to adopt as soon as possible the higher value of the time preference rate. Results are available upon request.

26Prescott (2002) finds welfare gains of similar magnitude when Japan or France adopt the tax policy or the production efficiency of the USA.
5.6 Further restrictions on the use of fiscal policy instruments

One could argue that the values of tax-spending policy instruments cannot differ substantially from those in the historical data (for various political economy reasons). Therefore, we now redo the main computations restricting the magnitude of feedback coefficients in the policy rules so as all tax-spending policy instruments cannot change by more than, say, 10 percentage points from their averages in the data. The new results are reported in Appendix E. Although obviously feedback policy coefficients are now smaller, the best fiscal policy mix again implies that we should earmark public spending for the reduction of public debt and, at the same time, cut taxes to mitigate the recessionary effects of debt consolidation. The only difference is that now, since cuts in income (capital and labor) taxes are restricted, we should also cut consumption taxes.

In the same Appendix (Appendix E), we compare results when the fiscal authorities use the optimal fiscal mix to results when they are restricted to use one fiscal instrument at a time only. The message from the new impulse response functions is that the reduction in public debt is more gradual when we use all fiscal instruments, and this allows a smaller fall in private consumption, than when the fiscal authorities are restricted to use one instrument at a time only. This is intuitive: a policy mix gives more choices.

6 Sensitivity analysis

This section checks the sensitivity of the above results. We start with changes in parameter values and then study robustness to more substantial, modeling changes. To save on space, we will selectively provide some results only (a full set of results is available upon request from the authors).

6.1 Changes in parameter values

We start with the value of the public debt threshold parameter, $d$, in the interest-rate premium equation (15). Recall that so far we have set $d = 0.9$. Our qualitative results do not depend on
this value. For instance, in Appendix F, we present the main results with $d = 0.8$ and $d = 1$. In this case, as said above, we need to recalibrate the value of $\psi$ so as to hit the data again; the new values are respectively $\psi = 0.0319$ and $\psi = 0.108$.

Our results are also robust to changes in the assumed value of net foreign debt in the steady state of the reformed economy. Recall that so far we have solved for the reformed economy assuming a zero net foreign debt position at steady state, $\bar{f} = 0$. Our main results remain unchanged when we instead set $\bar{f} = 0.1$ and $\bar{f} = 0.2109$ in the reformed steady state (where $0.2109$ is the average value of the country’s foreign debt in the data). Results for these two cases are presented in Appendix G.

Our qualitative results are also robust to changes in other model parameters. For instance, we have experimented with changes in the values of the Calvo parameter in the firm’s problem, $\theta$, and the adjustment cost parameters on foreign private assets/debt, $\phi^h$, foreign public debt, $\phi^g$, and physical capital, $\xi$. We report that our main results do not change when we set $\theta$ at, say, 0, 0.1 or 0.9 (we also report that as price stickiness, $\theta$, rises, the optimal fiscal reaction to public debt becomes milder), when $0.2 \leq \phi^h \leq 0.5$ and $0 \leq \xi \leq 2$, while the value of $\phi^g$ has not been found to be important. Similarly, our results remain unchanged for $0.8 \leq \gamma \leq 0.99$, which measures the sensitivity of exports to changes in the terms of trade in equation (20). Also, our main results do not depend on the value of $\chi_g$, namely, how much agents value public consumption spending in the utility function. The values of the labor supply parameters, $\chi_n$ and $\eta$, are not crucial either; nevertheless, we report that when $\eta = 0.5$, the reaction of the labor tax rate to the debt target is zero, i.e. $\gamma^n_l = 0$, while when $\eta = 2$, we get $\gamma^n_l = 0.3761$. That is, as $\eta$ (namely, the inverse of the Frisch elasticity) rises, the reaction of the labor tax rate to the debt target becomes stronger which, in turn, means that the net cut in the labor tax rate should be smaller. As said, the above results are available upon request.

6.2 Changes in policy variables

Our results are also robust to the specific way we model the fiscal policy instruments. For instance, the main results remain unaffected when we allow for persistence in the feedback policy rules, (16)-(19), in the sense that, for example in the case of the consumption tax rate, we use:
\[
\tau_t^c = (1 - \rho^{r^c}) \tau_t^c + \rho^{r^c} \tau_{t-1}^c + \gamma_l^c (l_{t-1} - l) + \gamma_y^c (y_t^H - y^H)
\] (22)

where \(0 \leq \rho^{r^c} \leq 1\) is an autoregressive policy parameter and the initial value of the policy instrument is its data average value as reported in Tables 1 and 2. Actually, we have also allowed \(\rho^{r^c}\) to be optimally chosen, jointly with the other feedback policy coefficients, and the main results again do not change. Interestingly, the optimized value of these autoregressive policy parameters are found to be relatively small meaning that it is better to adjust the policy instrument(s) relatively soon.

Also, following several related papers (see e.g. Coenen et al., 2008, Forni et al., 2010a, and Erceg and Lindé, 2013), we have experimented with time-varying debt policy targets. Thus, instead of using a constant over time debt policy target, \(l\), like in equations (16)-(19) above, we assume that the target, defined now as \(l_t^*\), follows an AR(1) process of the form:

\[
l_t^* = (1 - \rho^l) l + \rho^l l_{t-1}^*
\] (23)

where \(0 \leq \rho^l \leq 1\) is an autoregressive policy parameter and the initial value of the target is given by its data average value in Tables 1 and 2. We report that our main results remain the same under this new specification.

Our results are also robust to adding more macroeconomic indicators in the feedback policy rules (like inflation or terms of trade). We have also experimented with changes in some exogenous policy instruments, which have been kept constant so far, like the fraction of public debt held by domestic agents relative to foreign investors, \(\lambda\). The latter has so far been kept constant and equal to its average value on the data, 0.64. When we experiment with \(\lambda = 0.54\) and \(\lambda = 0.74\), the results do not change.

6.3 Allowing for new shocks

Our results are also robust to allowing for a more volatile economy. This can be captured by increasing the standard deviation of the existing TFP shock and/or by adding new shocks. Specifically, regarding new shocks, we have experimented with adding shocks to the fiscal
policy rules in subsection 2.7 to the time-varying debt policy target presented in subsection 6.2 above, or to the world interest rate in equation (15). The main results again do not change. In particular, regarding shocks to the world interest rate, and following the specification of García-Cicco et al. (2010), we augment equation (15) by:

$$Q_t = Q_t^* + \psi \left( e^2 - 1 \right) + e^{e^{\mu_{t-1}} - 1}$$

where

$$\log (\mu_t^q) = \rho^q \log (\mu_{t-1}^q) + \varepsilon_t^q$$

where $0 \leq \rho^q \leq 1$ is a parameter and $\varepsilon_t^q$ is an iid shock. In our experiments, we set 0.9845 for $\rho^q$ and 0.0487 for the standard deviation of $\varepsilon_t^q$. The new results are reported in Appendix H.

As can be seen, the main messages remain the same. This is not surprising: the key driver of transition dynamics is policy reforms, like debt consolidation, rather than cyclical fluctuations generated by exogenous shocks.

### 6.4 Transition to the Ramsey steady state

So far, we have studied the optimal transition of the economy from the status quo to an arbitrarily reformed steady state. A potential criticism (see e.g. Schmitt-Grohé and Uribe, 2009, chapter 3) might be that the accuracy of approximation (in our paper, first- and second-order) is poor because, when the policymaker chooses the feedback policy coefficients in such cases, he/she may want to use his/her choices in order to influence the mean of variables, rather than to stabilize the economy around the assumed steady state. A way of checking whether this affects the main results is to approximate the economy around the Ramsey steady state, meaning that the steady state policy targets in the feedback rules (16)-(19) are the Ramsey steady state values of the corresponding variables.

Therefore, this subsection examines the optimal transition of the economy from the status quo to the Ramsey steady state and compares this to the case studied so far, in which we computed the optimal transition to an arbitrarily reformed steady state. By optimal transition,

---

27 These estimates follow from OLS regressions of (25) where $\mu_t^q$ denotes the deviation of the 10-year German bond rate from its average value on the data. The data for the 10-year German bond are from Eurostat, 2001-2013.
we again mean that we select the feedback policy coefficients so as to maximize the household’s welfare criterion as above. The difference is that now the steady state, around which we approximate, is the steady state of the Ramsey second-best policy problem.\footnote{Schmitt-Grohé and Uribe (2005, 2007) and Kliem and Kriwoluzky (2014), on the other hand, select the feedback policy coefficients so as to minimize the distance from the Ramsey solution.}

The first step is to solve the Ramsey second-best policy problem. In this problem, when we use the so-called dual approach, the government chooses the time-paths of all endogenous variables of the DE system plus the fiscal policy instruments taken as given at the DE level (see e.g. Schmitt-Grohé and Uribe, 2005, for a related problem). For simplicity, and following the same authors, we solve for the so-called timeless Ramsey equilibrium, which means that the optimality conditions in the initial periods do not differ from those in later periods. Besides, to make the comparison to the previous cases meaningful, we again assume, as we have done so far in the steady state of the arbitrarily reformed economy, that, in the steady state of the Ramsey problem, the public debt to GDP ratio is set at 0.9 and that the country’s net foreign debt position is zero.\footnote{Actually, setting some debt values at exogenous values is necessary to get a well-defined steady state system in the Ramsey problem. See also e.g. Schmitt-Grohé and Uribe (2004b). We provide details in Appendix I.} Details of the Ramsey policy problem are in Appendix I.

When we solve this Ramsey problem numerically using Dynare, the resulting steady state solution (reported in Appendix I) gives us an unrealistically high consumption tax rate and an equally large labour subsidy, i.e. $\tau^n < 0$. This is a well-known property of the Ramsey problem, when the policymaker has also access to consumption taxes (see the early papers by Lansing, 1999, and Coleman, 2000, and many others since then).\footnote{Another related well-known property is that, at least in the baseline neoclassical growth model without market frictions, the Ramsey government can implement the first-best allocation if he/she has access to consumption, labor and capital taxes.} In addition, our DE economy does not seem to converge to this unrestricted Ramsey steady state. To overcome these problems, and given the difficulty of setting negative labor tax rates in reality, we solve the Ramsey problem by not letting the government to choose the labor tax rate, which is now constrained to remain fixed at, say, zero ($\tau^n = 0$). Actually, most of the above mentioned literature on Ramsey policy with consumption taxes works similarly. The steady state solution of this "restricted" Ramsey problem is reported in Table 7.

[Table 7 here]
As can be seen, the implied utility and output are higher in the Ramsey steady state than in all other cases studied so far. This is as expected. It is also worth noticing that, in this solution, the capital tax rate is very low (but not zero since there are several market imperfections in the model) and the consumption tax rate is well defined (although high, as one would expect for the reasons discussed above).

The next step is to compute the optimal transition from the status quo steady state in Table 2 to the Ramsey steady state in Table 7. Results for the optimal fiscal policy mix and the associated welfare over different time horizons are reported in Tables 8 and 9 respectively. These two tables need to be compared to Tables 4 and 6 respectively. As can be seen, the main results are not affected. Namely, it is again optimal to earmark public spending for the reduction of public debt and, at the same time, to use the tax rates for the stimulation of the real economy. The difference is that since changes in the labor tax are now restricted (the Ramsey labor tax rate has been set to zero) it is also optimal to make use of the consumption tax rate. Namely, it is optimal, in the short term, to cut not only the capital tax rate, but also the consumption tax rate; in later periods, the consumption tax rate rises substantially converging to its high Ramsey steady state value in Table 7. Observe that now, as shown in Table 9, the case with debt consolidation is superior to the reference case without debt consolidation over all time horizons, even in the short term. Thus, an intuitive general message is that a proper use of fiscal policy can mitigate (in our case, avoid) the short-term pain from debt consolidation. But, of course, as is typically the case, the optimal Ramsey values for the fiscal policy instruments are far away from their values in the data.

[Table 8 here]

[Table 9 here]

6.5 The interest-rate premium as a function of net foreign debt

So far, we have assumed that the sovereign interest-rate premium in equation (15) is a function of the the public debt to GDP ratio. We now assume that the premium is a function of the country’s net foreign debt ratio, \( \tilde{f} \equiv \frac{(1-\delta)TT^{1-\nu}d^{-TT^\nu}f^h}{x^H}. \) The latter is another obvious candidate for the emergence of country interest-rate premia. In particular, equation (15)
changes to (assuming a zero threshold parameter in this case)\textsuperscript{31}
\[ Q_t - Q^* = \psi \left( e^{(1-\gamma)\alpha T^1 T^{1-\nu}\nu - T F^1 F^{1-\nu}\nu f h} - 1 \right) \]  \hspace{1cm} (26)

The new results are summarized in Tables 10 and 11. As can be seen, by comparing Tables 10 and 11 to Tables 4 and 6 respectively, the main messages remain the same.

[Table 10 here]
[Table 11 here]

7 What would have happened under flexible exchange rates?

This section resolves the baseline model developed in section 2 under the fiction of flexible exchange rates, other things equal. Again, the initial conditions for the state variables will be those of the steady state solution of the status quo model. In terms of modelling, the only difference from the model in section 2 is that now the exchange rate becomes an endogenous variable. Thus, \( R_t \) and \( S_t \) exchange places. The former was endogenous in section 2, while now it is the latter that becomes endogenous with the former being free to follow a national Taylor-type rule for the nominal interest rate. In particular, we postulate:

\[ \log \left( \frac{R_t}{\overline{R}} \right) = \phi_\pi \log \left( \frac{\Pi_t}{\overline{\Pi}} \right) + \phi_y \log \left( \frac{y_t}{\overline{y}} \right) + \phi_\varepsilon \log \left( \frac{\varepsilon_t}{\overline{\varepsilon}} \right) \]  \hspace{1cm} (27)

where \( \phi_\pi, \phi_y, \phi_\varepsilon \geq 0 \) are feedback monetary policy coefficients on price inflation, output and exchange rate depreciation respectively as deviations from their steady state values. This is like the fiscal policy rules above. All feedback (monetary and fiscal) policy coefficients are again computed optimally, as described in section 4.

Since money is neutral in the long run (and \( \Pi^H = 1 \)), a switch to flexible exchange rates does not affect the solution of real variables in the steady state. Any differences will thus arise in the transition only, during which money is not neutral because of Calvo-type nominal fixities. Results are reported in Tables 12 and 13, which need to be compared to Tables 4 and 6 respectively (Appendix J also presents impulse response functions). The computed values of

\textsuperscript{31}The value of \( \psi \) is recalibrated in the same way as explained in subsections 3.1 and 6.1 above (namely, to hit the net foreign debt position in the data).
\( \phi_n \) and \( \phi_y \) are as typically found in the related literature (see e.g. Schmitt-Grohé and Uribe, 2007), while the high value of \( \phi_r \) implies that exchange rate stabilization is desirable. But, in the context of our paper, the key result is that the optimal fiscal policy mix remains as above. Notice also that the associated welfare is only slightly higher than that without monetary policy independence in Table 4. In other words, to the extent that feedback policy coefficients are selected optimally, the loss of monetary policy independence is not a big loss, at least in this class of New Keynesian models with Calvo- or Rotemberg-type nominal stickiness. This is in line with the related literature (see Schmitt-Grohé and Uribe, 2015).

8 Concluding remarks and possible extensions

This paper has studied fiscal policy action in a New Keynesian model of a small open economy facing debt-elastic interest-rate premia and not being able to use monetary policy. Our analysis was based on optimized, simple and implementable feedback policy rules for various categories of tax rates and public spending.

Since the main results have been listed in the Introduction already, we close with some possible extensions. Here, we have focused on the macroeconomic, or aggregate, implications of alternative debt consolidation policies leaving out the issue of distributional implications. It would be interesting to add heterogeneity both in terms of economic agents within the country and in terms of countries. For instance, within each country, we could distinguish between those who have access to financial markets and those who just work and consume (the so-called rule-of-thumb consumers); or between those working in the private sector and those working in the public sector. For instance, Schmitt-Grohé and Uribe (2015) give the following explanation for the small differences in macro performance under fixed and flexible exchange rates in this class of models: increases in unemployment during recessions are roughly offset by rises in work hours during expansions, so that the average level of employment, and hence welfare, are affected relatively little by the exchange rate regime (we report that, in our model, impulse response functions for hours of work and real wages are very similar under flexible and fixed exchange rates). It therefore seems that one has to add extra forms of nominal fixities to make flexible exchange rates, and hence the use of independent monetary policy, more desirable. For instance, Schmitt-Grohé and Uribe (2015) add downward nominal wage rigidity in a small open economy model and also assume a labour contract according to which employment is demand determined during recessions but demand-supply determined during booms. This implies that aggregate fluctuations cause higher unemployment on average, so that having an extra instrument for stabilization, like independent monetary policy, becomes useful.

\[\text{Table 12 here}\]

\[\text{Table 13 here}\]
the public sector (public employees). It would be also interesting to use a two-country model, where countries can differ in, say, fiscal imbalances and/or time preferences and so study the asymmetric cross-border effects of national stabilization and debt consolidation policies. We leave these extensions for future work.
References


9 Tables

Table 1: Baseline parameter values and policy variables

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a$</td>
<td>0.38</td>
<td>share of capital</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.9708</td>
<td>rate of time preference</td>
</tr>
<tr>
<td>$\nu$</td>
<td>0.5</td>
<td>home goods bias parameter at home</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.04</td>
<td>rate of capital depreciation</td>
</tr>
<tr>
<td>$\phi$</td>
<td>6</td>
<td>price elasticity of demand</td>
</tr>
<tr>
<td>$\eta$</td>
<td>1</td>
<td>inverse of Frisch labour elasticity</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>1</td>
<td>elasticity of intertemporal substitution</td>
</tr>
<tr>
<td>$\nu^a$</td>
<td>0.5</td>
<td>home goods bias parameter abroad</td>
</tr>
<tr>
<td>$\theta$</td>
<td>0.5</td>
<td>price rigidity parameter</td>
</tr>
<tr>
<td>$\psi$</td>
<td>0.0505</td>
<td>interest-rate premium parameter</td>
</tr>
<tr>
<td>$\chi_n$</td>
<td>3.66</td>
<td>preference parameter related to work effort</td>
</tr>
<tr>
<td>$\chi_a$</td>
<td>0.1</td>
<td>preference parameter related to public spending</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.9</td>
<td>threshold parameter of public debt as share of output</td>
</tr>
<tr>
<td>$\rho^a$</td>
<td>0.9479</td>
<td>persistence of TFP (1980-2014)</td>
</tr>
<tr>
<td>$\sigma_a$</td>
<td>0.007636</td>
<td>standard deviation of TFP (1980-2014)</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.9</td>
<td>terms of trade elasticity of foreign imports</td>
</tr>
<tr>
<td>$\xi$</td>
<td>0.3</td>
<td>adjustment cost parameter on physical capital</td>
</tr>
<tr>
<td>$\phi^a$</td>
<td>0.3</td>
<td>adjustment cost parameter on foreign public debt</td>
</tr>
<tr>
<td>$\phi^p$</td>
<td>0.3</td>
<td>adjustment cost parameter on private foreign assets/debt</td>
</tr>
<tr>
<td>$\tau^c$</td>
<td>0.1756</td>
<td>consumption tax rate</td>
</tr>
<tr>
<td>$\tau^k$</td>
<td>0.3118</td>
<td>capital tax rate</td>
</tr>
<tr>
<td>$\tau^n$</td>
<td>0.421</td>
<td>labour tax rate</td>
</tr>
<tr>
<td>$s^g$</td>
<td>0.2222</td>
<td>government spending on goods/services as share of GDP</td>
</tr>
<tr>
<td>$s^f$</td>
<td>0.2326</td>
<td>government transfers as share of GDP</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>0.64</td>
<td>fraction of total public debt held by domestic agents</td>
</tr>
<tr>
<td>$\tau_{c^c}$</td>
<td>1.01</td>
<td>exports to imports ratio</td>
</tr>
</tbody>
</table>

Table 2: Status quo steady state solution

<table>
<thead>
<tr>
<th>Variables</th>
<th>Description</th>
<th>Steady-state solution</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>$u$</td>
<td>period utility</td>
<td>0.8217</td>
<td>-</td>
</tr>
<tr>
<td>$r^k$</td>
<td>real return to physical capital</td>
<td>0.0908</td>
<td>-</td>
</tr>
<tr>
<td>$w$</td>
<td>real wage rate</td>
<td>1.1138</td>
<td>-</td>
</tr>
<tr>
<td>$n$</td>
<td>hours worked</td>
<td>0.331281</td>
<td>0.2183</td>
</tr>
<tr>
<td>$y^H$</td>
<td>output</td>
<td>0.712326</td>
<td>-</td>
</tr>
<tr>
<td>$TT$</td>
<td>terms of trade</td>
<td>0.994923</td>
<td>-</td>
</tr>
<tr>
<td>$Q - Q^*$</td>
<td>interest rate premium</td>
<td>0.011</td>
<td>0.011</td>
</tr>
<tr>
<td>$\frac{TT}{y^H}$</td>
<td>consumption as share of GDP</td>
<td>0.6335</td>
<td>0.5961</td>
</tr>
<tr>
<td>$\frac{f}{y^H}$</td>
<td>physical capital as share of GDP</td>
<td>3.4872</td>
<td>-</td>
</tr>
<tr>
<td>$\frac{TT}{y^H}$</td>
<td>private foreign assets as share of GDP</td>
<td>0.1813</td>
<td>0.1039</td>
</tr>
<tr>
<td>$\frac{TT}{y^H}$</td>
<td>total public debt as share of GDP</td>
<td>1.0965</td>
<td>1.098</td>
</tr>
<tr>
<td>$f$</td>
<td>country’s net foreign debt as share of GDP</td>
<td>0.2134</td>
<td>0.2109</td>
</tr>
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</table>
Table 3: Steady state utility and output in the reformed economy

<table>
<thead>
<tr>
<th>residual fiscal instrument</th>
<th>steady state utility</th>
<th>steady state output</th>
</tr>
</thead>
<tbody>
<tr>
<td>$s^g$</td>
<td>0.9125</td>
<td>0.82367</td>
</tr>
<tr>
<td>$\tau^c$</td>
<td>0.9227</td>
<td>0.82367</td>
</tr>
<tr>
<td>$\tau^k$</td>
<td>0.9311</td>
<td>0.834774</td>
</tr>
<tr>
<td>$\tau^n$</td>
<td>0.9290</td>
<td>0.83139</td>
</tr>
<tr>
<td>$s^l$</td>
<td>0.9180</td>
<td>0.817941</td>
</tr>
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</table>

Table 4: Optimal fiscal reaction to debt and output with debt consolidation

<table>
<thead>
<tr>
<th>fiscal instruments</th>
<th>optimal reaction to debt</th>
<th>optimal reaction to output</th>
</tr>
</thead>
<tbody>
<tr>
<td>$s^g_t$</td>
<td>$\gamma^g_t = 0.5009$</td>
<td>$\gamma^y_t = 0$</td>
</tr>
<tr>
<td>$\tau^c_t$</td>
<td>$\gamma^c_t = 0$</td>
<td>$\gamma^c_y = 0$</td>
</tr>
<tr>
<td>$\tau^k_t$</td>
<td>$\gamma^k_t = 0.0031$</td>
<td>$\gamma^k_y = 2.2569$</td>
</tr>
<tr>
<td>$\tau^n_t$</td>
<td>$\gamma^n_t = 0.0753$</td>
<td>$\gamma^n_y = 2.1360$</td>
</tr>
</tbody>
</table>

Notes: (i) $\tau^k_t$ is the residual instrument at the reformed steady state. (ii) At all $t$, $R_t \geq 1$, $0 < s^g_t$, $\tau^c_t$, $\tau^k_t$, $\tau^n_t < 1$. (iii) Restriction on $\gamma^k_t$ so as $0.11 < \tau^k_t < 0.51$. (iv) Lifetime utility $V_0 = 79.98864$. 

40
Table 5: Statistics implied by Table 4

<table>
<thead>
<tr>
<th>Elasticity to</th>
<th>Elasticity to</th>
<th>Min/max</th>
<th>5 periods</th>
<th>10 periods</th>
<th>20 periods</th>
<th>Data</th>
</tr>
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<tr>
<td>liabilities</td>
<td>output</td>
<td></td>
<td>average</td>
<td>average</td>
<td>average</td>
<td>average</td>
</tr>
<tr>
<td>$s^t_i$</td>
<td>-2.52%</td>
<td>0%</td>
<td>0.1073/0.2352</td>
<td>0.1204</td>
<td>0.1383</td>
<td>0.1694</td>
</tr>
<tr>
<td>$r^t_i$</td>
<td>0%</td>
<td>0%</td>
<td>0.1756/0.1756</td>
<td>0.1756</td>
<td>0.1756</td>
<td>0.1756</td>
</tr>
<tr>
<td>$r^t_{yi}$</td>
<td>0.01%</td>
<td>5.13%</td>
<td>0.1443/0.3118</td>
<td>0.1901</td>
<td>0.2165</td>
<td>0.2470</td>
</tr>
<tr>
<td>$r^t_{yn}$</td>
<td>0.2%</td>
<td>3.61%</td>
<td>0.2956/0.4379</td>
<td>0.3383</td>
<td>0.3607</td>
<td>0.3851</td>
</tr>
</tbody>
</table>

Notes: As in Table 4.

Table 6: Welfare over different time horizons with, and without, debt consolidation

<table>
<thead>
<tr>
<th></th>
<th>2 periods</th>
<th>4 periods</th>
<th>10 periods</th>
<th>20 periods</th>
<th>30 periods</th>
</tr>
</thead>
<tbody>
<tr>
<td>with consolid.</td>
<td>1.3576</td>
<td>2.8154</td>
<td>7.4589</td>
<td>15.1321</td>
<td>22.1331</td>
</tr>
<tr>
<td>without consolid.</td>
<td>(1.6251)</td>
<td>(3.1791)</td>
<td>(7.4481)</td>
<td>(13.4127)</td>
<td>(18.1904)</td>
</tr>
<tr>
<td>welfare gain/loss</td>
<td>-0.0862</td>
<td>-0.0717</td>
<td>0.001</td>
<td>0.0959</td>
<td>0.1310</td>
</tr>
</tbody>
</table>

Notes: As in Table 4.

33 The values of the optimized feedback policy coefficients without debt consolidation are $\gamma^d_t = 0.2622, \gamma^c_t = 0.3303, \gamma^{d,j}_{yi} = 0.5819, \gamma^{d,j}_{yn} = 0.2510, \gamma^d_y = 0.3157, \gamma^c_y = 0.5772, \gamma^{d,j}_y = 0, \gamma^a_y = 0.0162$. 

41
Table 7: Ramsey steady state solution with \( \tau^n = 0 \)

<table>
<thead>
<tr>
<th>Variables</th>
<th>Description</th>
<th>Steady-state solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>( u )</td>
<td>period utility</td>
<td>1.0950</td>
</tr>
<tr>
<td>( y^H )</td>
<td>output</td>
<td>1.1162</td>
</tr>
<tr>
<td>( TT )</td>
<td>terms of trade</td>
<td>1.01</td>
</tr>
<tr>
<td>( \tau^k )</td>
<td>&quot;optimal&quot; capital tax rate</td>
<td>0.0500</td>
</tr>
<tr>
<td>( \tau^c )</td>
<td>&quot;optimal&quot; consumption tax rate</td>
<td>0.810199</td>
</tr>
<tr>
<td>( Q - Q^* )</td>
<td>interest rate premium</td>
<td>0</td>
</tr>
<tr>
<td>( TT^{1-v} \frac{c}{y^H} )</td>
<td>consumption as share of GDP</td>
<td>0.5443</td>
</tr>
<tr>
<td>( \frac{k}{y^H} )</td>
<td>physical capital as share of GDP</td>
<td>5.8387</td>
</tr>
<tr>
<td>( \frac{TT^{1-v} f^h}{y^H d} )</td>
<td>private foreign assets as share of GDP</td>
<td>0.3240</td>
</tr>
<tr>
<td>( \frac{TT^{1-v} f^d}{y^H d} )</td>
<td>total public debt as share of GDP</td>
<td>0.9</td>
</tr>
<tr>
<td>( f )</td>
<td>total foreign debt as share of GDP</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 8: Optimal fiscal reaction to debt and output with debt consolidation and a Ramsey steady state

<table>
<thead>
<tr>
<th></th>
<th>optimal reaction to debt</th>
<th>optimal reaction to output</th>
</tr>
</thead>
<tbody>
<tr>
<td>( s^d_t )</td>
<td>( \gamma^d_t = 0.6515 )</td>
<td>( \gamma^d_y = 0.0102 )</td>
</tr>
<tr>
<td>( \tau^c )</td>
<td>( \gamma^c_t = 0 )</td>
<td>( \gamma^c_y = 1.9621 )</td>
</tr>
<tr>
<td>( \tau^k )</td>
<td>( \gamma^k_t = 0 )</td>
<td>( \gamma^k_y = 0.6183 )</td>
</tr>
<tr>
<td>( \tau^n )</td>
<td>( \gamma^n_t = 0.0086 )</td>
<td>( \gamma^n_y = 0.0002 )</td>
</tr>
</tbody>
</table>

Notes: (i)-(iii) as in Table 4. (iv) \( V_0 = 92.4789 \). (v) Optimal degree of persistence in fiscal instruments, \( \rho = 0.2533 \).
### Table 9: Welfare over different time horizons with, and without, debt consolidation and a Ramsey steady state

<table>
<thead>
<tr>
<th></th>
<th>2 periods</th>
<th>4 periods</th>
<th>10 periods</th>
<th>20 periods</th>
<th>30 periods</th>
</tr>
</thead>
<tbody>
<tr>
<td>with consolid.</td>
<td>1.7351</td>
<td>3.3455</td>
<td>8.2102</td>
<td>16.6584</td>
<td>24.6462</td>
</tr>
<tr>
<td>without consolid.</td>
<td>(1.6251)</td>
<td>(3.1791)</td>
<td>(7.4481)</td>
<td>(13.4127)</td>
<td>(18.1904)</td>
</tr>
<tr>
<td>welfare gain/loss</td>
<td>0.0378</td>
<td>0.0346</td>
<td>0.0761</td>
<td>0.1888</td>
<td>0.2789</td>
</tr>
</tbody>
</table>

Notes: As in Table 8.

### Table 10: Optimal fiscal reaction to debt and output with debt consolidation and when the premium depends on foreign debt

<table>
<thead>
<tr>
<th>fiscal instruments</th>
<th>optimal reaction to debt</th>
<th>optimal reaction to output</th>
</tr>
</thead>
<tbody>
<tr>
<td>$s^f_t$</td>
<td>$\gamma^f_t = 0.5322$</td>
<td>$\gamma^y_y = 0.0005$</td>
</tr>
<tr>
<td>$\tau^c_t$</td>
<td>$\gamma^c_t = 0$</td>
<td>$\gamma^c_y = 0.2912$</td>
</tr>
<tr>
<td>$\tau^k_t$</td>
<td>$\gamma^k_t = 0$</td>
<td>$\gamma^k_y = 2.9810$</td>
</tr>
<tr>
<td>$\tau^n_t$</td>
<td>$\gamma^n_t = 0.283$</td>
<td>$\gamma^n_y = 2.9578$</td>
</tr>
</tbody>
</table>

Notes: (i)-(iii) as in Table 4. (iv) $V_0 = 79.9909$.

---

34 The values of the optimized feedback policy coefficients without debt consolidation are as in Table 6.
Table 11: Welfare over different time horizons with, and without, debt consolidation and when the premium depends on foreign debt

<table>
<thead>
<tr>
<th></th>
<th>2 periods</th>
<th>4 periods</th>
<th>10 periods</th>
<th>20 periods</th>
<th>30 periods</th>
</tr>
</thead>
<tbody>
<tr>
<td>with consolid.</td>
<td>1.3667</td>
<td>2.8256</td>
<td>7.4659</td>
<td>15.1503</td>
<td>22.1528</td>
</tr>
<tr>
<td>without consolid</td>
<td>(1.6251)</td>
<td>(3.1793)</td>
<td>(7.4468)</td>
<td>(13.4065)</td>
<td>(18.1785)</td>
</tr>
<tr>
<td>welfare gain/loss</td>
<td>-0.0834</td>
<td>-0.0698</td>
<td>0.0018</td>
<td>0.0974</td>
<td>0.1635</td>
</tr>
</tbody>
</table>

Notes: As in Table 10.

Table 12: Optimal reaction to inflation, depreciation, debt and output with debt consolidation (optimal monetary and fiscal policy mix) under flexible exchange rates

<table>
<thead>
<tr>
<th>Monetary Policy</th>
<th>Fiscal Policy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Instrument</td>
<td>optimal reaction to inflation, output and depreciation</td>
</tr>
<tr>
<td>$R_t$</td>
<td>$\phi_x = 1.36$</td>
</tr>
<tr>
<td>$\tau^l_t$</td>
<td>$\tau^l_k$</td>
</tr>
<tr>
<td>$\tau^l_n$</td>
<td>$\gamma^l_n = 0.0744$</td>
</tr>
</tbody>
</table>

Notes: (i)-(iii) as in Table 4. (iv) $V_0 = 79.9954$.

\[ ^{35} \text{In the reference case without debt consolidation, the optimal feedbacks are } \gamma^g_l = 0.3303, \gamma^g_y = 0.0033, \gamma^k_l = 0.7693, \gamma^k_i = 0.2089, \gamma^k_y = 0.3042, \gamma^g_k = 0.4519, \gamma^k_y = 0.0041, \gamma^k_n = 0.0045. \]
Table 13: Welfare over different time horizons with, and without, debt consolidation under flexible exchange rates

<table>
<thead>
<tr>
<th></th>
<th>2 periods</th>
<th>4 periods</th>
<th>10 periods</th>
<th>20 periods</th>
<th>30 periods</th>
</tr>
</thead>
<tbody>
<tr>
<td>with consolid.</td>
<td>1.3482</td>
<td>2.7976</td>
<td>7.4448</td>
<td>15.1411</td>
<td>22.1486</td>
</tr>
<tr>
<td>without consolid.</td>
<td>(1.6247)</td>
<td>(3.1786)</td>
<td>(7.4451)</td>
<td>(13.4043)</td>
<td>(18.1768)</td>
</tr>
<tr>
<td>welfare gain/loss</td>
<td>-0.0890</td>
<td>-0.0750</td>
<td>-0.00005</td>
<td>0.0969</td>
<td>0.1634</td>
</tr>
</tbody>
</table>

Notes: As in Table 12.

10 Figures

Figure 1: IRFs under debt consolidation with the optimal fiscal mix

Notes: IRFs are in levels and converge to the reformed steady-state, while the solid horizontal line indicates the point of departure (status-quo value).

36 In the case without debt consolidation, the optimal feedbacks are $\phi_s = 1.3972$, $\phi_c = 0$, $\phi_k = 0.0002$, $\gamma^g = 0.2515$, $\gamma^c = 0.0011$, $\gamma^k = 0.6417$, $\gamma^n = 0.3298$, $\gamma^g = 0.3572$, $\gamma^c = 0.3132$, $\gamma^k = 0.0169$, $\gamma^n = 0.0001$. 
Appendix to "Fiscal consolidation in an open economy with sovereign premia and without monetary policy independence"

Apostolis Philippopoulos *
(Athens University of Economics and Business, and CESifo)

Petros Varthalitis
(University of Glasgow)

Vanghelis Vassilatos
(Athens University of Economics and Business)

December 25, 2016

1 Appendix A: Households

This Appendix presents and solves the problem of the household in some detail. There are $i = 1, 2, \ldots, N$ identical domestic households who act competitively.

1.1 Household’s problem

Each household $i$ maximizes expected lifetime utility given by:

$$E_0 \sum_{t=0}^{\infty} \beta^t U(c_{i,t}, n_{i,t}, g_t)$$

where $c_{i,t}$ is $i$’s consumption bundle, $n_{i,t}$ is $i$’s hours of work, $g_t$ is per capita public spending, $0 < \beta < 1$ is the time preference rate and $E_0$ is the rational expectations operator conditional on the information set.

The consumption bundle, $c_{i,t}$, is defined as:

---

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\[ c_{i,t} = \left( \frac{c_{i,t}^H}{\nu} \left( \frac{c_{i,t}^F}{c_{i,t}^H} \right)^{1-\nu} \right)^{\frac{1}{\nu}} \]  

(2)

where \( c_{i,t}^H \) and \( c_{i,t}^F \) denote the composite domestic good and the composite foreign good respectively, and \( \nu \) is the degree of preference for the domestic good.

We define the two composites, \( c_{i,t}^H \) and \( c_{i,t}^F \), as:

\[ c_{i,t}^H = \sum_{h=1}^{N} \left[ c_{i,t}^H(h) \right]^\frac{\phi}{\phi-1} \]  

(3)

\[ c_{i,t}^F = \sum_{f=1}^{N} \left[ c_{i,t}^F(f) \right]^\frac{\phi}{\phi-1} \]  

(4)

where \( \phi > 0 \) is the elasticity of substitution across goods produced in the domestic country.

The period budget constraint of each \( i \) expressed in real terms is (see e.g. Benigno and Thoenissen, 2008, for similar modeling):

\[
(1 + \tau_t^i) \left[ \frac{P_t^H}{P_t} c_{i,t}^H + \frac{P_t^F}{P_t} c_{i,t}^F \right] + \frac{P_t^H}{P_t} x_{i,t} + b_{i,t} + \frac{S_t P_t^* f_{i,t}^h + \phi^h}{2} \left( \frac{S_t P_t^* f_{i,t}^h - SP^*}{P} f^h \right)^2 \\
= \left( 1 - \tau_t^k \right) \left[ r_t^k \frac{P_t^H}{P_t} k_{i,t-1} + \bar{\omega}_{i,t} \right] + (1 - \tau_t^n) w_t n_{i,t} + R_{t-1} \frac{P_t}{P_t} b_{i,t-1} + \\
+ Q_{t-1} \frac{S_t P_t^*}{P_t} P_{t-1} f_{i,t-1}^h - \tau_{i,t}^l
\]

(5)

where \( P_t \) is the consumer price index (CPI), \( P_t^H \) is the price index of home tradables, \( P_t^F \) is the price index of foreign tradables (expressed in domestic currency), \( x_{i,t} \) is \( i \)'s domestic investment, \( b_{i,t} \) is \( i \)'s end-of-period real domestic government bonds, \( f_{i,t}^h \) is \( i \)'s end-of-period real internationally traded assets denominated in foreign currency, \( r_t^k \) is the real return to inherited domestic capital, \( k_{i,t-1} \), \( \bar{\omega}_{i,t} \) is \( i \)'s real dividends received by domestic firms, \( w_t \) is the real wage rate, \( R_{t-1} \geq 1 \) is the gross nominal return to domestic government bonds between \( t-1 \) and \( t \), \( Q_{t-1} \geq 1 \) is the gross nominal return to international assets between \( t-1 \) and \( t \), \( \tau_{i,t}^l \) are real lump-sum taxes if positive (or transfers if negative) to each household, and \( \tau_t^c, \tau_t^k, \tau_t^n \) are tax rates on consumption, capital income and labour income respectively. The parameter \( \phi^h \geq 0 \) captures adjustment costs related to private foreign assets, where variables without time subscripts denote steady state values.

Each household \( i \) also faces the budget constraints:
\[ P_t c_{i,t} = P_t^H c_{i,t}^H + P_t^F c_{i,t}^F \]  \hspace{1cm} (6)

\[ P_t^H c_{i,t}^H = \sum_{h=1}^{N} P_t^H(h) c_{i,t}^H(h) \]  \hspace{1cm} (7)

\[ P_t^F c_{i,t}^F = \sum_{f=1}^{N} P_t^F(f) c_{i,t}^F(f) \]  \hspace{1cm} (8)

where \( P_t^H(h) \) is the price of each variety \( h \) produced at home and \( P_t^F(f) \) is the price of each variety \( f \) produced abroad (expressed in domestic currency).

Finally, the law of motion of physical capital for household \( i \) is:

\[ k_{i,t} = (1 - \delta) k_{i,t-1} + x_{i,t} - \frac{\xi}{2} \left( \frac{k_{i,t}}{k_{i,t-1}} - 1 \right)^2 k_{i,t-1} \]  \hspace{1cm} (9)

where \( 0 < \delta < 1 \) is the depreciation rate of capital and \( \xi \geq 0 \) is a parameter capturing adjustment costs related to physical capital.

For our numerical solutions, we use the usual functional form:

\[ u_{i,t}(c_{i,t}, n_{i,t}, m_{i,t}, g_t) = \frac{c_{i,t}^{1-\sigma}}{1 - \sigma} - \chi_n \frac{n_{i,t}^{1+\eta}}{1 + \eta} + \chi_g \frac{g_t^{1-\zeta}}{1 - \zeta} \]  \hspace{1cm} (10)

where \( \chi_n, \chi_g, \sigma, \eta, \zeta \) are preference parameters.

### 1.2 Household’s optimality conditions

Each household \( i \) acts competitively taking prices and policy as given. Following the literature, to solve the household’s problem, we follow a two-step procedure. We first suppose that the household determines its desired consumption of composite goods, \( c_{i,t}^H \) and \( c_{i,t}^F \), and, in turn, chooses how to distribute its purchases of individual varieties, \( c_{i,t}^H(h) \) and \( c_{i,t}^F(f) \).

The first-order conditions of each \( i \) include the budget constraints written above and also:

\[ \frac{\partial u_{i,t}}{\partial c_{i,t}} = \frac{\partial u_{i,t+1}}{\partial c_{i,t+1}} \frac{P_t}{P_t^H(1+\tau^H_t)} \]  \hspace{1cm} (11)

\[ = \beta E_t \frac{\partial u_{i,t+1}}{\partial c_{i,t+1}} \frac{\partial c_{i,t+1}^H}{\partial c_{i,t+1}^H} \frac{P_{t+1}}{P_{t+1}^H(1+\tau_{t+1}^H)} R_t \frac{P_t}{P_{t+1}} \]  \hspace{1cm} (12)
\[
\frac{\partial u_{it}}{\partial c_{i,t}} \frac{\partial c_{i,t}}{\partial c_{i,t}} \frac{1}{1 + \tau_{t+1}} \left\{ \frac{1 + \xi \left( \frac{k_{i,t}}{k_{i,t-1}} - 1 \right)}{(1 - \delta) - \frac{\xi}{2} \left( \frac{k_{i,t+1}}{k_{i,t}} - 1 \right)^2 + \xi \left( \frac{k_{i,t+1}}{k_{i,t}} - 1 \right) \frac{k_{i,t+1}}{k_{i,t}}} \right\} = \beta E_t \frac{\partial u_{it+1}}{\partial c_{i,t+1}} \frac{\partial c_{i,t+1}}{\partial c_{i,t+1}} \frac{1}{1 + \tau_{t+1}} \left\{ \frac{1 + \xi \left( \frac{k_{i,t+1}}{k_{i,t-1}} - 1 \right)}{(1 - \delta) - \frac{\xi}{2} \left( \frac{k_{i,t+1}}{k_{i,t}} - 1 \right)^2 + \xi \left( \frac{k_{i,t+1}}{k_{i,t}} - 1 \right) \frac{k_{i,t+1}}{k_{i,t}}} \right\} 
\]

(13)

\[
\frac{\partial u_{it}}{\partial n_{i,t}} = \frac{\partial u_{it}}{\partial c_{i,t}} \frac{\partial c_{i,t}}{\partial c_{i,t}} \frac{P_t}{1 - \nu} \frac{P_t}{1 + \tau_t^k} (1 - \tau_t^n) w_t
\]

(14)

\[
\frac{c_{i,t}^H}{c_{i,t}^F} = \frac{\nu}{1 - \nu} \frac{P_t^F}{P_t^H}
\]

(15)

\[
c_{i,t}^H(h) = \left[ \frac{P_t^H(h)}{P_t^H} \right]^{-\phi} c_{i,t}^H
\]

(16)

\[
c_{i,t}^F(f) = \left[ \frac{P_t^F(f)}{P_t^F} \right]^{-\phi} c_{i,t}^F
\]

(17)

Equations (11)-(13) are respectively the Euler equations for domestic bonds, foreign assets and domestic capital and (14) is the optimality condition for work hours. Finally, (15) shows the optimal allocation between domestic and foreign goods, while (16) and (17) show the optimal demand for each variety of domestic and foreign goods respectively; these conditions are used in the firm’s optimization problem below.

1.3 Implications for price bundles

Equations (15), (16) and (17), combined with the household’s budget constraints, imply that the three price indexes are:

\[
P_t = (P_t^H)^\nu (P_t^F)^{1-\nu}
\]

(18)

\[
P_t^H = \left[ \sum_{h=1}^{N} [P_t^H(h)]^{1-\phi} \right]^{\frac{1}{1-\phi}}
\]

(19)

\[
P_t^F = \left[ \sum_{f=1}^{N} [P_t^F(f)]^{1-\phi} \right]^{\frac{1}{1-\phi}}
\]

(20)

2 Appendix B: Firms

This Appendix presents and solves the problem of the firm in some detail. There are \( h = 1, 2, \ldots, N \) domestic firms. Each firm \( h \) produces a differentiated good of variety \( h \) under monopolistic competition in its own product market facing Calvo-type nominal price fixities.
2.1 Demand for the firm’s product

Demand for firm $h$’s product, $y_t^H(h)$, comes from domestic households’ consumption and investment, $C_t^H(h)$ and $X_t(h)$, where $C_t^H(h) \equiv \sum_{i=1}^{N} c_{i,t}^H(h)$ and $X_t(h) \equiv \sum_{i=1}^{N} x_{i,t}(h)$, from the domestic government, $G_t(h)$, and from foreign households’ consumption, $C_t^{F*}(h) \equiv \sum_{i=1}^{N^*} c_{i,t}^{F*}(h)$. Thus, the demand for each domestic firm’s product is:

$$y_t^H(h) = C_t^H(h) + X_t(h) + G_t(h) + C_t^{F*}(h) \quad (21)$$

Using demand functions like those derived by solving the household’s problem above, we have:

$$c_{i,t}^H(h) = \left[ \frac{P_t^H(h)}{p_t^H} \right]^{-\phi} c_{i,t}^H \quad (22)$$

$$x_{i,t}(h) = \left[ \frac{P_t^H(h)}{p_t^H} \right]^{-\phi} x_{i,t} \quad (23)$$

$$G_t(h) = \left[ \frac{P_t^H(h)}{p_t^H} \right]^{-\phi} G_t \quad (24)$$

$$c_{i,t}^{F*}(h) = \left[ \frac{P_t^{F*}(h)}{p_t^{F*}} \right]^{-\phi} c_{i,t}^{F*} \quad (25)$$

where, using the law of one price, we have in (25):

$$\frac{P_t^{F*}(h)}{p_t^{F*}} = \frac{P_t^H(h)}{s_t} = \frac{P_t^H(h)}{p_t^H} \quad (26)$$

Since, at the economy level, aggregate demand is:

$$Y_t^H = C_t^H + X_t + G_t + C_t^{F*} \quad (27)$$

the above equations imply that the demand for each firm’s product is:

$$y_t^H(h) = \left[ \frac{P_t^H(h)}{p_t^H} \right]^{-\phi} Y_t^H \quad (28)$$

2.2 The firm’s problem

The real profit of each firm $h$ is (see e.g. Benigno and Thoenissen, 2008, for similar modeling):
\[ \bar{\omega}_t(h) = \frac{P_t^H(h)}{P_t} y_t^H(h) - \frac{P_t^H}{P_t} r_t^k k_{t-1}(h) - w_t n_t(h) \]  

(29)

The production technology is:

\[ y_t^H(h) = A_t[k_{t-1}(h)]^\alpha [n_t(h)]^{1-\alpha} \]  

(30)

where \( A_t \) is an exogenous stochastic TFP process whose motion is defined below.

Profit maximization is also subject to the demand for its product as derived above:

\[ y_t^H(h) = \left[ \frac{P_t^H(h)}{P_t^H} \right]^{-\phi} Y_t^H \]  

(31)

In addition, firms choose their prices facing a Calvo-type nominal fixity. In each period, firm \( h \) faces an exogenous probability \( \theta \) of not being able to reset its price. A firm \( h \), which is able to reset its price, chooses its price \( P_t^#(h) \) to maximize the sum of discounted expected nominal profits for the next \( k \) periods in which it may have to keep its price fixed. This is modeled next.

### 2.3 Firm’s optimality conditions

To solve the firm’s problem, we follow a two-step procedure. We first solve a cost minimization problem, where each firm \( h \) minimizes its cost by choosing factor inputs given technology and prices. The solution will give a minimum nominal cost function, which is a function of factor prices and output produced by the firm. In turn, given this cost function, each firm, which is able to reset its price, solves a maximization problem by choosing its price.

The solution to the cost minimization problem gives the input demand functions:

\[ w_t = m c_t(h)(1 - a) \frac{y_t(h)}{n_t(h)} \]  

(32)

\[ \frac{P_t^H}{P_t} r_t^k = m c_t(h) a \frac{y_t(h)}{k_{t-1}(h)} \]  

(33)

where \( m c_t(h) = \frac{\psi'_t(h)}{P_t} \) denotes the real marginal cost.

Then, the firm chooses its price to maximize nominal profits written as (see also e.g.
Rabanal, 2009, for similar modeling):

\[ E_t \sum_{k=0}^{\infty} \theta^k \Xi_{t,t+k} \left\{ P_t^\#(h) y_t^H(h) - \Psi_{t+k} \left( y_{t+k}^H(h) \right) \right\} \]

where \( \Xi_{t,t+k} \) is a discount factor taken as given by the firm, \( y_t^H(h) = \left[ \frac{P_t^\#(h)}{P_{t+k}^H} \right]^{-\phi} y_{t+k}^H \) and \( \Psi_t(h) \) denotes the minimum nominal cost function for producing \( y_t^H(h) \) at \( t \) so that \( \Psi_t^H(h) \) is the associated marginal cost.

The first-order condition gives:

\[ E_t \sum_{k=0}^{\infty} \theta^k \Xi_{t,t+k} \left[ \frac{P_t^\#(h)}{P_{t+k}^H} \right]^{-\phi} \left\{ P_t^\#(h) \left( \frac{P_t^\#(h)}{P_{t+k}^H} \right) - \frac{\phi}{\phi-1} \Psi_{t+k}^H(h) \right\} = 0 \quad (34) \]

Dividing by \( P_{t+k}^H \), we have:

\[ E_t \sum_{k=0}^{\infty} \theta^k \Xi_{t,t+k} \left[ \frac{P_t^\#(h)}{P_{t+k}^H} \right]^{-\phi} \left\{ \frac{P_t^\#(h)}{P_{t+k}^H} - \frac{\phi}{\phi-1} m_{t+k}^H \left( \frac{P_{t+k}^H}{P_t^H} \right) \right\} = 0 \quad (35) \]

Summing up, the behaviour of each firm \( h \) is summarized by (32), (33) and (35). A recursive expression of the equation above is presented below.

Notice, finally, that each firm \( h \), which can reset its price in period \( t \), solves an identical problem, so \( P_t^\#(h) = P_t^\# \) is independent of \( h \), and each firm \( h \), which cannot reset its price, just sets its previous period price \( P_t^H(h) = P_{t-1}^H(h) \). Thus, the evolution of the aggregate price level is given by:

\[ (P_t^H)^{1-\phi} = \theta (P_{t-1}^H)^{1-\phi} + (1-\theta) \left( P_t^\# \right)^{1-\phi} \quad (36) \]

### 3 Appendix C: Government budget constraint

This Appendix presents in some detail the government budget constraint and the menu of fiscal policy instruments. Recall that we have assumed a cashless economy for simplicity.

Then, the period budget constraint of the government written in aggregate and nominal quantities is:

\[ B_t + S_t F_t^\theta = P_t^\# \frac{S_t F_t^\theta}{P_t^H} - \frac{S_{t-1} F_{t-1}^\theta}{P_{t-1}^H} + R_{t-1} B_{t-1} + Q_{t-1} S_{t-1} F_{t-1}^\theta + P_t^H G_t - \tau^t_t (P_t^H C_t^H + P_t^F C_t^F) - \tau^t_t (\tilde{r}_t^k P_t^H K_{t-1} + P_t^\tilde{\Omega}_t) - \tau^t_t W_t \tilde{N}_t - T^\theta_t \quad (37) \]

where \( B_t \) is the end-of-period nominal domestic public debt and \( F_t^\theta \) is the end-of-period nominal
foreign public debt expressed in foreign currency so it is multiplied by the exchange rate, $S_t$.

The parameter $\phi^g \geq 0$ captures adjustment costs related to public foreign debt. The rest of the notation is as above. Note that $B_t = \sum_{i=1}^{N} B_{i,t}$, $C_t^H = \sum_{i=1}^{N} c_{i,t}^H$, $C_t^F = \sum_{i=1}^{N} c_{i,t}^F$, $K_{t-1} = \sum_{i=1}^{N} k_{i,t-1}$, $\Omega_t = \sum_{i=1}^{N} \omega_{i,t}$, $\bar{N}_t = \sum_{i=1}^{N} n_{i,t}$ and $T_t^l$ denotes the nominal value of lump-sum taxes.

Let $D_t = B_t + S_t F_t^g$ denote the total nominal public debt at the end of the period. This can be held both by domestic private agents, $\lambda_t D_t$, and by foreign private agents, $S_t F_t^g \equiv (1 - \lambda_t) D_t$, where $0 \leq \lambda_t \leq 1$. Then, dividing by the price level and the number of households, the budget constraint in real and per capita terms is:

$$d_t = \frac{\phi^g}{T} [(1 - \lambda_t) d_t - (1 - \lambda) d_t^2] + R_{t-1} \frac{P_{t-1}}{P_t} \lambda_{t-1} d_{t-1} + Q_t - S_t P^* \frac{P_{t+1}}{P_t} \frac{P_{t+1}}{P_{t-1}} \frac{P_{t+1}}{P_t} (1 - \lambda_{t-1}) d_{t-1}$$

$$+ \frac{P^H}{P_t} g_t - \tau_t^c (\frac{P^H}{P_t} c^H + \frac{P^F}{P_t} c^F) - \tau_t^k (r_t^k \frac{P^H}{P_t} D_{t-1} + \bar{\omega}_t) - \tau_t^m w_t n_t - \tau_t^l$$

(38)

where, as said in the main text, in each period, one of the policy instruments $(\tau_t^c, \tau_t^k, \tau_t^m, g_t, \tau_t^l, \lambda_t, d_t)$ follows residually to satisfy the government budget constraint.

4 Appendix D: Decentralized equilibrium (DE)

This Appendix presents in some detail the DE system. Following the related literature, we work in steps.

4.1 Equilibrium equations

The DE is summarized by the following equations (quantities are in per capita terms):

$$\frac{\partial u_t}{\partial c_t} \frac{\partial c_t}{\partial c_t^H} \frac{1}{(1 + \tau_t^c) \frac{P_t}{P_t^H}} = \beta E_t \frac{\partial u_{t+1}}{\partial c_{t+1}} \frac{\partial c_{t+1}}{\partial c_{t+1}^H} \frac{1}{(1 + \tau_{t+1}^c) \frac{P_{t+1}}{P_{t+1}^H}} \frac{P_{t+1}}{P_{t+1}}$$

(39)

$$= \beta \frac{\partial u_{t+1}}{\partial c_{t+1}} \frac{\partial c_{t+1}}{\partial c_{t+1}^H} \frac{1}{(1 + \tau_{t+1}^c) \frac{P_{t+1}}{P_{t+1}^H}} \frac{S_{t+1} P_{t+1}^*}{S_{t+1} P_{t+1}^*} \frac{P_{t+1}^*}{P_{t+1}^*}$$

(40)

$$= \beta E_t \frac{\partial u_{t+1}}{\partial c_{t+1}} \frac{\partial c_{t+1}}{\partial c_{t+1}^H} \frac{1}{(1 + \tau_{t+1}^c) \frac{P_{t+1}}{P_{t+1}^H}} \frac{1}{(1 - \delta) - \frac{\xi}{2} \left( \frac{k_{t+1}}{k_t} - 1 \right)^2}$$

(41)

$$= \beta E_t \frac{\partial u_{t+1}}{\partial c_{t+1}} \frac{\partial c_{t+1}}{\partial c_{t+1}^H} \frac{1}{(1 + \tau_{t+1}^c) \frac{P_{t+1}}{P_{t+1}^H}} \left\{ (1 - \delta) - \frac{\xi}{2} \left( \frac{k_{t+1}}{k_t} - 1 \right)^2 \right\}$$

$$- \frac{\partial u_t}{\partial n_t} = \frac{\partial u_t}{\partial c_t} \frac{\partial c_t}{\partial c_t^H} \frac{P_t}{P_t^H} (1 + \tau_t^c) w_t$$

(42)
\begin{align*}
\frac{c_t^H}{c_t^F} &= \frac{\nu}{1 - \nu} \frac{P_t^F}{P_t^H} \\
k_t &= (1 - \delta)k_{t-1} + x_t - \frac{\xi}{2} \left( \frac{k_t}{k_{t-1}} - 1 \right)^2 k_{t-1} \\
c_t &\equiv \left( c_t^H \right)^\nu \left( c_t^F \right)^{1-\nu} \\
w_t &= mc_t(1 - a)A_tA_t^{a-1}n_t^{-a} \\
\frac{P_t^H}{P_t} \tau_t^k &= mc_tA_tA_t^{a-1}n_t^{-a} \\
\tilde{w}_t &= \frac{P_t^H}{P_t} y_t^H - \frac{P_t^H}{P_t} \tau_t^k k_{t-1} - w_t n_t \\
E_t \sum_{k=0}^{\infty} \theta^k \left[ \xi_{t+k} \right] \left[ \frac{P_{t+k}^H}{P_t^H} \right]^{-\phi} y_{t+k}^H \left\{ \frac{P_{t+k}^H}{P_t} - \frac{\phi}{\phi - 1} mc_{t+k} \frac{P_{t+k}^H}{P_t} \right\} &= 0 \\
y_t^H &= \frac{1}{\left[ \frac{P_t^H}{P_t^H} \right]^{-\phi}} A_tA_t^{a-1}n_t^{-a} \\
d_t &= R_{t-1} \frac{P_{t-1}^H}{P_{t-1}} A_{t-1} d_{t-1} + Q_{t-1} \frac{S_{t-1}^*}{P_{t-1}} \frac{P_{t-1}^H}{P_{t-1}} A_{t-1} \frac{P_{t-1}^H}{P_{t-1}} (1 - \lambda_{t-1}) d_{t-1} + \frac{P_t^H}{P_t} g_t \\
-\tau_t^c \left( \frac{P_t^H}{P_t} c_t^H + \frac{P_t^F}{P_t} c_t^F \right) - \tau_t^k \left( \frac{P_t^H}{P_t} \tau_t^k k_{t-1} + \tilde{w}_t \right) - \tau_t^n w_t n_t - \tau_t^l + \frac{\phi^l}{2} [(1 - \lambda_t) d_t - (1 - \lambda) d_t]^2 \\
y_t^H &= c_t^H + x_t + g_t + c_t^F \\
-\frac{P_t^H}{P_t} c_t^F + \frac{P_t^F}{P_t} c_t^F + \frac{\phi^F}{2} [(1 - \lambda_t) d_t - (1 - \lambda) d_t]^2 + \frac{\phi^P}{2} \left( \frac{S_{t-1}^*}{P_{t-1}} f_{t-1}^h - \frac{S_t^*}{P_t} f_{t-1}^h \right)^2 \\
Q_{t-1} \frac{S_{t-1}^*}{P_{t-1}} \frac{P_{t-1}^H}{P_{t-1}} \left( \frac{(1 - \lambda_{t-1}) d_{t-1}}{S_{t-1}^* f_{t-1}^h} - f_{t-1}^h \right) &= (1 - \lambda_t) d_t - \frac{S_t^*}{P_t} f_{t-1}^h \\
(P_t^H)^{1-\phi} &= \theta P_{t-1}^{1-\phi} + (1 - \theta) \left( P_{t-1}^H \right)^{1-\phi} \\
P_t &= (P_t^H)^\nu \left( P_t^F \right)^{1-\nu} \\
P_t^F &= S_t P_t^H \\
P_t^H &= (P_t^H)^\nu \left( \frac{P_t^H}{S_t} \right)^{1-\nu^*} \\
(P_t^H)^{-\phi} &= \theta \left( P_{t-1}^H \right)^{-\phi} + (1 - \theta) \left( P_{t-1}^H \right)^{-\phi}
\end{align*}


$$Q_t = Q_t^* + \psi \left( \frac{d_t}{P_t^H} \frac{1}{\sqrt{T_t}} - \frac{d}{3} \right)$$

(59)

$$l_t \equiv \frac{R_t \lambda_t d_t + Q_t \frac{S_{t+1}}{S_t} (1 - \lambda_t) d_t}{P_t^H \frac{y_t}{P_t^H}}$$

(60)

where $f_t^g = \frac{(1-\lambda_t) d_t}{y_t^H}$, $\Xi_{t+k} \equiv \beta^k \frac{e_{t+k}}{c_t} \frac{P_{t+k}}{P_t^H} \frac{\tau_t^H}{\tau_{t+k}}$, $y_t^H = \left[ \frac{N}{h=1} \left( y_t^H(h) \frac{d_t}{\phi} \right)^{\frac{1}{\phi}} \right]^{-1}$ and $\widetilde{P}_t^H = \left( \frac{\sum_{h=1}^N [P_t(h)]^{-\phi}}{\phi} \right)^{\frac{1}{\phi}}$.

Thus, $(\frac{\widetilde{P}_t^H}{P_t^H})^{-\phi}$ is a measure of price dispersion.

We thus have 22 equations in 22 variables, $\{y_t^H, c_t, c_t^H, c_t^F, n_t, x_t, k_t, f_t^H, P_t^F, P_t, P_t^H, P_t^\#, \widetilde{P}_t^H, \omega_t, m_t, \omega_t, \tau_t^H, Q_t, d_t, P_t^*, R_t, l_t\}_{t=0}^\infty$. This is given the independently set monetary and fiscal policy instruments, $\{S_t, \tau_t^H, \tau_t^F, \gamma_t, \tau_t, \lambda_t\}_{t=0}^\infty$, the rest-of-the-world variables, $\{c_t^F, P_t^*, P_t^{H*}\}_{t=0}^\infty$, technology, $\{A_t\}_{t=0}^\infty$, and initial conditions for the state variables.

In what follows, we shall transform the above equilibrium conditions. In particular, following the related literature, we rewrite them, first, by expressing price levels in inflation rates, secondly, by writing the firm’s optimality conditions in recursive form and, thirdly, by introducing a new equation that helps us to compute expected discounted lifetime utility. Finally, we will present the final transformed system that is solved numerically.

### 4.2 Variables expressed in ratios

We first express prices in rate form. We define 7 new variables, which are the gross domestic CPI inflation rate $\Pi_t \equiv \frac{P_t}{P_{t-1}}$, the gross foreign CPI inflation rate $\Pi_t^* \equiv \frac{P_t^*}{P_{t-1}}$, the gross domestic goods inflation rate $\Pi_t^H \equiv \frac{P_t^H}{P_{t-1}}$, the auxiliary variable $\Theta_t \equiv \frac{r_t^H}{P_{t-1}^H}$, the price dispersion index $\Delta_t \equiv \left[ \frac{P_t^H}{P_{t-1}^H} \right]^{-\phi}$, the gross rate of exchange rate depreciation, $\epsilon_t \equiv \frac{S_t}{S_{t-1}}$ and the terms of trade $TT_t \equiv \frac{P_t^F}{P_t^H} = \frac{S_t P_t^H}{S_{t-1}}$.\(^1\) In what follows, we use $\Pi_t$, $\Pi_t^*$, $\Pi_t^H$, $\Theta_t$, $\Delta_t$, $\epsilon_t$, $TT_t$ instead of $P_t$, $P_t^*$, $P_t^H$, $P_t^\#$, $\widetilde{P}_t$, $S_t$, $P_t$ respectively.

Also, for convenience and comparison with the data, we express fiscal and public finance variables as shares of nominal output, $P_t H y_t^H$. In particular, using the definitions above, real government spending, $g_t$, can be written as $g_t = s_t^g y_t^H$, while real government transfers, $\tau_t^l$, can be written as $\tau_t^l = s_t^l y_t^H T T_t^\nu - 1$, where $s_t^g$ and $s_t^l$ denote respectively the output shares of government spending and government transfers.

\(^1\)Thus, $\frac{TT_t}{TT_{t-1}} = \frac{\frac{S_t P_t^H}{S_{t-1}}}{\frac{S_{t-1} P_{t-1}^H}{S_{t-1}}} = \frac{e_t}{\epsilon_{t-1}}$.  

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4.3 Equation (49) expressed in recursive form

We now replace equation (49), from the firm’s optimization problem, with an equivalent equation in recursive form. In particular, following Schmitt-Grohé and Uribe (2007), we look for a recursive representation of the form:

\[ E_t \sum_{k=0}^{\infty} \theta^k z_{t+k} \left[ \frac{P_t^H}{P_{t+k}} \right]^{-\phi} y_{t+k}^{-\phi} \left( P_t^H - \frac{\phi}{(\phi - 1)} mc_{t+k} P_{t+k} \right) = 0 \]  

We define two auxiliary endogenous variables:

\[ z_1^1 = E_t \sum_{k=0}^{\infty} \theta^k z_{t+k} \left[ \frac{P_t^H}{P_{t+k}} \right]^{-\phi} y_{t+k}^{-\phi} \left( \frac{P_t^H}{P_{t+k}} \right) \]  

\[ z_2^1 = E_t \sum_{k=0}^{\infty} \theta^k z_{t+k} \left[ \frac{P_t^H}{P_{t+k}} \right]^{-\phi} y_{t+k}^{-\phi} \left( \frac{P_t^H}{P_{t+k}} \right) \]

Using these two auxiliary variables, \( z_1^1 \) and \( z_2^1 \), as well as equation (61), we come up with two new equations which enter the dynamic system and allow a recursive representation of (61).

Thus, in what follows, we replace equation (49) with:

\[ z_1^1 = \frac{\phi}{(\phi - 1)} z_2^1 \]  

where we also have the two new equations (and the two new endogenous variables, \( z_1^1 \) and \( z_2^1 \)):

\[ z_1^1 = \Theta_t^{-\phi} y_t TT_t^{1-\phi} + \beta \theta E_t \frac{c_{t+1}^{\sigma}}{c_t^{\sigma}} \frac{1 + \tau_{t+1}^c}{1 + \tau_{t+1}^c} \left( \frac{\Theta_t}{\Theta_{t+1}} \right)^{1-\phi} \left( \frac{1}{\Pi_{t+1}^H} \right)^{1-\phi} z_1^1_{t+1} \]  

\[ z_2^1 = \Theta_t^{-\phi} y_t mc_t + \beta \theta E_t \frac{c_{t+1}^{\sigma}}{c_t^{\sigma}} \frac{1 + \tau_{t+1}^c}{1 + \tau_{t+1}^c} \left( \frac{\Theta_t}{\Theta_{t+1}} \right)^{-\phi} \left( \frac{1}{\Pi_{t+1}^H} \right)^{-\phi} z_2^1_{t+1} \]

4.4 Lifetime utility written as a first-order difference equation

Since we want to compute social welfare, we follow Schmitt-Grohé and Uribe (2007) by defining a new endogenous variable, \( V_t \), whose motion is given by:

\[ V_t = \frac{c_t^{1-\sigma}}{1 - \sigma} - \chi_p \rho_t^{1+\eta} + \chi_s \left( s_t y_t^H \right)^{1-\zeta} - \zeta E_t V_{t+1} \]  

where \( V_t \) is household’s expected discounted lifetime utility at time \( t \).

Thus, in what follows, we add equation (67) and the new variable \( V_t \) to the equilibrium
system. Note that, in the welfare numerical solutions reported, we add a constant number, 2, to each period’s utility; this makes the welfare numbers easier to read.

4.5 Equilibrium equations transformed

Using the above, the transformed DE system is summarized by:

\[ V_t = \frac{c_t^{1-\sigma}}{1 - \sigma} - \chi_n n_t^{1+\eta} + \chi_g \left( s_t g_t H_t \right)^{1-\zeta} + \beta E_t V_{t+1} \]  
\[ (68) \]

\[ \beta E_t c_{t+1}^{1-\sigma} (1+\tau_{t+1})^{-1} R_t \frac{1}{n_{t+1}} = \]
\[ c_t^{1-\sigma} (1+\tau_t)^{-1} \]  
\[ (69) \]

\[ \beta E_t c_{t+1}^{1-\sigma} (1+\tau_{t+1})^{-1} Q_t T_t^{\nu-\nu+1} \frac{1}{n_{t+1}} = \]
\[ c_t^{1-\sigma} (1+\tau_t)^{-1} \left[ 1 + \phi^R \left( T_{t+1}^{\nu+1} - T_{t+1}^{\nu+1} f_t^h \right) \right] \]  
\[ (70) \]

\[ \beta c_{t+1}^{1-\sigma} T_{t+1}^{\nu-1} \frac{1}{(1+\tau_{t+1})} \left\{ 1 - \delta - \frac{\xi}{2} \left( \frac{k_{t+1}}{k_t} - 1 \right) \right\} + \xi \left( \frac{k_{t+1}}{k_t} - 1 \right) \frac{k_{t+1}}{k_t} + (1 - \tau_{t+1}) r_{t+1}^{1-\nu} \right\} = \]
\[ c_t^{1-\sigma} T_t^{\nu-1} \frac{1}{(1+\tau_t)} \left[ 1 + \xi \left( \frac{k_t}{k_{t+1}} - 1 \right) \right] \]  
\[ (71) \]

\[ \chi_n n_t^{\eta} = c_t^{1-\sigma} \frac{(1-\tau_t^\nu) w_t}{(1+\tau_t^\nu)} \]  
\[ (72) \]

\[ \frac{c_t^H}{c_t^F} = \frac{\nu}{1-\nu} T_t \]  
\[ (73) \]

\[ k_t = (1 - \delta) k_{t-1} + x_t - \frac{\xi}{2} \left( \frac{k_t}{k_{t-1}} - 1 \right)^2 k_{t-1} \]  
\[ (74) \]

\[ c_t \equiv \frac{\left( c_t^H \right)^\nu (c_t^F)^{1-\nu}}{\left( \nu \right)^\nu (1-\nu)^{1-\nu}} \]  
\[ (75) \]

\[ w_t = mc_t (1-a) A_t k_{t-1}^{a-1} n_t^{-a} \]  
\[ (76) \]

\[ \frac{1}{T_t^{1-\nu}} r_t^k = mc_t a A_t k_{t-1}^{a-1} n_t^{1-a} \]  
\[ (77) \]

\[ \tilde{w}_t = \frac{1}{T_t^{1-\nu}} H_t - \frac{1}{T_t^{1-\nu}} r_t^k k_{t-1} - w_t n_t \]  
\[ (78) \]

\[ z_t^1 = \frac{\phi}{(\phi-1)} z_t^2 \]  
\[ (79) \]
\[ y_t^H = \frac{1}{\Delta_t} A_t k_{t-1}^a n_t^{1-a} \]  
(80)

\[ d_t = \frac{\phi^g}{2} [(1 - \lambda_t) d_t - (1 - \lambda) d_t^2 + R_{t-1} \frac{1}{\Pi_t} \lambda_{t-1} d_{t-1} +
+ Q_{t-1} T T_t^{\nu_t-1} \frac{1}{\Pi_t} \frac{1}{TT_{t-1}^{\nu_t-1}} (1 - \lambda_{t-1}) d_{t-1} + TT_t^{\nu_t-1} s_t^g y_t^h -
- \tau_t^c (\frac{1}{TT_t^\nu} c_t^H + TT_t^\nu c_t^F) - \tau_t^k (\frac{1}{TT_t^\nu} k_{t-1} + \omega_t) - \tau_t^n w_t n_t - TT_t^{\nu_t-1} s_t^I y_t^H \]  
(81)

\[ y_t^H = c_t^H + x_t + s_t^g y_t^H + c_t^{F^*} \]  
(82)

\[ (1 - \lambda_t) d_t - TT_t^{\nu_t+\nu_t-1} f_t^h = -TT_t^{\nu_t-1} c_t^{F^*} + TT_t^\nu c_t^F +
+ Q_{t-1} TT_t^{\nu_t+\nu_t-1} \frac{1}{\Pi_t} \left( \frac{1}{TT_{t-1}^{\nu_t-1}} (1 - \lambda_{t-1}) d_{t-1} - f_t^h \right) \]  
(83)

\[ (\Pi_t^H)^{1-\phi} = \theta + (1 - \theta) (\Theta_t \Pi_t^H)^{1-\phi} \]  
(84)

\[ \frac{\Pi_t}{\Pi_t^H} = \left( \frac{TT_t}{TT_{t-1}} \right)^{1-\nu} \]  
(85)

\[ \frac{TT_t}{TT_{t-1}} = \frac{e_t \Pi_t^{H^*}}{\Pi_t^H} \]  
(86)

\[ \frac{\Pi_t^*}{\Pi_t^{H^*}} = \left( \frac{TT_{t-1}}{TT_t} \right)^{1-\nu^*} \]  
(87)

\[ \Delta_t = \theta \Delta_{t-1} (\Pi_t^H)^{\phi} + (1 - \theta) (\Theta_t)^{-\phi} \]  
(88)

\[ Q_t = Q_t^* + \psi \left( e^{\frac{d_t}{\Pi_t^{H^*}} - \frac{d_t}{\Pi_t^H}} - 1 \right) \]  
(89)

\[ z_t^1 = \Theta_t^{1-\phi} y_t T T_t^{\nu_t-1} + \beta \theta E_t \frac{c_t^{\nu_t-1}}{c_t^\nu} \frac{1 + \tau_t^c}{1 + \tau_t^{\nu_t+\nu_t-1}} \left( \frac{\Theta_t}{\Theta_{t+1}} \right)^{1-\phi} \left( \frac{1}{\Pi_{t+1}^H} \right)^{1-\phi} z_{t+1}^1 \]  
(90)

\[ z_t^2 = \Theta_t^{-\phi} y_t m c_t + \beta \theta E_t \frac{c_t^{\nu_t+\nu_t-1}}{c_t^\nu} \frac{1 + \tau_t^c}{1 + \tau_t^{\nu_t+\nu_t-1}} \left( \frac{\Theta_t}{\Theta_{t+1}} \right)^{-\phi} \left( \frac{1}{\Pi_{t+1}^H} \right)^{-\phi} z_{t+1}^2 \]  
(91)

\[ l_t = \frac{R_t s_t d_t + Q_t \epsilon_{t+1} + (1 - \lambda_t) d_t}{TT_t^{\nu_t-1} y_t^H} \]  
(92)

We thus have 25 equations in 25 variables, \{V_t, y_t^H, c_t, c_t^H, c_t^F, n_t, x_t, k_t, f_t^h, TT_t, \Pi_t, \Pi_t^H, \Theta_t, \Delta_t, w_t, m, c_t, \omega_t, r_t^F, Q_t, d_t, \Pi_t^*, z_t^1, z_t^2, R_t, l_t\}_{t=0}^\infty. This is given the independently set policy instruments, \{\epsilon_t, \tau_t^c, \tau_t^k, \tau_t^n, s_t^a, s_t^l, \lambda_t\}_{t=0}^\infty, the rest-of-the-world variables, \{c_t^{F^*}, Q_t^*, \Pi_t^{H^*}\}_{t=0}^\infty, technology, \{A_t\}_{t=0}^\infty, and initial conditions for the state variables.
4.6 Equations and unknowns (given feedback policy coefficients)

We can now define the final equilibrium system. It consists of the 25 equations of the transformed DE presented in the previous subsection, the 4 policy rules in subsection 2.7 in the main text, and the equation for domestic exports in subsection 2.8 in the main text. Thus, we have a system of 30 equations. By using 2 auxiliary variables, we transform it to a first-order,$^2$ so we end up with 32 equations in 32 variables, \{\(y^H_t, c_t, c^F_t, x_t, n_t, f^H_t, d_t, k_t, \bar{\omega}_t, mc_t, \Pi_t, \Pi^*_t, \Theta_t, \Delta_t, TT_t, w_t, r^k_t, Q_t, k_{\ell}, z_1^t, z_2^t, V_t, R_t; \tau^c_t, s^g_t, \tau^k_t, \tau^n_t; k\text{lead}_t, TT_lag_t, c^F_t\}_{t=0}^{\infty}$. This is given the exogenous variables, \{\(Q^*_t, \Pi^H_t, A_t, \epsilon_t, s_{1_t}^1, \lambda_t\)\}_{t=0}^{\infty}, and initial conditions for the state variables. The 32 endogenous variables are distinguished in 24 control variables, \{\(y^H_t, c_t, c^F_t, x_t, n_t, \bar{\omega}_t, mc_t, \Pi_t, \Pi^*_t, \Theta_t, TT_t, w_t, r^k_t, z_1^t, z_2^t, V_t, k\text{lead}_t, c^F_t, \tau^c_t, s^g_t, \tau^k_t, \tau^n_t\)\}_{t=0}^{\infty}, and 8 state variables, \{\(f^H_{t-1}, d_{t-1}, k_{t-1}, \Delta_{t-1}, Q_{t-1}, R_{t-1}, l_{t-1}, TT_lag_t\)\}_{t=0}^{\infty}. All this is given the values of feedback policy coefficients in the policy rules, which are chosen optimally in the computational part of the paper.

4.7 Status quo steady state

In the steady state of the above model economy, if \(\epsilon_t = 1\), our equations imply \(\Pi^H_t = \Pi^{H*}_t = \Pi^*_t = \Pi\). If we also set \(\Pi^{H*}_t = 1\) (this is the exogenous component of foreign inflation), then \(\Pi^H_t = \Pi^{H*}_t = \Pi^*_t = \Pi = 1\); this in turn implies \(\Delta = \Theta = 1\). Also, in order to solve the model numerically, we need a value for the exogenous exports, \(c^F_t\); we assume \(c^F_t = 1.01c^F\) as it is the case in the data. Finally, since from the Euler for bonds, \(1 = \frac{\beta R}{\Pi}\), we residually get \(R = \frac{1}{\beta}\). (Note that these conditions will also hold in all steady state solutions throughout the paper.)

The steady state system is (in the labor supply condition, we use \(\frac{\nu c}{c^F} TT^{1-v} = 1\)):

\[
1 = \beta[1 - \delta + \left(1 - \tau^k\right) r^k] \quad \text{(93)}
\]

\[
1 = \beta Q \quad \text{(94)}
\]

\[
\chi_n n^n = c^{-\sigma} \frac{\nu c}{c^H} TT^{1-v} \frac{(1 - \tau^n)}{(1 + \tau^n)} \quad \text{(95)}
\]

\[
\frac{c^H}{c^F} = \frac{\nu}{1 - \nu} TT \quad \text{(96)}
\]

\[
x = \delta k \quad \text{(97)}
\]

\(^2\text{In particular, when we use the Schmitt-Grohé and Uribe (2004a) matlab routines, we add 2 auxiliary endogenous variables, }k\text{lead and }TT\text{lag, to reduce the dynamic system into a first-order one.}\)
\[ c = \frac{(c^H)^\nu (c^F)^{1-\nu}}{(\nu)^\nu (1-\nu)^{1-\nu}} \] (98)

\[ w = mc(1-a)Ak^a n^{-a} \] (99)

\[ r^k = TT^{1-v}mcA k^{a-1} n^{1-a} \] (100)

\[ \bar{\omega} = \frac{1}{TT^{1-v}y^H} - \frac{1}{TT^{1-v}r^k k - wn} \] (101)

\[ y^H = Ak^a n^{1-a} \] (102)

\[ d = Qd + TT^{\nu-1}s^g y^H - \tau^c (\frac{1}{TT^{1-v}c^H} + TT^\nu c^F) - \tau^k (\frac{1}{TT^{1-v}k + \bar{\omega}}) - \tau^a wn - TT^{\nu-1}s^f y^H \] (103)

\[ y^H = c^H + x + s^g y^H + c^{F*} \] (104)

\[ (1 - \lambda)d - TT^{\nu^*+\nu-1} f^h = -TT^{\nu-1}c^{F*} + TT^\nu c^F + QTT^{\nu^*+\nu-1} \left( \frac{1}{TT^{\nu^*+\nu-1}} (1 - \lambda)d - f^h \right) \] (105)

\[ Q = Q^* + \psi \left( e^{\left( \frac{d}{TT^{\nu^*-\nu^*}} - \tilde{\beta} \right)} - 1 \right) \] (106)

\[ z^1 = \frac{\phi}{(\phi - 1)} z^2 \] (107)

\[ z^1 = TT^{\nu-1}y^H + \beta \theta z^1 \] (108)

\[ z^2 = y^H mc + \beta \theta z^2 \] (109)

We thus have 17 equations in 17 variables, \( c, c^H, c^F, k, x, n, y^H, TT, mc, \bar{\omega}, d, z^1, z^2, Q, f^h, r^k, w \). The numerical solution of this system is in Table 2 in the main text.

5 Appendix E: The reformed economy

This Appendix presents the reformed economy as defined in subsection 4.1 in the main text. The equations describing the reformed economy are as in the previous Appendix except that now we set \( Q = Q^* \), so that the public debt ratio is determined by the no premium condition,
\( \frac{dT}{TT^{\nu - 1}y^H} \equiv \bar{d} \) or \( d \equiv \bar{a}TT^{\nu - 1}y^H \). Note also that since \( Q = Q^* \), we set \( \beta = \frac{1}{Q^*} \) in the parameterization stage.

In what follows, we present steady state solutions of this economy. We will start with the case in which the country’s net foreign debt position is unrestricted so that \( \tilde{f} \equiv \frac{(1-\lambda)TT^{1-\nu}dTT^{\nu - 1}f}{y^H} \) is endogenously determined. In turn, we will study the case in which we set the country’s net foreign debt position equal to zero (meaning that the trade is balanced) so that \( \tilde{f} = 0 \) in the new reformed steady state.

### 5.1 Steady state of the reformed economy with unrestricted foreign debt

Now \( Q = Q^* \) so that \( d \equiv \bar{a}TT^{\nu - 1}y^H \). The steady state system is:

\[
1 = \beta [1 - \delta + (1 - \tau^k)r^k] 
\]

\[
\chi_n n^n = c^{-\sigma} \frac{\nu c}{c^H TT^{1-\nu}} \frac{(1 - \tau^n)}{(1 + \tau^n)} w \quad (110)
\]

\[
\frac{c^H}{c^F} = \frac{\nu}{1 - \nu} TT \quad (112)
\]

\[
x = \delta k \quad (113)
\]

\[
c = \frac{(c^H)^\nu (c^F)^{1-\nu}}{(\nu)^\nu (1 - \nu)^{1-\nu}} \quad (114)
\]

\[
y^H = Ak^a n^{1-a} \quad (115)
\]

\[
y^H = c^H + x + s^g y^H + c^F^* \quad (116)
\]

\[
r^k = TT^{1-\nu} mca Ak^{a-1} n^{1-a} \quad (117)
\]

\[
w = mc(1 - a) Ak^a n^{-a} \quad (118)
\]

\[
\bar{\omega} = \frac{1}{TT^{1-\nu}} y^H - \frac{1}{TT^{1-\nu}} r^k k - wn \quad (119)
\]
$$z^1 = \frac{\phi}{(\phi - 1)} z^2$$  \hfill (120)  

$$z^1 = yTT^{v-1} + \beta \theta z^1$$  \hfill (121)  

$$z^2 = ymc + \beta \theta z^2$$  \hfill (122)  

$$d = dTT^{v-1}y^H$$  \hfill (123)  

\begin{align*}
d &= Q*d + TT^{v-1}s^qy^H - \tau c\left(\frac{1}{TT^{v-1}c^H}c^H + TT^v c^F\right) - \\
&- \tau k\left(\frac{1}{TT^{v-1}}r^kk + \bar{\omega}\right) - \tau^m wn - TT^{v-1}s^l y^H_l \hfill (124)
\end{align*}  

\begin{align*}(1 - \lambda)d - TT^{v*+v-1}f^h &= -TT^{v-1}c^{F*} + TT^v c^F + \\
QTT^{v*+v-1} \left(\frac{1}{TT^{v*+v-1}}(1 - \lambda)d - f^h\right) \hfill (125)
\end{align*}  

We thus have 16 equations in $c$, $c^H$, $c^F$, $k$, $x$, $n$, $y^H$, $TT$, $mc$, $\bar{\omega}$, $d$, $z^1$, $z^2$, $r^k$, $w$, $f^h$. The numerical solution of this system is presented in Table E.1. Notice that, for the reasons explained in the text, private foreign debt, $-f^h > 0$, and hence the country’s net foreign debt-to-GDP ratio, $\bar{f}$, are extremely high in this case.
Table E1: Steady state solution of the reformed economy
with unrestricted $\bar{f}$

<table>
<thead>
<tr>
<th>Variables</th>
<th>Description</th>
<th>Steady-state solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>$u$</td>
<td>period utility</td>
<td>0.9458</td>
</tr>
<tr>
<td>$y^H$</td>
<td>output</td>
<td>0.858759</td>
</tr>
<tr>
<td>$TT$</td>
<td>terms of trade</td>
<td>0.838459</td>
</tr>
<tr>
<td>$Q - Q^*$</td>
<td>interest rate premium</td>
<td>0</td>
</tr>
<tr>
<td>$TT^{1-\nu}c_{y^H}$</td>
<td>consumption as share of GDP</td>
<td>0.5522</td>
</tr>
<tr>
<td>$\frac{k}{y^T}$</td>
<td>physical capital as share of GDP</td>
<td>4.2291</td>
</tr>
<tr>
<td>$\frac{TT^\nu f^h}{y^T}$</td>
<td>private foreign assets as share of GDP</td>
<td>-4.5742</td>
</tr>
<tr>
<td>$\frac{TT^{1-\nu}d}{y^T}$</td>
<td>total public debt as share of GDP</td>
<td>0.9</td>
</tr>
<tr>
<td>$\bar{f} \equiv \frac{(1-\lambda)TT^{1-\nu} - TT^\nu f^h}{y^T}$</td>
<td>total foreign debt as share of GDP</td>
<td>4.8982</td>
</tr>
</tbody>
</table>

5.2 Steady state of the reformed economy with restricted foreign debt

Now $Q = Q^*$, so that $d \equiv \bar{d}TT^{\nu-1}y^H$ as above, and, in addition, $\bar{f} \equiv \frac{(1-\lambda)TT^{1-\nu} - TT^\nu f^h}{y^T} = 0$, so that $(1 - \lambda) dTT^{1-\nu} - TT^\nu f^h = 0$ or $f^h \equiv \frac{(1-\lambda)TT^{1-\nu}}{TT^\nu}$. In this case, the equation for the balance-of-payments (125) simplifies to:

$$TTc^F = c^{F^*}$$ (126)

Therefore, using this equation in the place of (125), we have 16 equations in $c, c^H, c^F, k, x, n, y^H, TT, mc, \bar{\omega}, d, z_1, z_2, r^k, w$ and one of the tax-spending policy instruments, $s^\theta, \tau^c, \tau^k, \tau^n$.

Note that in turn $f^h$ follows residually from $f^h \equiv \frac{(1-\lambda)TT^{1-\nu}}{TT^\nu}$. The numerical solution of this system under each public financing instrument is presented below, while a summary of these solutions appears in Table 3 in the main text.
Table E2: Steady state solution of the reformed economy with $\bar{f} = 0$ when the residual fiscal instrument is $s^g$

<table>
<thead>
<tr>
<th>Variables</th>
<th>Description</th>
<th>Steady-state solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>$u$</td>
<td>period utility</td>
<td>0.9125</td>
</tr>
<tr>
<td>$r^k$</td>
<td>real return to physical capital</td>
<td>0.0749</td>
</tr>
<tr>
<td>$w$</td>
<td>real wage rate</td>
<td>1.2442</td>
</tr>
<tr>
<td>$n$</td>
<td>hours worked</td>
<td>0.3403</td>
</tr>
<tr>
<td>$y^H$</td>
<td>output</td>
<td>0.8237</td>
</tr>
<tr>
<td>$TT$</td>
<td>terms of trade</td>
<td>1.01</td>
</tr>
<tr>
<td>$s^g$</td>
<td>capital tax rate</td>
<td>0.2306</td>
</tr>
<tr>
<td>$Q - Q^*$</td>
<td>interest rate premium</td>
<td>0</td>
</tr>
<tr>
<td>$TT^{1-\nu} \frac{c}{y^H}$</td>
<td>consumption as share of GDP</td>
<td>0.6002</td>
</tr>
<tr>
<td>$\frac{k}{y^H}$</td>
<td>physical capital as share of GDP</td>
<td>4.2291</td>
</tr>
<tr>
<td>$TT^{\nu^*} \frac{f^h}{y^H}$</td>
<td>private foreign assets as share of GDP</td>
<td>0.3240</td>
</tr>
<tr>
<td>$\frac{TT^{1-\nu}d}{y^H}$</td>
<td>total public debt as share of GDP</td>
<td>0.9</td>
</tr>
<tr>
<td>$\bar{f} \equiv \frac{(1-\lambda)dTT^{1-\nu} - TT^{\nu^*}f^h}{y^H}$</td>
<td>total foreign debt as share of GDP</td>
<td>0</td>
</tr>
</tbody>
</table>
Table E3: Steady state solution of the reformed economy with $\bar{f} = 0$ when the residual fiscal instrument is $\tau^c$

<table>
<thead>
<tr>
<th>Variables</th>
<th>Description</th>
<th>Steady-state solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>$u$</td>
<td>period utility</td>
<td>0.9227</td>
</tr>
<tr>
<td>$r^k$</td>
<td>real return to physical capital</td>
<td>0.0749</td>
</tr>
<tr>
<td>$w$</td>
<td>real wage rate</td>
<td>1.2442</td>
</tr>
<tr>
<td>$n$</td>
<td>hours worked</td>
<td>0.3403</td>
</tr>
<tr>
<td>$y^H$</td>
<td>output</td>
<td>0.8237</td>
</tr>
<tr>
<td>$TT$</td>
<td>terms of trade</td>
<td>1.01</td>
</tr>
<tr>
<td>$\tau^c$</td>
<td>capital tax rate</td>
<td>0.1593</td>
</tr>
<tr>
<td>$Q - Q^*$</td>
<td>interest rate premium</td>
<td>0</td>
</tr>
<tr>
<td>$TT^{1-\nu} \frac{c}{y^H}$</td>
<td>consumption as share of GDP</td>
<td>0.6086</td>
</tr>
<tr>
<td>$\frac{k}{y^H}$</td>
<td>physical capital as share of GDP</td>
<td>4.2291</td>
</tr>
<tr>
<td>$TT^{\nu^*} \frac{f^h}{y^H}$</td>
<td>private foreign assets as share of GDP</td>
<td>0.3240</td>
</tr>
<tr>
<td>$\frac{TT^{1-\nu} d}{y^H}$</td>
<td>total public debt as share of GDP</td>
<td>0.9</td>
</tr>
<tr>
<td>$\bar{f} \equiv \frac{(1-\lambda)dTT^{1-\nu} - TT^{\nu^*} f^h}{y^H}$</td>
<td>total foreign debt as share of GDP</td>
<td>0</td>
</tr>
</tbody>
</table>
Table E4: Steady state solution of the reformed economy

with $\tilde{f} = 0$ when the residual fiscal instrument is $\tau^k$

<table>
<thead>
<tr>
<th>Variables</th>
<th>Description</th>
<th>Steady-state solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>$u$</td>
<td>period utility</td>
<td>0.9311</td>
</tr>
<tr>
<td>$r^k$</td>
<td>real return to physical capital</td>
<td>0.0729</td>
</tr>
<tr>
<td>$w$</td>
<td>real wage rate</td>
<td>1.2649</td>
</tr>
<tr>
<td>$n$</td>
<td>hours worked</td>
<td>0.3393</td>
</tr>
<tr>
<td>$y^H$</td>
<td>output</td>
<td>0.8348</td>
</tr>
<tr>
<td>$TT$</td>
<td>terms of trade</td>
<td>1.01</td>
</tr>
<tr>
<td>$\tau^k$</td>
<td>capital tax rate</td>
<td>0.2930</td>
</tr>
<tr>
<td>$Q - Q^*$</td>
<td>interest rate premium</td>
<td>0</td>
</tr>
<tr>
<td>$TT^{1-\nu} \frac{c}{y^H}$</td>
<td>consumption as share of GDP</td>
<td>0.6040</td>
</tr>
<tr>
<td>$\frac{k}{y^H}$</td>
<td>physical capital as share of GDP</td>
<td>4.3448</td>
</tr>
<tr>
<td>$TT^{\nu^*} \frac{f^h}{y^H}$</td>
<td>private foreign assets as share of GDP</td>
<td>0.3240</td>
</tr>
<tr>
<td>$\frac{TT^{1-\nu}d}{y^H}$</td>
<td>total public debt as share of GDP</td>
<td>0.9</td>
</tr>
<tr>
<td>$\tilde{f} \equiv \frac{(1-\lambda)dTT^{1-\nu}-TT^{\nu^*}f^h}{y^H}$</td>
<td>total foreign debt as share of GDP</td>
<td>0</td>
</tr>
</tbody>
</table>
Table E5: Steady state solution of the reformed economy

with $\bar{f} = 0$ when the residual fiscal instrument is $\tau^n$

<table>
<thead>
<tr>
<th>Variables</th>
<th>Description</th>
<th>Steady-state solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>$u$</td>
<td>period utility</td>
<td>0.9290</td>
</tr>
<tr>
<td>$r^k$</td>
<td>real return to physical capital</td>
<td>0.0749</td>
</tr>
<tr>
<td>$w$</td>
<td>real wage rate</td>
<td>1.2442</td>
</tr>
<tr>
<td>$n$</td>
<td>hours worked</td>
<td>0.3435</td>
</tr>
<tr>
<td>$y^H$</td>
<td>output</td>
<td>0.8314</td>
</tr>
<tr>
<td>$TT$</td>
<td>terms of trade</td>
<td>1.01</td>
</tr>
<tr>
<td>$\tau^n$</td>
<td>capital tax rate</td>
<td>0.4018</td>
</tr>
<tr>
<td>$Q - Q^*$</td>
<td>interest rate premium</td>
<td>0</td>
</tr>
<tr>
<td>$TT^{1-\nu} \frac{c}{y^H}$</td>
<td>consumption as share of GDP</td>
<td>0.6086</td>
</tr>
<tr>
<td>$\frac{k}{y^H}$</td>
<td>physical capital as share of GDP</td>
<td>4.2291</td>
</tr>
<tr>
<td>$TT^{1-\nu} \frac{f^h}{y^H}$</td>
<td>private foreign assets as share of GDP</td>
<td>0.3240</td>
</tr>
<tr>
<td>$TT^{1-\nu} \frac{d}{y^H}$</td>
<td>total public debt as share of GDP</td>
<td>0.9</td>
</tr>
<tr>
<td>$\bar{f} \equiv \frac{(1-\lambda)dTT^{1-\nu}TT^{\nu}f^h}{y^H}$</td>
<td>total foreign debt as share of GDP</td>
<td>0</td>
</tr>
</tbody>
</table>

5.3 Restricted changes in fiscal policy instruments

We now repeat the main computations restricting the magnitude of feedback coefficients in the policy rules so as all tax-spending policy instruments cannot change by more than 10 percentage points from their averages in the data (we report that the results also do not change when we allow for 5 percentage points only).

Restricted results for optimal feedback policy coefficients, the implied statistics and welfare over various time horizons are reported in Tables E6, E7 and E8 which correspond to Tables 4, 5 and 6 respectively in the main text.
Table E6: Optimal reaction to debt and output with debt consolidation

<table>
<thead>
<tr>
<th>fiscal instruments</th>
<th>optimal reaction to debt</th>
<th>optimal reaction to output</th>
</tr>
</thead>
<tbody>
<tr>
<td>$s^g_t$</td>
<td>$\gamma^g_t = 0.4$</td>
<td>$\gamma^g_y = 0.0011$</td>
</tr>
<tr>
<td>$\tau^c_t$</td>
<td>$\gamma^c_t = 0.0005$</td>
<td>$\gamma^c_y = 0.4865$</td>
</tr>
<tr>
<td>$\tau^k_t$</td>
<td>$\gamma^k_t = 0.0026$</td>
<td>$\gamma^k_y = 0.9496$</td>
</tr>
<tr>
<td>$\tau^n_t$</td>
<td>$\gamma^n_t = 0.0023$</td>
<td>$\gamma^n_y = 0.95$</td>
</tr>
</tbody>
</table>

Note: $V_0 = 79.9336$

Table E7: Statistics implied by Table E6

<table>
<thead>
<tr>
<th>Elasticity to liabilities</th>
<th>Elasticity to output</th>
<th>Min/max</th>
<th>5 periods average</th>
<th>10 periods average</th>
<th>20 periods average</th>
<th>Data average</th>
</tr>
</thead>
<tbody>
<tr>
<td>$s^g_t$</td>
<td>-2.016%</td>
<td>-0.0035%</td>
<td>0.1310/0.2273</td>
<td>0.1473</td>
<td>0.1576</td>
<td>0.1760</td>
</tr>
<tr>
<td>$\tau^c_t$</td>
<td>0.0032%</td>
<td>1.97%</td>
<td>0.1352/ 0.1799</td>
<td>0.1492</td>
<td>0.1545</td>
<td>0.1614</td>
</tr>
<tr>
<td>$\tau^k_t$</td>
<td>0.0093%</td>
<td>2.16%</td>
<td>0.2143/ 0.3014</td>
<td>0.2418</td>
<td>0.2520</td>
<td>0.2654</td>
</tr>
<tr>
<td>$\tau^n_t$</td>
<td>0.0062%</td>
<td>1.61%</td>
<td>0.3423/0.4295</td>
<td>0.3698</td>
<td>0.38</td>
<td>0.3934</td>
</tr>
</tbody>
</table>

Table E8: Welfare over different time horizons

<table>
<thead>
<tr>
<th></th>
<th>2 periods</th>
<th>4 periods</th>
<th>10 periods</th>
<th>20 periods</th>
<th>30 periods</th>
</tr>
</thead>
<tbody>
<tr>
<td>with consolid.</td>
<td>1.4862</td>
<td>3.0164</td>
<td>7.7221</td>
<td>15.3294</td>
<td>22.2939</td>
</tr>
<tr>
<td>without consolid.</td>
<td>(1.6251)</td>
<td>(3.1791)</td>
<td>(7.4481)</td>
<td>(13.4127)</td>
<td>(18.1904)</td>
</tr>
<tr>
<td>welfare gain/loss</td>
<td>-0.0458</td>
<td>-0.0327</td>
<td>0.0267</td>
<td>0.1075</td>
<td>0.1693</td>
</tr>
</tbody>
</table>

Inspection of the new tables, and comparison to their counterparts in the main text, implies that the main qualitative results do not change. Namely, debt consolidation is again preferable to non-debt consolidation after the first ten years. Also, although obviously feedback policy coefficients are now smaller, the best fiscal policy mix again implies that we should earmark public spending for the reduction of public debt and, at the same time, cut taxes to mitigate the recessionary effects of debt consolidation. The only difference is that now, since cuts in income taxes are restricted, we should also cut consumption taxes.

### 5.4 IRFs with one fiscal instrument at a time

The figures below compare debt consolidation results when the fiscal authorities can use all instruments jointly to the case in which they are restricted to use one instrument at a time.
only. Results are always with optimized rules. To save on space, we focus on results for the public debt to GDP ratio and private consumption.

Figure E1: IRFs of public debt to GDP (comparison of alternative fiscal policies)

Notes: As in Figure 1.
Figure E2: IRFs of consumption (comparison of alternative fiscal policies)

Notes: As in Figure 1.
6 Appendix F: Robustness to the public debt threshold parameter

6.1 The case with a lower public debt threshold

We now assume $\bar{d} \equiv 0.8$. This implies $\psi \equiv 0.0319$ to hit the data. Then, we have the solution:

<table>
<thead>
<tr>
<th>Variables</th>
<th>Description</th>
<th>Steady-state solution</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>$u$</td>
<td>period utility</td>
<td>0.8217</td>
<td>-</td>
</tr>
<tr>
<td>$r^k$</td>
<td>real return to physical capital</td>
<td>0.0908</td>
<td>-</td>
</tr>
<tr>
<td>$w$</td>
<td>real wage rate</td>
<td>1.1140</td>
<td>-</td>
</tr>
<tr>
<td>$n$</td>
<td>hours worked</td>
<td>0.3313</td>
<td>0.2183</td>
</tr>
<tr>
<td>$y^H$</td>
<td>output</td>
<td>0.7124</td>
<td>-</td>
</tr>
<tr>
<td>$TT$</td>
<td>terms of trade</td>
<td>0.9945</td>
<td>-</td>
</tr>
<tr>
<td>$Q - Q^*$</td>
<td>interest rate premium</td>
<td>0</td>
<td>0.011</td>
</tr>
<tr>
<td>$TT^{1-\nu} \frac{c}{y^H}$</td>
<td>consumption as share of GDP</td>
<td>0.6334</td>
<td>0.5961</td>
</tr>
<tr>
<td>$\frac{k}{y^H}$</td>
<td>physical capital as share of GDP</td>
<td>3.4872</td>
<td>-</td>
</tr>
<tr>
<td>$TT^{1-\nu} \frac{f^h}{y^H}$</td>
<td>private foreign assets as share of GDP</td>
<td>0.1749</td>
<td>0.1039</td>
</tr>
<tr>
<td>$\frac{TT^{1-\nu}d}{y^H}$</td>
<td>total public debt as share of GDP</td>
<td>1.0965</td>
<td>1.0828</td>
</tr>
<tr>
<td>$\tilde{f} \equiv \frac{(1-\lambda)dTT^{1-\nu} - TT^{\nu^*} f^h}{y^H}$</td>
<td>total foreign debt as share of GDP</td>
<td>0.2194</td>
<td>0.2109</td>
</tr>
</tbody>
</table>

26
### Table F2: Reformed steady state solution

<table>
<thead>
<tr>
<th>Variables</th>
<th>Description</th>
<th>Steady-state solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>$u$</td>
<td>period utility</td>
<td>0.9326</td>
</tr>
<tr>
<td>$r^k$</td>
<td>real return to physical capital</td>
<td>0.0727</td>
</tr>
<tr>
<td>$w$</td>
<td>real wage rate</td>
<td>1.2673</td>
</tr>
<tr>
<td>$n$</td>
<td>hours worked</td>
<td>0.3394</td>
</tr>
<tr>
<td>$y^H$</td>
<td>output</td>
<td>0.8367</td>
</tr>
<tr>
<td>$TT$</td>
<td>terms of trade</td>
<td>1.01</td>
</tr>
<tr>
<td>$\tau^k$</td>
<td>capital tax rate</td>
<td>0.2908</td>
</tr>
<tr>
<td>$Q - Q^*$</td>
<td>interest rate premium</td>
<td>0</td>
</tr>
<tr>
<td>$TT^{1-\nu}\frac{c}{y^H}$</td>
<td>consumption as share of GDP</td>
<td>0.6035</td>
</tr>
<tr>
<td>$\frac{k}{y^H}$</td>
<td>physical capital as share of GDP</td>
<td>4.3583</td>
</tr>
<tr>
<td>$TT^{\nu}\frac{f_h}{y^H}$</td>
<td>private foreign assets as share of GDP</td>
<td>0.2880</td>
</tr>
<tr>
<td>$TT^{1-\nu}\frac{d}{y^H}$</td>
<td>total public debt as share of GDP</td>
<td>0.8</td>
</tr>
<tr>
<td>$\bar{f} \equiv \frac{(1-\lambda)dTT^{1-\nu}TT^{\nu}f_h}{y^H}$</td>
<td>total foreign debt as share of GDP</td>
<td>0</td>
</tr>
</tbody>
</table>

### Table F3: Optimal reaction to debt and output with debt consolidation

(optimal fiscal policy mix)

<table>
<thead>
<tr>
<th>fiscal instruments</th>
<th>optimal reaction to debt</th>
<th>optimal reaction to output</th>
</tr>
</thead>
<tbody>
<tr>
<td>$s_t^g$</td>
<td>$\gamma^g_l = 0.4338$</td>
<td>$\gamma^g_y = 0$</td>
</tr>
<tr>
<td>$\tau^c_l$</td>
<td>$\gamma^c_l = 0$</td>
<td>$\gamma^c_y = 0.0279$</td>
</tr>
<tr>
<td>$\tau^k_l$</td>
<td>$\gamma^k_l = 0$</td>
<td>$\gamma^k_y = 2.2569$</td>
</tr>
<tr>
<td>$\tau^n_l$</td>
<td>$\gamma^n_l = 0.0001$</td>
<td>$\gamma^n_y = 2.136$</td>
</tr>
</tbody>
</table>

Notes: $V_0 = 80.1038$.  

27
6.2 The case with a higher public debt threshold

Next, we assume \( \tilde{d} \equiv 1 \). This implies \( \psi \equiv 0.108 \) to hit the data. Then, we have the solution:

<table>
<thead>
<tr>
<th>Variables</th>
<th>Description</th>
<th>Steady-state solution</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>( u )</td>
<td>period utility</td>
<td>0.8217</td>
<td>-</td>
</tr>
<tr>
<td>( r^k )</td>
<td>real return to physical capital</td>
<td>0.0908092</td>
<td>-</td>
</tr>
<tr>
<td>( w )</td>
<td>real wage rate</td>
<td>1.11373</td>
<td>-</td>
</tr>
<tr>
<td>( n )</td>
<td>hours worked</td>
<td>0.331273</td>
<td>0.2183</td>
</tr>
<tr>
<td>( y^H )</td>
<td>output</td>
<td>0.712311</td>
<td>-</td>
</tr>
<tr>
<td>( TT )</td>
<td>terms of trade</td>
<td>0.995009</td>
<td>-</td>
</tr>
<tr>
<td>( Q - Q^* )</td>
<td>interest rate premium</td>
<td>0.011</td>
<td>0.011</td>
</tr>
<tr>
<td>( TT^{1-\nu} \frac{c}{y^H} )</td>
<td>consumption as share of GDP</td>
<td>0.6335</td>
<td>0.5961</td>
</tr>
<tr>
<td>( \frac{k}{y^H} )</td>
<td>physical capital as share of GDP</td>
<td>3.4872</td>
<td>-</td>
</tr>
<tr>
<td>( TT^{u^*} \frac{f^h}{y^H} )</td>
<td>private foreign assets as share of GDP</td>
<td>0.1827</td>
<td>0.1039</td>
</tr>
<tr>
<td>( TT^{1-\nu} \frac{d}{y^H} )</td>
<td>total public debt as share of GDP</td>
<td>1.0965</td>
<td>1.0828</td>
</tr>
<tr>
<td>( \tilde{f} \equiv \frac{(1-\lambda)dTT^{1-\nu} - TT^{u^*} f^h}{y^H} )</td>
<td>total foreign debt as share of GDP</td>
<td>0.2122</td>
<td>0.2109</td>
</tr>
</tbody>
</table>
### Table F5: Reformed steady state solution

<table>
<thead>
<tr>
<th>Variables</th>
<th>Description</th>
<th>Steady-state solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>$u$</td>
<td>period utility</td>
<td>0.9296</td>
</tr>
<tr>
<td>$r^k$</td>
<td>real return to physical capital</td>
<td>0.0731</td>
</tr>
<tr>
<td>$w$</td>
<td>real wage rate</td>
<td>1.2625</td>
</tr>
<tr>
<td>$n$</td>
<td>hours worked</td>
<td>0.3391</td>
</tr>
<tr>
<td>$y^H$</td>
<td>output</td>
<td>0.8328</td>
</tr>
<tr>
<td>$TT$</td>
<td>terms of trade</td>
<td>1.01</td>
</tr>
<tr>
<td>$\tau^k$</td>
<td>capital tax rate</td>
<td>0.2951</td>
</tr>
<tr>
<td>$Q - Q^*$</td>
<td>interest rate premium</td>
<td>0</td>
</tr>
<tr>
<td>$TT^{1-\nu} \frac{c}{y^H}$</td>
<td>consumption as share of GDP</td>
<td>0.6045</td>
</tr>
<tr>
<td>$\frac{k}{y^H}$</td>
<td>physical capital as share of GDP</td>
<td>4.3314</td>
</tr>
<tr>
<td>$TT^{1-\nu} \frac{f^h}{y^H}$</td>
<td>private foreign assets as share of GDP</td>
<td>0.3600</td>
</tr>
<tr>
<td>$TT^{1-\nu} \frac{d}{y^H}$</td>
<td>total public debt as share of GDP</td>
<td>1</td>
</tr>
<tr>
<td>$\tilde{f} \equiv \frac{(1-\lambda)\delta TT^{1-\nu} - TT^{1-\nu} \frac{f^h}{y^H}}{y^H}$</td>
<td>total foreign debt as share of GDP</td>
<td>0</td>
</tr>
</tbody>
</table>

### Table F6: Optimal reaction to debt and output with debt consolidation

(optimal fiscal policy mix)

<table>
<thead>
<tr>
<th>fiscal instruments</th>
<th>optimal reaction to debt</th>
<th>optimal reaction to output</th>
</tr>
</thead>
<tbody>
<tr>
<td>$s^g_t$</td>
<td>$\gamma^g_t = 0.4588$</td>
<td>$\gamma^g_y = 0.0017$</td>
</tr>
<tr>
<td>$\tau^c_t$</td>
<td>$\gamma^c_t = 0$</td>
<td>$\gamma^c_y = 0.0018$</td>
</tr>
<tr>
<td>$\tau^k_t$</td>
<td>$\gamma^k_t = 0.0014$</td>
<td>$\gamma^k_y = 2.2569$</td>
</tr>
<tr>
<td>$\tau^n_t$</td>
<td>$\gamma^n_t = 0.1306$</td>
<td>$\gamma^n_y = 1.5629$</td>
</tr>
</tbody>
</table>

Notes: $V_0 = 79.8482$. 
7 Appendix G: Robustness to the net foreign debt position in the reformed steady state

7.1 The case with 0.1 net foreign debt ratio in the reformed steady state

When we set \( f = 0.1 \) in the reformed steady state, the results are:

Table G1: Steady state solution of the reformed economy with \( f = 0.1 \)

<table>
<thead>
<tr>
<th>Variables</th>
<th>Description</th>
<th>Steady-state solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>( u )</td>
<td>period utility</td>
<td>0.9315</td>
</tr>
<tr>
<td>( y^H )</td>
<td>output</td>
<td>0.835228</td>
</tr>
<tr>
<td>( T )</td>
<td>terms of trade</td>
<td>1.00615</td>
</tr>
<tr>
<td>( Q - Q^* )</td>
<td>interest rate premium</td>
<td>0</td>
</tr>
<tr>
<td>( TT^{1-\nu} \frac{c}{y^H} )</td>
<td>consumption as share of GDP</td>
<td>0.6029</td>
</tr>
<tr>
<td>( \frac{k}{y^H} )</td>
<td>physical capital as share of GDP</td>
<td>4.3425</td>
</tr>
<tr>
<td>( TT^{\nu^*} \frac{f^h}{y^H} )</td>
<td>private foreign assets as share of GDP</td>
<td>0.2240</td>
</tr>
<tr>
<td>( \frac{TT^{1-\nu} d}{y^H} )</td>
<td>total public debt as share of GDP</td>
<td>0.9</td>
</tr>
<tr>
<td>( \bar{f} \equiv \frac{(1-\lambda) TT^{1-\nu} d - TT^{\nu^*} f^h}{y^H} )</td>
<td>total foreign debt as share of GDP</td>
<td>0.1</td>
</tr>
</tbody>
</table>

Table G2: Optimal reaction to debt and output with debt consolidation (optimal fiscal policy mix)

<table>
<thead>
<tr>
<th>fiscal instruments</th>
<th>optimal reaction to debt</th>
<th>optimal reaction to output</th>
</tr>
</thead>
<tbody>
<tr>
<td>( s^g_t )</td>
<td>( \gamma^g_t = 0.6725 )</td>
<td>( \gamma^g_y = 0.0020 )</td>
</tr>
<tr>
<td>( \tau^c_t )</td>
<td>( \gamma^c_t = 0 )</td>
<td>( \gamma^c_y = 0.7435 )</td>
</tr>
<tr>
<td>( \tau^k_t )</td>
<td>( \gamma^k_t = 0.0011 )</td>
<td>( \gamma^k_y = 2.2572 )</td>
</tr>
<tr>
<td>( \tau^n_t )</td>
<td>( \gamma^n_t = 0.0015 )</td>
<td>( \gamma^n_y = 2.14 )</td>
</tr>
</tbody>
</table>

Notes: \( V_0 = 80.2538 \).
7.2 The case with 0.2109 net foreign debt ratio in the reformed steady state

When we set \( \tilde{f} \equiv \frac{(1-\lambda)TT^{1-\nu}d - TT^{\nu}f^h}{y^H} = 0.2109 \) (as in the data) in the reformed steady state, the results are:

Table G3: Steady state solution of the reformed economy

<table>
<thead>
<tr>
<th>Variables</th>
<th>Description</th>
<th>Steady-state solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>( u )</td>
<td>period utility</td>
<td>0.9320</td>
</tr>
<tr>
<td>( y^H )</td>
<td>output</td>
<td>0.8358</td>
</tr>
<tr>
<td>( TT )</td>
<td>terms of trade</td>
<td>1.002</td>
</tr>
<tr>
<td>( Q - Q^* )</td>
<td>interest rate premium</td>
<td>0</td>
</tr>
<tr>
<td>( TT^{1-\nu} \frac{c}{y^H} )</td>
<td>consumption as share of GDP</td>
<td>0.6017</td>
</tr>
<tr>
<td>( \frac{k}{y^H} )</td>
<td>physical capital as share of GDP</td>
<td>4.3397</td>
</tr>
<tr>
<td>( TT^{\nu} \frac{f^h}{y^H} )</td>
<td>private foreign assets as share of GDP</td>
<td>0.1050</td>
</tr>
<tr>
<td>( \frac{TT^{1-\nu}d}{y^H} )</td>
<td>total public debt as share of GDP</td>
<td>0.9</td>
</tr>
<tr>
<td>( \tilde{f} \equiv \frac{(1-\lambda)TT^{1-\nu}d - TT^{\nu}f^h}{y^H} )</td>
<td>total foreign debt as share of GDP</td>
<td>0.2109</td>
</tr>
</tbody>
</table>

Table G4: Optimal reaction to debt and output with debt consolidation (optimal fiscal policy mix)

<table>
<thead>
<tr>
<th>fiscal instruments</th>
<th>optimal reaction to debt</th>
<th>optimal reaction to output</th>
</tr>
</thead>
<tbody>
<tr>
<td>( s_i^g )</td>
<td>( \gamma_i^g = 0.5558 )</td>
<td>( \gamma_y^g = 0.0114 )</td>
</tr>
<tr>
<td>( \tau_i^c )</td>
<td>( \gamma_i^c = 0.0094 )</td>
<td>( \gamma_y^c = 1.3091 )</td>
</tr>
<tr>
<td>( \tau_i^k )</td>
<td>( \gamma_i^k = 0.0159 )</td>
<td>( \gamma_y^k = 1.3759 )</td>
</tr>
<tr>
<td>( \tau_i^n )</td>
<td>( \gamma_i^n = 0 )</td>
<td>( \gamma_y^n = 2.14 )</td>
</tr>
</tbody>
</table>

Notes: \( V_0 = 80.3931 \).
8 Appendix H: Robustness to world interest-rate shocks

Using the new specification presented in the main text, the results are:

Table H1: Optimal reaction to debt and output with debt consolidation

<table>
<thead>
<tr>
<th>Fiscal instruments</th>
<th>Optimal reaction to debt</th>
<th>Optimal reaction to output</th>
</tr>
</thead>
<tbody>
<tr>
<td>( s_t^g )</td>
<td>( \gamma_t^g = 0.2068 )</td>
<td>( \gamma_t^g = 0 )</td>
</tr>
<tr>
<td>( \tau_t^c )</td>
<td>( \gamma_t^c = 0.0002 )</td>
<td>( \gamma_t^c = 0.2097 )</td>
</tr>
<tr>
<td>( \tau_t^k )</td>
<td>( \gamma_t^k = 0 )</td>
<td>( \gamma_t^k = 2.2569 )</td>
</tr>
<tr>
<td>( \tau_t^n )</td>
<td>( \gamma_t^n = 0 )</td>
<td>( \gamma_t^n = 2.1360 )</td>
</tr>
</tbody>
</table>

Notes: \( V_0 = 79.5457 \)

Table H2: Welfare over different time horizons with, and without, debt consolidation (optimal fiscal policy mix)

<table>
<thead>
<tr>
<th></th>
<th>2 periods</th>
<th>4 periods</th>
<th>10 periods</th>
<th>20 periods</th>
<th>30 periods</th>
</tr>
</thead>
<tbody>
<tr>
<td>with consolid.</td>
<td>1.4589</td>
<td>2.8359</td>
<td>7.0660</td>
<td>14.4129</td>
<td>21.9026</td>
</tr>
<tr>
<td>without consolid.</td>
<td>(1.5236)</td>
<td>(2.9533)</td>
<td>(6.8521)</td>
<td>(12.6868)</td>
<td>(17.8657)</td>
</tr>
<tr>
<td>Welfare gain/loss</td>
<td>-0.0216</td>
<td>-0.0237</td>
<td>0.0208</td>
<td>0.0963</td>
<td>0.1662</td>
</tr>
</tbody>
</table>

9 Appendix I: Ramsey policy problem

The Ramsey government chooses the paths of policy instruments to maximize the household’s expected discounted lifetime utility, \( V_t \), subject to the equations of the DE. As shown in Appendix D above, the DE system contains 24 equations (we do not include the equation for \( V_t \)).

As said in the main text, we solve for a "timeless" Ramsey equilibrium, meaning that the resulting equilibrium conditions are time invariant (see also e.g. Schmitt-Grohé and Uribe, 2005). If we follow the dual approach to the Ramsey policy problem, the government chooses the paths of \( \{y_t^H, c_t, c_t^H, c_t^F, n_t, x_t, k_t, f_t^H, TT_t, \Pi_t, \Pi_t^H, \Theta_t, \Delta_t, w_t, mc_t, \omega_t, r_t^h, Q_t, d_t, \Pi_t^s, \} \).

The values of the optimized feedback policy coefficients when we start from, and return to, the same status quo steady state are \( \gamma_t^d = 0.0109, \gamma_t^g = 0.5206, \gamma_t^h = 0, \gamma_t^n = 0.3032, \gamma_t^q = 0.0129, \gamma_t^c = 0, \gamma_t^k = 2.2564, \gamma_t^y = 0.0236 \).
$z_t^1, z_t^2, R_t, l_t \in \mathbb{I}_{t=0}^\infty$, which are the 24 endogenous variables of the DE system, plus the paths of the three tax rates, $\{\tau_t^c, \tau_t^k, \tau_t^n\}_{t=0}^\infty$. Notice that public debt, $\{d_t\}_{t=0}^\infty$, is also among the choice variables. Also notice that, in solving this optimization problem, we take as given the rate of exchange rate depreciation, $\{\epsilon_t\}_{t=0}^\infty$, public spending on goods/services, $\{s_t^g\}_{t=0}^\infty$, lump-sum government transfers, $\{s_t^l\}_{t=0}^\infty$, and the share of public debt issued for the domestic market, $\{\lambda_t\}_{t=0}^\infty$. We treat these variables as exogenous for different reasons: in a currency union model, $\epsilon_t = 1$ all the time; we take $\{s_t^g\}_{t=0}^\infty$ as given for simplicity; we take $\{s_t^l\}_{t=0}^\infty$ as given because, if the Ramsey government can choose a lump-sum policy instrument, then its problem becomes trivial; finally, we set $\lambda_t$ exogenously and equal to its data average value (as we have done so far anyway) because, if it is chosen by the Ramsey government, its optimality condition will be identical to the household’s optimality condition for financial assets/debt in a Ramsey steady state without sovereign interest-rate premia, so the Ramsey steady state system will be indeterminate (in particular, in the steady state, the government’s optimality condition for $\lambda_t$ is reduced to $1 = \beta Q$, which is also the household’s optimality condition for foreign assets/debt). For the very same reason (namely, the optimality conditions of the household and the government with respect to $f_t^h$ are both reduced to $1 = \beta Q$ in the Ramsey steady state without sovereign interest-rate premia), we set $f_t^h$ so as to satisfy $f = 0$ in the reformed steady state (as we have done so far anyway in the reformed steady state). Thus, from $\tilde{f} \equiv \frac{(1-\lambda)\bar{d}T^{1-\nu}TT^{\nu}f^h}{y^H} = 0$, it follows $f^h = (1 - \lambda) \frac{\bar{d}TTT^{\nu-1}y^H}{y^H}$. Working in this way, and including Ramsey multipliers, we end up with a well-defined system consisting of 51 equations in 51 endogenous variables (the system is available upon request from the authors). The numerical solution of this system in the steady state is presented in Table II (we do not report solutions for Ramsey multipliers). In this solution, as said before, $Q = Q^* = R = \frac{1}{\beta}$, and $f^h = (1 - \lambda) \frac{\bar{d}TTT^{\nu-1}y^H}{y^H}$. (Note that nominal variables, like $R$, do not affect the real allocation, and hence utility, at steady state.)
<table>
<thead>
<tr>
<th>Variables</th>
<th>Description</th>
<th>Steady-state solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>$u$</td>
<td>period utility</td>
<td>1.2215</td>
</tr>
<tr>
<td>$y^H$</td>
<td>output</td>
<td>1.56176</td>
</tr>
<tr>
<td>$TT$</td>
<td>terms of trade</td>
<td>1.01</td>
</tr>
<tr>
<td>$\tau^c$</td>
<td>&quot;optimal&quot; consumption tax rate</td>
<td>26.5726</td>
</tr>
<tr>
<td>$\tau^k$</td>
<td>&quot;optimal&quot; capital tax rate</td>
<td>0.00272056</td>
</tr>
<tr>
<td>$\tau^n$</td>
<td>&quot;optimal&quot; consumption tax rate</td>
<td>-26.4976</td>
</tr>
<tr>
<td>$Q - Q^*$</td>
<td>interest rate premium</td>
<td>0</td>
</tr>
<tr>
<td>$TT^{1-\nu} \frac{c}{\bar{y}^H}$</td>
<td>consumption as share of GDP</td>
<td>0.5327</td>
</tr>
<tr>
<td>$\frac{k}{\bar{y}^H}$</td>
<td>physical capital as share of GDP</td>
<td>6.1284</td>
</tr>
<tr>
<td>$TT^{\nu} \frac{f_h}{\bar{y}^H}$</td>
<td>private foreign assets as share of GDP</td>
<td>0.3240</td>
</tr>
<tr>
<td>$\frac{TT^{1-\nu} \frac{d}{\bar{y}^H}}{y^H}$</td>
<td>total public debt as share of GDP</td>
<td>0.9</td>
</tr>
<tr>
<td>$\tilde{f} \equiv \frac{(1-\lambda)dTT^{1-\nu} - TT^{\nu} f_h}{y^H}$</td>
<td>total foreign debt as share of GDP</td>
<td>0</td>
</tr>
</tbody>
</table>

Notice, as also said in the text, and as is well established in the Ramsey literature with optimally chosen consumption taxes, that the solution in Table I1 is characterized by an extremely large consumption tax rate, $\tau^c = 26.5726$, and an equally large labor subsidy, $\tau^n = -26.4976 < 0$. Hence, following the same literature, we rule out a subsidy to labor by setting $\tau^n_t = 0$ at all $t$. This gives the solution in Table I2 (this Table is also presented in the main text, but we repeat it here for the reader’s convenience):
Table I2: Ramsey steady state solution (restricted $\tau^n_t = 0$)

<table>
<thead>
<tr>
<th>Variables</th>
<th>Description</th>
<th>Steady-state solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>$u$</td>
<td>period utility</td>
<td>1.0950</td>
</tr>
<tr>
<td>$y^H$</td>
<td>output</td>
<td>1.11622</td>
</tr>
<tr>
<td>$TT$</td>
<td>terms of trade</td>
<td>1.01</td>
</tr>
<tr>
<td>$\tau^k$</td>
<td>&quot;optimal&quot; capital tax rate</td>
<td>0.0500465</td>
</tr>
<tr>
<td>$\tau^c$</td>
<td>&quot;optimal&quot; consumption tax rate</td>
<td>0.810199</td>
</tr>
<tr>
<td>$Q - Q^*$</td>
<td>interest rate premium</td>
<td>0</td>
</tr>
<tr>
<td>$TT^{1-\nu} c / y^H$</td>
<td>consumption as share of GDP</td>
<td>0.5443</td>
</tr>
<tr>
<td>$k / y^H$</td>
<td>physical capital as share of GDP</td>
<td>5.8387</td>
</tr>
<tr>
<td>$TT^{\nu} f^h / y^H$</td>
<td>private foreign assets as share of GDP</td>
<td>0.3240</td>
</tr>
<tr>
<td>$TT^{1-\nu} f^d / y^H$</td>
<td>total public debt as share of GDP</td>
<td>0.9</td>
</tr>
<tr>
<td>$\tilde{f} \equiv \frac{(1-\lambda)d TT^{1-\nu} - TT^n f^h}{y^H}$</td>
<td>total foreign debt as share of GDP</td>
<td>0</td>
</tr>
</tbody>
</table>
10 Appendix J: Flexible exchange rates

The impulse response functions (IRFs) under flexible exchange rates are shown in Figure J1 (the zero-lower bound for the nominal interest rate is not violated):

Figure J1: IRFs under flexible exchange rates

Notes: As in Figure 1.