Simulating the Wake Down-stream of a Horizontal-axis Tidal Turbine Using a Modified Vorticity Transport Model

Lada Vybulkova, Marco Vezza, and Richard Brown

Abstract

In order to decrease the need for fossil fuels, the alternative energy resources must be not only economically viable but also sustainable in the long-term. One of the most promising alternatives is the marine renewable energy resource. The relatively young marine energy industry is presented with two challenges. Firstly, to deliver a continuous reliable power supply and secondly, to minimize potentially harmful effects of the power extraction on the marine environment. The requirement to understand the interactions between a tidal turbine and the surrounding flow environment motivated the present work. A tidal turbine mounted on the seabed induces a wake that extends far down-stream of the device. As the direction of tidal flow changes so does the position of the wake with respect to the device The detailed study of the turbine wake has been conducted by means of computer simulations. An existing finite-volume computer model called the Vorticity Transport Model has been modified to suit the purpose of simulating the wake of a horizontal-axis tidal turbine subjected to a non-uniform flow typical of that close to the seabed. High-resolution computer simulations suggest that a progressive fragmentation of the vortical structure occurs during the development of the wake of a tidal turbine. The predicted fragmentation generates small-scale unsteady flow phenomena beyond five rotor diameters down-stream of the device in the area previously thought unaffected by the presence of a tidal turbine. The effects of non-uniform flow on the vorticity structure down-stream of a tidal turbine and the fragmentation process are analysed in this work.

Index Terms

Tidal, turbine, wake, computer, simulation, marine, environment.

I. Introduction

The variety of devices designed to harness energy from tides continues to expand every year. The enthusiasm for marine renewable energy technology world-wide has been fuelled by optimistic estimates of the tidal energy

Lada Vybulkova and Richard Brown are with the Department of Mechanical and Aerospace Engineering, University of Strathclyde, Glasgow G1 1XJ, United Kingdom. E-mail: lada.vybulkova@strath.ac.uk.

Marco Vezza is with School of Engineering, University of Glasgow, Glasgow G12 8QQ, United Kingdom.

resources. These estimates have suggested that the amount of power available in tidal currents and waves could be, in combination with other renewable sources of power, sufficient to provide a realistic energy alternative to fossil fuels. The UK Total Resource alone was estimated at approximately 110 TWh annually, a large proportion of which is from presently known locations. A comprehensive overview of the recent developments in the technology used to extract power from tides can be found in the study by Bahaj [1].

Initially, the research related to tidal turbines was mostly focused on enhancing the power extracting capabilities of the devices. The rotor wake has been considered of secondary importance and the area of interest has been limited to the vicinity of the rotor.

A small but growing number of research studies focus on the wake structure to explore the interaction of wakes in tidal arrays. One such study applied the Large-Eddy Simulation (LES) technique to examine the interactions between the wakes produced by multiple rotors in a turbulent flow to assess the changes in power production due to the turbulent inflow [2]. Another study investigated the effect of a simple non-uniform inflow model on the performance and wake structure of tidal turbines using a BEM representation of the rotor [3]. The representation of the rotor in [3] was based on the assumption that the influence of the moving blades on the flow becomes evenly distributed over a circular region over time.

The majority of computational studies on the wake of tidal turbines have used the velocity-pressure formulation of the equations of the fluid motion to advance velocity and pressure in time. A comprehensive summary of the work done on the subject, both by experiments and using CFD, is presented by Tedds et al. [4]. Tedds et al. analysed the structure of the mean velocity field in the area between $3R$ and $14R$ down-stream of a horizontal-axis turbine (R denotes the radius of the tidal turbine), using an Acoustic Doppler Velocimeter (ADV). In [4] it is stated that "*previous modelling approaches, which neglected swirl effects and modelled the turbine by absorption discs, may significantly over-predict the TKE decay rate of HATT wakes*" and the authors recommend further experimental investigation of the structure of turbine wakes. Indeed, only a few of the computational studies have considered three-dimensional phenomena, which are crucial to understanding of the small-scale physics in the wake. The detailed structure of the wake of a tidal turbine deployed in a non-uniform tidal stream is the focus of the present research.

The most important three-dimensional feature of the flow through the rotor of a tidal turbine is a series of helical vortex filaments down-stream of the device. The presence of the rotor in the flow generates a vorticity field, which is trailed from the entire span of each blade of the tidal turbine, producing a sheet-like vortex structure in the flow immediately behind each blade. The sheet then rolls up to form concentrated vortices. The vortices induce a local velocity field of a magnitude that is proportional to the circulation around the blade [5]. Whether the local flow velocity surpasses the velocity of the background flow in the neighbourhood of the rotor depends on the flow conditions and on the parameters of the tidal turbine. The magnitude of the induced velocity can be amplified when the device is mounted close to the seabed, as a result of the interaction between the vortex filaments and the ground plane [6]. The local effect of the vortex-induced velocities on the flow contributes to the impact of a tidal turbine on the marine environment.

The technique used in the present work simulates directly the evolution of the vorticity field which is referred to as vorticity transport. Although this technique is not new, it has been utilized to study the wake of a tidal turbine only recently. Pinon et al. [7] have implemented a particle representation of the wake vorticity down-stream of a horizontal-axis tidal turbine in a uniform current. The numerical results indicate however a high level of numerical dissipation in comparison with experimental data [7].

The rotor wake has been previously subjected to thorough investigation in the field of helicopter aerodynamics. The findings of studies which employed the Vorticity Transport Model (VTM) have been repeatedly confirmed by experiments [8], [9], [10]. According to Fletcher et al. [10], the numerical diffusion of vorticity within the flow field surrounding the turbine is kept at a very low level by using a Riemann problem technique based on the Weighted Average Flux method to advance the Vorticity Transport Equation (1) through time [10], [11]. Fletcher states: "*This approach allows the structure of the wake to be captured at significantly larger wake ages, without significant spatial smearing of the wake structure, than is possible when using more conventional computational fluid dynamics (CFD) techniques based on the pressure-velocity-density formulation of the Navier-Stokes equations*". The structure of the vorticity field for a typical case simulated by the VTM by Phillips [8] and has been validated through comparison with experimental data from Lee et al. [12], providing confidence that the experience gained from the past wake studies can be applied to the analysis of the wake of a tidal turbine. The VTM, which has been modified to study the wake of a tidal turbine in a non-uniform flow, has been used in the present research. The modified VTM provides a detailed representation of the fluid dynamics that takes place within the wake of a rotor.

II. Hydrodynamic modelling

A series of high-resolution computer simulations has been conducted using the VTM. The VTM solves the equations of motion of an incompressible fluid in velocity-vorticity form. The evolution of the vorticity field, defined as the curl of the velocity field, is governed by

the *Vorticity Transport Equation*

$$\frac{\partial \omega}{\partial t} + (\mathbf{v} \cdot \nabla)\omega - (\omega \cdot \nabla)\mathbf{v} = S_\omega, \tag{1}$$

which results from taking the curl of the unsteady, incompressible

Navier-Stokes equations

$$\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} = -\frac{\nabla p}{\rho} + \nu \nabla^2 \mathbf{v} \tag{2}$$

$$\nabla \cdot \mathbf{v} = 0, \tag{3}$$

where the source of the vorticity field S_ω represents the influence of the tidal turbine on the flow.

The seawater is approximated as an incompressible fluid with a constant density, hence $\nabla \cdot \mathbf{v} = 0$. As a result, the rotational part of the velocity field can be calculated as a solution of the Poisson equation

$$\nabla^2 \mathbf{v} = -\nabla \times \omega. \tag{4}$$

The velocity field is composed of the inflow velocity and the velocity induced by the presence of the tidal turbine. When the inflow is approximated as uniform, the inflow velocity is irrotational and the only vorticity present in the flow is induced by the presence of the tidal turbine.

The VTM has been modified to include a non-uniform model. This model uses an approximation of a tidal flow in which the magnitude of the flow velocity decreases with increasing proximity to the seabed. The approximation of the tidal flow (also referred to as *free-stream*) velocity is a steady logarithmic velocity profile, defined as $\mathbf{v}_{fs} = (V_{in}(z), 0, 0)$ and

$$\frac{\partial \mathbf{v}_{fs}}{\partial t} = 0. \tag{5}$$

The approximation of the tidal flow velocity as time-invariant relies on the assumption that the changes of the inflow velocity during the period of the wake development are very small in comparison with the tidal cycle. Indeed, the variation of the inflow velocity with time can be omitted from consideration since the typical time-scale of the wake development is around 90 s (and the helical structure typically starts disintegrating within 50 s).

The incorporation of the new inflow model into the VTM has brought additional modelling challenges compared with the uniform inflow case since the non-uniform free-stream contains vorticity which influences the vorticity induced by the presence of a tidal turbine in the flow. The free-stream vorticity is defined as $\omega_{fs} = \nabla \times \mathbf{v}_{fs}$ and is considered steady over the period of the wake development. Since the free-stream contains vorticity, the free-stream velocity is rotational by definition, and the governing vorticity transport equation can be rewritten as follows

$$\frac{\partial \omega_I}{\partial t} + (\mathbf{v} \cdot \nabla)\, \omega_I - (\omega_I \cdot \nabla)\, \mathbf{v} = S_\omega - (\mathbf{v} \cdot \nabla)\omega_{fs} + (\omega_{fs} \cdot \nabla)\mathbf{v}, \tag{6}$$

where the terms on the right hand side containing ω_{fs} can be perceived as additional sources of the induced vorticity field $\omega_I \equiv \omega - \omega_{fs}$.

In order to study the effects of the free-stream vorticity on the development of the wake of a tidal turbine, the induced vorticity field is treated as a composition of two vector fields ω_w and ω^*. These vorticity fields are defined as the solutions of the two equations

$$\frac{\partial \omega_w}{\partial t} + (\mathbf{v} \cdot \nabla)\omega_w - (\omega_w \cdot \nabla)\mathbf{v} = S_\omega \tag{7}$$

and

$$\frac{\partial \omega^*}{\partial t} + (\mathbf{v} \cdot \nabla)\omega^* - (\omega^* \cdot \nabla)\mathbf{v} = -(\mathbf{v} \cdot \nabla)\omega_{fs} + (\omega_{fs} \cdot \nabla)\mathbf{v}. \tag{8}$$

The vorticity component ω^*, called the tide induced wake vorticity (*TIW vorticity*), is interpreted as a reaction of the wake to the non-uniform inflow, since the TIW vorticity is zero when the inflow velocity is uniform. ω_w is called here the rotor induced wake vorticity (*RIW vorticity*). The total vorticity within the flow around the tidal turbine is equal to the superposition of the three vector fields, $\omega = \omega_w + \omega^* + \omega_{fs}$. The flow velocity is then obtained from the total vorticity as a solution of (4).

III. Results

The device used in the simulations is a horizontal-axis tidal turbine (HATT) shown in Figure 1.

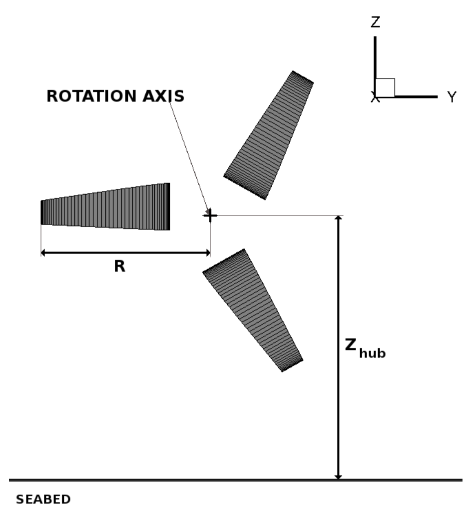

Fig. 1. The blades of the HATT.

TABLE I
PARAMETERS OF THE HATT

R [m]	N_b	z_{hub} [m]	Aerofoil	AR	Solidity	Rated power [kw]
5	3	7.5	NACA0012	4.65	0.205	150

The parameters of the HATT are listed in Table 1, where R is the rotor radius, N_b is the number of blades and AR= l^2/S is the aspect ratio of the blades. S is the surface area and l is the length of the blades, which have a twist of 18.58 degrees from root to tip. The solidity of the HATT is the ratio between the surface area of its blades and the overall rotor disc area, $S/\pi R^2$. The roots of the blades are located at $0.23R$ from the centre of the rotor. The HATT is rated for 150kW power output in a stream of 2 m/s.

The vorticity transport equation (6) is solved using the Finite Volume method implemented on a structured mesh composed of cells with uniform dimension $\Delta x = 0.04R$. The computational domain is shown in Figure 2. The initial condition for the velocity field which has been used to approximate tidal flow is illustrated in Figure 3.

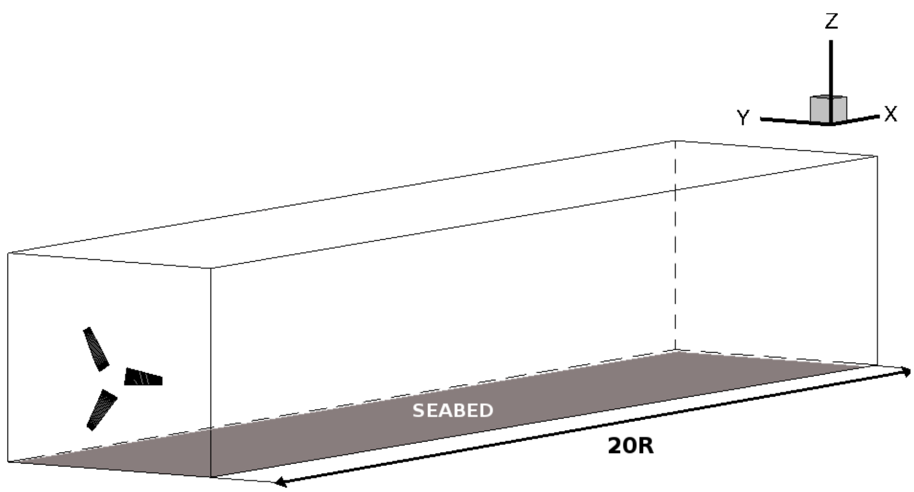

Fig. 2. The computational domain with the blades of the HATT.

Boundary conditions of zero curvature for the induced vorticity are set on all boundaries, except the bottom plane and the rotor plane. The boundary condition at the rotor plane has been defined as $\omega = \omega_{fs}$, which is consistent with the initial condition for the total vorticity field. The bottom boundary is modelled as impermeable by the method of images to ensure that the normal component of the wake induced velocity field is zero. The implementation of the method of images in the VTM is explained in [8].

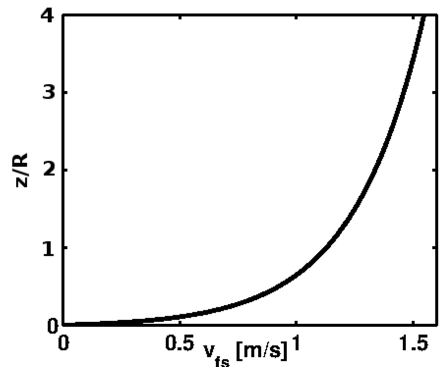

Fig. 3. The velocity profile used in the tidal inflow model.

One of the biggest advantages of using the VTM to predict the structure of the wake is the detail to which the vorticity field is calculated. The value of the presented results lies in the detailed assessment of small-scale flow phenomena, and this detailed assessment of the wake further than $12R$ down-stream of the turbine remains unchallenged by studies which have relied on conventional CFD techniques (for example [3] and [13]). Churchfield et al. [2] (who used a LES equipped CFD solver) states a limitation of their modelling approach: "*wall-modeled LES does not resolve down to where the actual physically-occurring peak in streamwise velocity variance occurs.*" The study [2] is an exception among other CFD studies, however, since it contains an investigation of the wake structure up to $12R$ down-stream of the rotor. Here, the wake structure up to $20R$ down-stream of the turbine is visualised by an iso-surface of the induced vorticity magnitude in Figures 4, 5, 6 and 7. The Figure 8 shows a section (which is perpendicular to the rotor plane and the bottom plane) through the induced vorticity field.

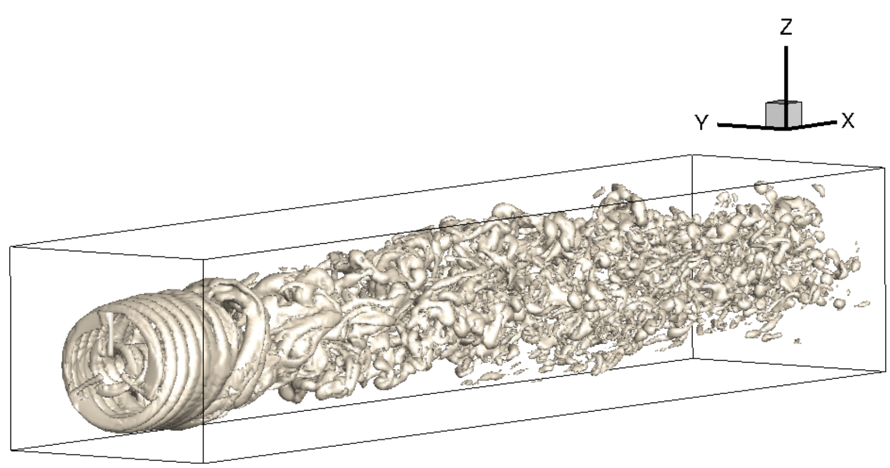

Fig. 4. The induced vorticity field in the computational domain illustrated by the iso-surface of 10% of the maximal magnitude of the induced vorticity.

Figure 5 illustrates the evolution of the induced vorticity field from an initial stage of the simulation until the wake is fully developed. The tip vortex filaments form a helical structure immediately down-stream of the HATT. The induced vorticity structure then advances down-stream propelled by the non-uniform flow. The helical structure becomes inclined further down-stream of the device and starts to fragment. The fragments of vorticity are then elevated from the seabed by the flow.

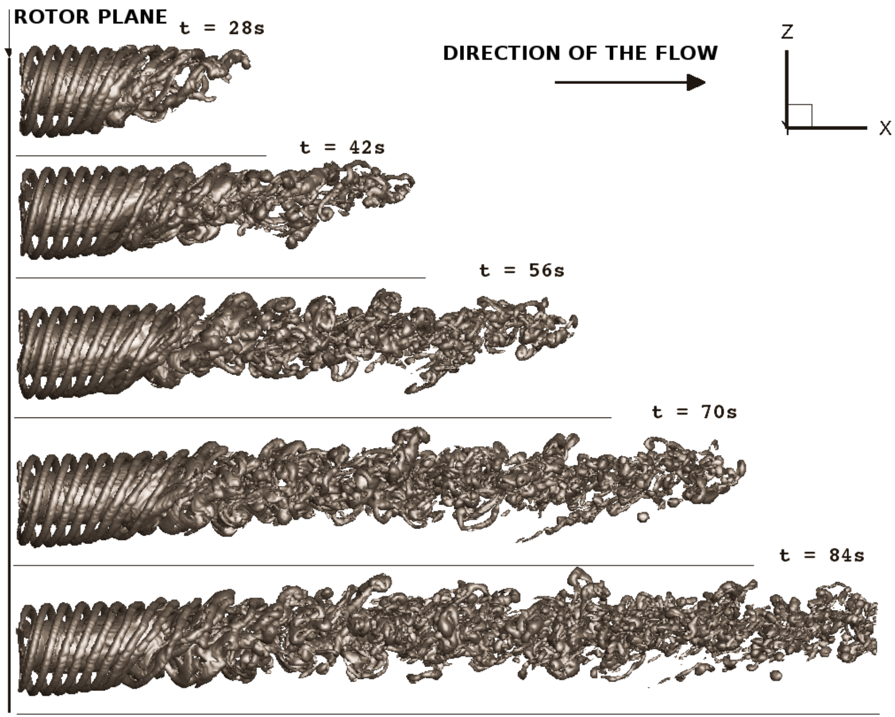

Fig. 5. The evolution of the induced vorticity field. The horizontal black lines denote the position of the seabed.

The role of the inflow model in the development of the HATT's wake is illustrated in Figure 6.

Over the duration of the simulation, the maximal magnitude of the TIW vorticity was less than 10% of the maximal magnitude of the RIW vorticity. However, although the quantitative change of the vorticity magnitude due to the non-uniform inflow is small, the effect on the structure of the vorticity field is considerably more significant.

The fragmented vorticity has been elevated away from the seabed in the case of the non-uniform inflow. Moreover, fragmentation of the helical structure of the wake commences earlier in this case, and the small scale structures are transported further from the seabed, never to return during the simulation. Such behaviour indicates the importance of the inflow in the assessment of the influence of the wake of a HATT on the surrounding environment.

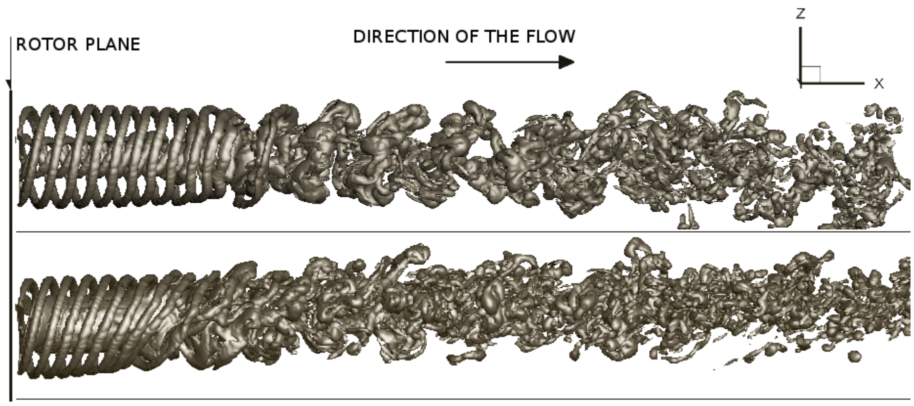

Fig. 6. Comparison of the induced vorticity field for the two different inflow models, $v_{tip}/|\mathbf{v}_{fs}(z_{hub})| = 4$, where v_{tip} is the rotational velocity of the blade tips. The horizontal black lines denote the position of the seabed. Top: Uniform inflow, bottom: A velocity profile.

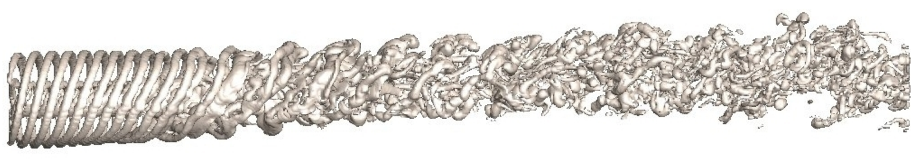

Fig. 7. The induced vorticity field when the wake has fully developed. The horizontal black line denotes the position of the seabed.

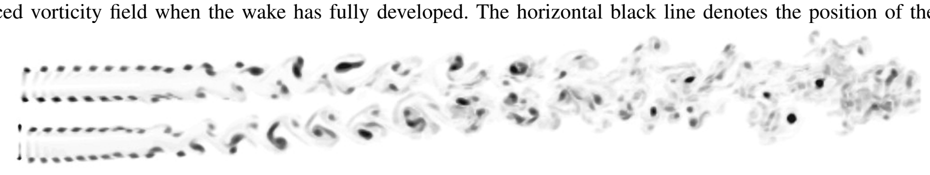

Fig. 8. Section through the induced vorticity field shown in Figure 7.

The striking difference between the wake of the HATT and the wake of a wind turbine is the inclination of the vortical structure down-stream of the HATT and its subsequent disintegration and uplift from the seabed due to the shear in the flow (given that the rotor is close to the seabed than is a wind turbine rotor to the ground).

IV. Conclusions

A finite-volume CFD model, the Vorticity Transport Model, has been enhanced for the purpose of simulating the wake of a horizontal-axis tidal turbine subjected to a non-uniform flow. High-resolution computer simulations suggest that a progressive fragmentation of the vortical structure occurs during the development of the wake of a tidal turbine. The predicted fragmentation generates small-scale unsteady flow phenomena beyond five rotor diameters down-stream of the device in the area previously thought unaffected by the presence of a tidal turbine. The effects of non-uniform flow on the vorticity structure down-stream of a tidal turbine and the fragmentation process have been analysed in this work. The role of the inflow in the wake evolution has been investigated by a comparison of the induced vorticity structure for two inflow models. In the case of a uniform inflow velocity the vorticity fragments have repeatedly approached the seabed down-stream of the HATT. When a non-uniform inflow model has been used to approximate the tidal flow the fragmented vortex structure has been elevated from the seabed down-stream of the HATT. Moreover, the fragmented vorticity has been predicted to remain clear of the seabed for the duration of the simulation. These results suggest that the inflow conditions have a profound influence on the development of the wake of the HATT.

V. Acknowledgments

This research was funded by James Watt Research and EPSRC scholarships as part of a PhD project at the University of Glasgow.

References

[1] Bahaj, A.S. “Marine current energy conversion: the dawn of a new era in electricity production”, *Phil. Trans. R. Soc.* , Vol. 371 (A), 2013.

[2] Churchfield, M.J., Li, Y. and Moriarty, P.J. “A Large-Eddy Simulation Study of Wake Propagation and Power Production in an Array of Tidal-Current Turbines”, *Proceedings of European Wave and Tidal Energy Conference*, 2011.

[3] Masters, I., Malki, R., Williams, A.J. and Croft, N.T. “The influence of flow acceleration on tidal stream turbine wake dynamics: A numerical study using a coupled BEMCFD model” *Applied Mathematical Modelling*, Vol. 37, pp. 7905-7918, June 2013.

[4] S.C. Tedds, S.C., Owen, I. and Poole, R.J. ”Near-wake characteristics of a model horizontal axis tidal stream turbine” *Renewable Energy*, Vol. 63, pp. 222 - 235, September 2013.

[5] Leishman J.G. *“Principles of Helicopter Aerodynamics”*, Cambridge University Press, 2006.

[6] Whitehouse, G.R and Brown, R.E. “Modelling Rotor Wakes in Ground Effect”, *Journal of the American Helicopter Society*, Vol.49 (3), pp. 238-249, July 2004.

[7] Pinon, G., Mycek, P., Germain, G. and Rivoalen, E. “Numerical simulation of the wake of marine current turbines with a particle method” *Renewable Energy*, Vol. 46, pp. 111-126, October 2012.

[8] Phillips, C. “Computational Study of Rotorcraft Aerodynamics in Ground Effect and Brownout”, *University of Glasgow PhD thesis*, 2010.

[9] Scheurich, F. “Modelling the Aerodynamics of Vertical-Axis Wind Turbines”, *University of Glasgow PhD thesis*, 2011.

[10] T.M. Fletcher, T.M., Brown,R., Kim, D. and Kwon, O.J “Predicting Wind Turbine Blade Loads using Vorticity Transport and RANS Methodologies”, *EWEC2009 conference proceedings*, 2009.

[11] Brown, R. E., and Line, A. J. “Efficient High-Resolution Wake Modelling Using the Vortex Transport Equation,” *AIAA Journal*, Vol. 43, No. 7, pp. 1434–1443, 2005.

[12] Lee, T.E., Leishman, J.G. and Ramasamy, M. ”Fluid Dynamics of Interacting Blade Tip Vortices With a Ground Plane”, *64th Annual Forum of the American Helicopter Society*, April 2008.

[13] AbdelSalam, A.M. and Ramalingam, V. “Wake prediction of horizontal-axis wind turbine using full-rotor modeling”, *Journal of Wind Engineering and Industrial Aerodynamics*, Volume 124, Pages 719, January 2014.