Self-fulfilling Mistakes: Characterization and Welfare*

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Abstract

This paper incorporates self-fulfilling mistakes into an otherwise classical decision-making framework. A behavioural agent makes a mistake when he fails to internalize all the consequences of his actions on himself. We show that Sen's axioms α and γ fully characterize choice data consistent with behavioral agents. These two axioms are weaker than Sen's axioms α and β that fully characterize rational agents. We offer a welfare benchmark that can be applied to existing behavioural economics models and show the conditions under which our welfare ranking can be used to infer welfare dominated (i.e. mistaken) choices using choice data alone.

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Standard normative economics employs the revealed preference approach to extract welfare measures from choice data. The *preferences revealed* from the individual's choices are assumed to be identical to the *normative preferences* representing the individual's best interest. There is, however, considerable empirical evidence that in an array of different situations, individuals do not appear to act in their own best interest, establishing a potential wedge between normative and revealed preferences.[1]

How should welfare analysis be performed if choices do not always reveal decision-makers' (DMs) best interest? One approach, advocated in an influential contribution by Bernheim and Rangel (2009) (hereafter BR), proposes a welfare criterion that can be applied even when observed choices are inconsistent: briefly, x is (strictly) unambiguously chosen over y if y is never chosen when x is available.[2] Hence, regardless of how poorly behaved choice correspondences may be, their criterion implies that every action chosen (within a welfare-relevant domain) is a (weak) welfare optimum (BR, observation 1, pg. 62). While this approach can exploit the coherent aspects of choice in a variety of behavioural models, it is silent about situations in which DMs may make mistakes in a systematic way, acting against their own best interest. That is, x may be unambiguously chosen over y, but still be against the DM's best interest. This is particularly relevant for models of addiction, projection bias, aspirations failure or overconfidence.

In light of the above, this paper studies DMs who, in contrast to rational ones, may not fully internalize all the consequences of their actions on themselves. We offer a general framework to model self-fulfilling mistakes, provide a choice-theoretic characterization of that framework, contrast it with the rational framework and propose a revealed welfare ranking able to identify dominated choice under a domain restriction.

Section 1 introduces the general framework, shows existence of a solution and describes some behavioural models encompassed in our framework. We model a DM who must choose between mutually exclusive actions. Each action impacts on payoffs, both directly and indirectly through its effect on a psychological state via a feedback function. We broadly interpret a psychological state as any pay-off relevant endogenous preference parameter that is assumed to be normatively irrelevant in a conventional account of rationality. This includes frames, reference points, beliefs, expectations, aspirations, states of mind, emotions, moods, deadlines, default options, etc. The DM's preferences rank both actions and psychological states. Mistakes (suboptimal behaviour) arise due to the DM's propensity to undertake actions without fully internalizing their consequences on own psychological

[1]Loewenstein and Ubel (2008) point out that in the "heat of the moment," people often take actions that they would not have intended to take and that they soon come to regret (Loewenstein, 1996). Koszegi and Rabin (2008) and Beshears et. al. (2008) review empirical evidence of systematic mistakes people make. Bernheim and Rangel (2007) record situations in which it is clear that people act against themselves: an anorexic's refusal to eat; people save less than what they would like; people fail to take advantage of low interest loans available through life insurance policies; they unsuccessfully attempt to quit smoking; they maintain substantial balances on high-interest credit cards; etc.

[2]Salant and Rubinstein (2008) make a similar point in their analysis of choice with exogenous frames.

states.[3]

We consider two types of decision making procedures: a Standard Decision Procedure (SDP) and Behavioural Decision Procedure (BDP). In a SDP, the DM fully internalizes the feedback from actions to psychological states, and chooses an action that maximizes his preferences over consistent pairs of actions and psychological states. This is equivalent to rational decision-making in a context with endogenous psychological states.[4] In a BDP, in contrast, a (behavioural) DM fails to internalize the effect of his action on his psychological state, and chooses an action taking as given his psychological state although psychological states and actions are required to be mutually consistent at a BDP outcome, i.e. the mistakes the DM makes are self-fulfilling. This is a form of bounded rational decision-making in a context where psychological states are endogenous.[5] Importantly, note that choices of behavioural DMs can be systematically coherent (in BR's sense) but yet be suboptimal.

Despite its simplicity, by appropriately specifying actions sets, psychological states, the feedback from actions to psychological states and the domain of preferences, our framework unifies seemingly disconnected models in the literature, from more recent positive behavioural economics models to older ones. In Section 1.3 we illustrate this feature by linking our framework to models of status-quo bias, reference-dependent consumption, time-inconsistent preferences, adaptive preferences, utility from anticipation and aspirations.

Section 2 introduces the axioms that fully characterize both decision procedures (BDP and SDP) based solely on choice data. We assume that neither the set of psychological states nor the feedback function are directly observable. Rather, they have to be inferred from choice behavior. We show that (i) Sen's (1971) axioms α and β fully characterize an admissible SDP,[6] (ii) observed choices are compatible with an BDP if and only they satisfy Sen's (1971) axioms α and γ,[7] and

[3]The key premise is that individuals are able to forecast the short-run consequences of their actions on preferences (e.g., the immediate, desirable nicotine rush from smoking) but they cannot forecast correctly the consequences of their actions on psychological states and their impact on preferences (e.g. nicotine dependence resulting from smoking). There is evidence that individuals misjudge either self or contingencies (Baumeister and Scher, 1988), fail to bear in mind that they will adapt (Ubel, 2005), overestimate regret or rejoicing (Sevdalis and Harvey, 2007), underestimate the use of coping strategies in the face of negative shocks (Lazarus and Flokman, 1984; Deci and Ryan, 2008), etc.

[4]Example of standard DMs are people who self-impose deadlines to overcome procrastination (Ariely and Wertenbroch, 2002), who limit the number of alternatives (limited focus) as a self-control device to avoid regret (Carrillo and Mariotti, 2000) or who choose an optimal aspiration level as motivator of effort (Heath et al., 1999).

[5]In the online Appendix, we extend our framework and allow for DMs who partially internalize the consequences of their actions. The main results of this paper still hold for this more general case.

[6]In an *admissible* SDP, the ranking over consistent decision states is transitive. A *consistent* decision state is a pair of an action and psychological state so that the psychological state is an element of the feedback function.

[7]Sen's axiom α states that the choice correspondence is (weakly) increasing as the choice set shrinks when all alternatives chosen in the larger set are also present in the smaller set. This axiom was also introduced by Chernoff (1954). Sen's axiom β states that when two actions are both chosen in a given set, and one of them is chosen in a larger set that includes the first set, then both are chosen in the larger set. Sen's axiom

(iii) whenever choice data satisfy Sen's axioms α and γ (but violate β), it is required at least two psychological states for such data to be rationalized as the outcome of an admissible BDP.[8]

The axiomatic characterization of SDPs and BDPs is relevant for two main reasons. First, it shows that regardless how seemingly disconnected some behavioural economic models may be, they are characterized by the same underlying choice structure. Second, the axiomatic characterization allows us to study the normative implications of boundedly rational decision-making basing our analysis on the information inferred from choice data.[9]

We devote Section 3 to discuss the insights for welfare analysis derived from our framework. We first define the appropriate welfare benchmark for our analysis as being the ranking over the set of actions induced by the ranking of consistent decision states. The choice of this ranking is based on the following two results: (i) by Proposition 2, this ranking coincides with revealed preferences in the sense of Samuelson (1938), and (ii) all admissible standard decision scenarios that rationalize a specific piece of choice data imply the same ranking over actions. Then, we show that, under a domain restriction on preferences, it is possible to reconstruct this welfare ranking from choice data when the DM solves a BDP and to infer the existence of welfare dominated choice. We finalize by applying our normative benchmark to some behavioural economics models reduced to our framework and by examining the implications of partial internalization of the impact of actions on psychological states. There we show that the welfare impact of a greater degree of internalisation may be ambiguous.

All proofs are contained in the appendix. The online appendix contains some extensions, mainly technical, of certain aspects of the model studied in this paper.

1 The General Framework

1.1 *The Model*

The primitives of the model consist of a set $A \subseteq \Re^K$ of actions, a set $P \subseteq \Re^L$ of psychological states and a function $\pi : A \to P$ modeling the feedback effect from actions to psychological states. It is assumed that $\pi(a)$ is non-empty and single-valued for each $a \in A$. A decision state is a pair of an action and psychological state (a, p) where $a \in A$ and $p \in P$. A *consistent* decision state is a decision state (a, p) such that $p = \pi(a)$.[10]

The preferences of the DM reflects some form of ex-post utility (interpreted as experienced utility) which depends on the chosen action a and the psychological state p. Following Harsanyi

γ states that if an action is chosen in each set in a class of sets, it must be also chosen in their union.

[8]In an *admissible* BDP, the ranking over actions for a given psychological state is transitive.

[9]See Manzini and Mariotti (2014) who contrast a modeless and a model-based approach to welfare economics in the presence of a boundedly rational decision-maker and make the case for the model-based approach.

[10]In the main body of the paper, it is assumed that $\pi(.)$ is single valued. The case where $\pi(.)$ is multi-valued is studied in the online Appendix.

(1954) we assume intrapersonal comparability of utility. That is, the DM is not only able to rank different elements in A for a given p, but he is also able to assess the subjective satisfaction he derives from an action when the psychological state is p with the subjective satisfaction he derives from another action when the psychological state is p'. In other words, we assume that the DM is able to rank elements in $A \times P$. Given intrapersonal comparability of utility, the preferences of the DM are denoted by $\succeq$, a binary relation ranking pairs of decision states in $(A \times P) \times (A \times P)$. A decision scenario is, thus, a collection $D = (A, P, \pi, \succeq)$.

We study two decision procedures:

1. Given a non-empty *feasible* set of actions $A' \subseteq A$, a *standard decision procedure* (SDP) is one where the DM chooses a consistent decision state (a, p), $a \in A'$ and $p = \pi(a)$. The outcomes of a SDP, denoted by $S \subseteq A \times P$, are

$$S = \left\{(a, p) : (a, \pi(a)) \succeq \left(a', \pi(a')\right) \text{ for all } a' \in A'\right\}.$$

2. Given a non-empty *feasible* set of actions $A' \subseteq A$, a *behavioural decision procedure* (BDP) is one where the DM takes as given the psychological state p when choosing $a \in A'$. For a fixed $p \in P$, define a preference relation $\succeq_p$ over A as follows:

$$a \succeq_p a' \Leftrightarrow (a, p) \succeq \left(a', p\right).$$

The outcomes of a BDP, denoted by $B \subseteq A \times P$, are

$$B = \left\{(a, p) : a \succeq_{p=\pi(a)} a' \text{ for all } a' \in A'\right\}.$$

In both, SDPs and BDPs, a decision outcome must be a consistent decision state where the action is chosen from some feasible set of actions. In a SDP, the DM internalizes that his psychological state is determined by his action via the feedback effect when choosing an action from the set of feasible actions. In a BDP, the DM takes the psychological state as given when he chooses an action from the set of feasible actions although the psychological state is required to be consistent with the action actually chosen by the DM.[11]

1.2 *Existence*

Motivated by the literature of behavioural economics, we prove existence of solutions to a SDP and a BDP allowing for preferences to be incomplete, non-convex and acyclic (i.e. not necessarily

[11]In subsection 1.3.2 below, we show that a BDP corresponds to the steady-state outcome of an adaptive preference adjustment mechanism.

transitive) and action sets to be non-convex.[12] We show (i) existence of a solution to a SDP applying Bergstrom (1975) and (ii) existence of a solution to a BDP extending Ghosal's (2011) result for normal-form games.[13] In what follows, we endow $\Re^K \times \Re^L$ with the standard topology.

Recall that the preferences of the DM are denoted by $\succeq$, a binary relation ranking pairs of decision states in $(A \times P) \times (A \times P)$. Let $\succ_p$ (resp. $\succ$) denote the strict (asymmetric) preference relation corresponding to $\succeq_p$ (resp. $\succeq$) i.e. $a \succ_p a'$ if and only if $a \succeq_p a'$ but $a' \not\succeq_p a$ (resp. $(a,p) \succ (a',p')$ if and only if $(a,p) \succeq (a',p')$ but $(a',p') \not\succeq (a,p)$). Define the sets $\succ_p (a) = \{a' \in A : a' \succ_p a\}$ (the upper section of $\succ_p$), $\succ_p^{-1} (a) = \{a' \in A : a \succ_p a'\}$ (the lower section of $\succ_p$). Define the sets $\succ (a,p) = \{(a',p') \in A \times P : (a',p') \succ (a,p)\}$ (the upper section of $\succ$), $\succ^{-1} (a,p) = \{(a',p') \in A : (a,p) \succ (a',p')\}$ (the lower section of $\succ$). Define a map $\Psi : P \to A$, where $\Psi(p) = \{a' \in A : \succ_p (a') = \emptyset\}$: for each $p \in P$, $\Psi(p)$ is the set of maximal elements of the preference relation $\succ_p$. Consider the following assumptions:

(A1) for each $p \in P$,

(i) $\succ_p$ is acyclic i.e., there is no finite set $\left\{a^1, ..., a^T\right\}$ such that $a^{t-1} \succ_p a^t$, $t = 2, ..., T$, $a^T \succ_p a^1$, or equivalently, $\succeq_p$ is P-acyclic,

(ii) $\succeq_p$ is complete,

(iii) $\succ_p^{-1} (a)$ is open relative to A i.e. $\succ_p$ has an open lower section,[14]

(A2) A, P are both compact lattices with the vector ordering and π is an increasing, continuous function,[15]

(A3) For each p, and a, a', (i) if $a \succeq_p \inf(a,a')$, then $\sup(a,a') \succeq_p a'$ (ii) if $a \succeq_p \sup(a,a')$ then $\inf(a,a') \succeq_p a'$ (quasi-supermodularity),

(A4) For each $a \geq a'$ and $p \geq p'$, (i) if $a \succeq_{p'} a'$, then $a \succeq_p a'$ and (ii) if $a' \succeq_p a$ then $a' \succeq_{p'} a$ (single-crossing property),

(A5) For each p and $a \geq a'$, (i) if $\succ_p (a') = \emptyset$ and $a \succeq_p a'$, then $\succ_p (a) = \emptyset$, and (ii) $\succ_p (a) = \emptyset$ and $a' \succeq_p a$, then $\succ_p (a') = \emptyset$ (monotone closure).

Assumptions A3-A4 were introduced by Milgrom and Shannon (1994) and A5 by Ghosal

[12] Mandler (2005) shows that incomplete preferences and intransitivity is required for "status quo maintenance" to be outcome rational. Tversky and Kahneman (1991) argue that reference-dependent preferences may not be convex.

[13] The seminal proof for existence of equilibria with incomplete preferences in Shafer and Sonnenschein (1975) requires convexity both to show existence of an optimal choice and to apply Kakutani's fix-point theorem.

[14] The continuity assumption, that $\succ_p$ has an open lower section, is weaker than the continuity assumption made by Debreu (1959) (who requires that preferences have both open upper and lower sections), which in turn is weaker than the assumption by Shafer and Sonnenschein (1975) (who assume that preferences have open graphs). Note that assuming $\succ_p$ has an open lower section is consistent with $\succ_p$ being a lexicographic preference ordering over A.

[15] A lattice is a partially ordered subset of $\Re^k$ with the vector ordering (the usual component wise ordering: $x \geq y$ if and only if $x_i \geq y_i$ for each $i = 1, .., K$, and $x > y$ if and only if both $x \geq y$ and $x \neq y$, and $x \gg y$ if and only if $x_i > y_i$ for each $i = 1, .., K$) which contains the supremum and infimum of any two of its elements. A lattice that is compact (in the usual topology) is a compact lattice.

(2011).[16] We are now in a position to state the following existence result:

PROPOSITION 1. *(i) Under assumptions A1-A5, a solution to a BDP exists. (ii) Suppose A is compact and* π is a continuous function. *If* $\succ$ *is acyclic and has an open lower section, a solution to a SDP exists.*[17]

Proof. See Appendix. ■

It is worth noting that the existence result in Proposition 1 above doesn't rely in an essential way on assumption A1 (i) of acyclicity. If, instead, we assumed that $\succeq_p$, for a fixed $p \in P$, was transitive, then assumption A5 of monotone closure could be dropped entirely. For later reference, we state this observation as the following result:

COROLLARY 1. *Suppose A is compact,* π *is a continuous function and* $\succeq_p$*, for a fixed* $p \in P$*, is transitive. Under assumptions A1 (ii), A1 (iii) and A2-A4, a solution to a BDP exists.*

Proof. Follows from Proposition 1 and Ghosal (2011). ■

Hence, from the viewpoint of Proposition 1, the assumption that $\succeq_p$ for a fixed $p \in P$ is transitive is a substitute for the assumption that $\succeq_p$ for a fixed $p \in P$ is acyclic and the assumption of monotone closure.

1.3 *Reduced Form Representation*

We define psychological states as endogenous features of the decision-making environment that the DM may (mistakenly) not internalize. In this part of the paper, we show, via a number of examples, how our framework is a reduced form representation of two very different sets of models with endogenous psychological states: (a) models in which psychological states are endogenous because they are determined by actions (e.g. emotions, aspirations, etc.), and (b) models in which endogenous psychological states are an equilibrium condition. Both notions of endogeneity are captured by our framework. For an example of the latter model, consider Tversky and Kahneman (1991)'s reference-dependent theory of riskless choice. In their framework, preferences do not only depend on consumption bundles but also on a reference consumption bundle which "usually corresponds to the decision-maker's current position" (Tversky and Kahneman, 1991, pp. 1046). In that sense, their idea of a reference point is interpreted in our framework as a psychological state defined in equilibrium as DM's current consumption.

A comprehensive list of all such models is beyond the scope of this paper. Nonetheless, in what follows, we take a few well-known behavioural economic models and make explicit the mapping to our framework by identifying the π, P and A of each model, and indicating which decision-making

[16] Consider a pair of actions such that the first action is greater (in the usual vector ordering) than the second action. For a fixed p, suppose the two actions are unranked by $\succ_p$. Then, assumption A5 requires that either both actions are maximal elements for $\succ_p$ or neither is maximal. The role played by this assumption in obtaining the monotone comparative statics is clarified in Ghosal (2011).

[17] Note that the conditions for the existence of a solution to a SDP are weaker than the conditions for the existence of a solution to a BDP.

procedure (i.e. SDP or BDP) they assume.[18]

1.3.1 *Dynamic Inconsistency(Strotz, 1956; Peleg and Yaari, 1973; Laibson, 1997)*

Here we illustrate how the standard models of time-inconsistent preferences (e.g. Strotz, 1956, Peleg and Yaari, 1973 and Laibson, 1997) can be reduced to our framework. Consider a three period problem $t = 1, 2, 3$ where at each t, the DM must choose action a_t, where $a_1 \in A_1$, $a_2 \in A_2(a_1)$, $a_3 \in A_3(a_1, a_2)$ and A_1, $A_2(a_1)$ for each a_1 and $A_3(a_1, a_2)$ for each a_1 and a_2 are non-empty sets of actions. Let $A_2 = \cup_{a_1 \in A_1} A_2(a_1)$ and $A_3 = \cup_{(a_1,a_2) \in A_1 \times A_2} A_3(a_1, a_2)$. The preferences of the DM over the action triple $(a_1, a_2, a_3) \in A_1 \times A_2 \times A_3$ are represented by: (i) $U_1 = u(a_1) + \beta\left[\delta u(a_2) + \delta^2 u(a_3)\right]$, (ii) $U_2 = u(a_2) + \beta\delta u(a_3)$, and (iii) $U_3 = u(a_3)$. Let

$$\widetilde{A}_3(a_1, a_2) = \arg\max_{a_3 \in A_3(a_1,a_2)} u(a_3), \qquad \widetilde{A}_2(a_1) = \arg\max_{a_2 \in A_1(a_1)} u(a_2) + \delta\beta u(\widetilde{A}_3(a_1, a_2)),$$

where it is assumed that both $\widetilde{A}_3(a_1, a_2)$ and $\widetilde{A}_2(a_1)$ are non-empty and single-valued.[19] Let

$$p \in P = A_2 \times A_3 \text{ and } p = (p_2, p_3) = \left(\widetilde{A}_2(a_1), \widetilde{A}_3\left(a_1, \widetilde{A}_2(a_1)\right)\right) = \pi(a_1).$$

P is the set of psychological states where a psychological state is an expectation of future actions at periods $t = 2, 3$ when calculating the optimal actions in period $t = 1$ with the feedback function $\pi(.)$ corresponding to the best responses of the two future selves to his action choice at $t = 1$.

A SDP is equivalent to a Strotz equilibrium where the DM at $t = 1$ solves

$$Max_{a_1 \in A_1} u(a_1) + \beta\left[\delta u\left(\widetilde{A}_2(a_1)\right) + \delta^2 u\left(\widetilde{A}_3\left(a_1, \widetilde{A}_2(a_1)\right)\right)\right].$$

A BDP is equivalent to the Nash equilibrium of the intra-self game proposed by Peleg and Yaari (1973) defined as (a_1^*, p^*) such that (i) $a_1^* \in \arg\max_{a_1 \in A_1} u(a_1) + \beta\left[\delta u(p_2^*) + \delta^2 u(p_3^*)\right]$, and (ii) $p^* = (p_2^*, p_3^*) = (\pi(a_1^*), \pi(a_1^*))$.

1.3.2 *Adaptive Preferences (von Weizsacker, 1971; Hammond, 1976; Pollak, 1978)*

In a number papers, von Weizsacker (1971), Hammond (1976) and Pollak (1978) study the steady

[18] Other models that have a reduced form representation in our framework include models of melioration (Herrnstein and Prelec, 1991), cognitive dissonance (Akerlof and Dickens, 1982), emotions (Bracha and Brown, 2007) and shrouded attributes (Gabaix and Laibson, 2006). In this latter case, for example, the psychological states can be interpreted as the (endogenous) costs of the add-ons (e.g. ink of a printer) that (behavioural) DMs fail to take into account at the moment of buying a base good (e.g. printer).

[19] In discussing the reduced form representation of time-inconsistent preferences (as well as in other models in this section), we assume uniqueness for expositional convenience. With non-uniqueness, it is necessary to extend the definition of a SDP and BDP to the case where $\pi(.)$ is no longer single-valued. We do this in the online Appendix.

states of adaptive preferences defined over consumption. Here we argue that the steady states of their models have a reduced form representation in our framework. As a by product, we also provide a dynamic interpretation of our framework.

Consider an adaptive preference adjustment mechanism where the preferences over actions at any t, denoted by $\succeq_{p_{t-1}}$, depend on the past psychological state. The statement $a \succeq_{p_{t-1}} a'$ means that the DM finds a at least as good as a' given the psychological state p_{t-1}. The DM takes as given the psychological state from the preceding period. Note that an outcome of a BDP corresponds to the steady state of an adjustment dynamics where the DM is myopic (i.e. does not anticipate that the psychological state at $t+1$ is affected by the action chosen at t). Let $h(p) = \{a \in A : a \succeq_p a',\, a' \in A\}$. For ease of exposition, assume that $h(p)$ is unique. Fix a $p_0 \in P$. A sequence of *short-run* outcomes is determined by the relations $a_t \in h(p_{t-1})$ and $p_t = \pi(a_t)$, $t = 1, 2, ...$ At each step, the DM chooses a myopic best-response.[20] *Long-run* outcomes are denoted by a pair (a, p) with $p = \pi(a)$ where a is defined to be the steady-state solution to the short-run outcome functions, i.e. $a = h(\pi(a))$. In other words, long-run behavior corresponds to a subset of the set of consistent decision states, namely those that are the outcome of a BDP.

In contrast, in a SDP, the DM is farsighted (i.e. anticipates that the psychological state at $t+1$ is affected by the action chosen at t). In this case, in each period t, the DM anticipates that p adjusts to a according to $\pi(\cdot)$ and taking this into account, chooses a. In a SDP therefore, at each t, the DM simply chooses between different consistent decision states: the outcome of a SDP at each t, is a pair (a_t, p_t) where $a_t \in \{a \in A : (a, \pi(a)) \succeq (a', \pi(a'))\,,$ for all $a' \in A\}$ and $p_t = \pi(a_t)$. Note that in this simple framework, at each period t, the DM anticipates that there is an instantaneous adjustment of the psychological state to the chosen action. Hence, the initial psychological state in period t, p_{t-1}, has no impact on the DM's choice. Moreover, with farsightedness, the dynamics of the preference adjustment mechanism is trivial as there is an instantaneous adjustment to the the steady-state outcome.[21]

Finally, note that a farsighted DM does not regret his choice. Suppose that $(a, p) \succ (a', p')$ and $(a, p) \prec (a', p)$ with $p = \pi(a)$ and $p' = \pi(a')$. Then the DM solving a SDP would choose action a, but in the subsequent period, when state p is realized he will not regret his choice although $(a, p) \prec (a', p)$, the DM will anticipate that if he chooses a' the psychological state will adjust to p' and, by assumption, $(a, p) \succ (a', p')$.

In the online Appendix we consider the case in which the DM anticipates changes in short-run

[20]Under the assumptions required to prove Proposition 1 (existence), $h(p)$ is increasing map of p so that the sequence of short-run outcomes is an (component-wise) increasing sequence (as by assumption contained in a compact set). Therefore it converges to its supremum, which is necessarily a BDP. So the existence result in Proposition 1 covers not only cases where a solution to a BDP (equivalently, a steady-state solution to the myopic preference adjustment mechanism) exists but also ensures that short-run outcomes converge to a BDP.

[21]Non-trivial dynamics would be associated with farsighted behavior if underlying preferences or action sets were time variant.

psychological states but not in the long-run. In next section, we extend our framework to one in which the psychological state is a vector of psychological states, and the DM correctly predicts the effect of his action on a subset of such vector and believes that he doesn't affect the complement.[22]

1.3.3 *Anticipatory Feelings (Caplin and Leahy, 2001)*

In Caplin and Leahy's (2001) model of anticipatory feelings, preferences do not only depend on consumption today, but also on the feeling of anticipation of future consumption. These (endogenous) feelings correspond to psychological states in our framework. We illustrate the link of Caplin and Leahy's (2001) and our framework by using a simple two-period deterministic version of their model.[23] Consider a DM who, at each $t = 1, 2$, chooses an action $a_1 \in A_1$ and $a_2 \in A_2(a_1)$. Let $A_2 = \cup_{a_1 \in A_1} A_2(a_1)$. An anticipatory feeling (e.g. anxiety) is a psychological state that depends on the anticipated action. Formally, they define a function (equivalent to π in our framework) $\mu : A_2 \to P$ that associates an action in period 2 to a psychological state. The instantaneous utility at $t = 1$ is $u_1(a_1, p)$ and the instantaneous utility at $t = 2$ is $u_2(a_2)$. The preferences of the DM at $t = 1$ are $u_1(a_1, p) + u_2(a_2)$ and the preferences of the DM at $t = 2$ are $u_2(a_2)$. Caplin and Leahy assume that the DM solves this problem by backward induction. First, given a_1 and p, the DM solves $Max\ u_2(a_2)$ s.t. $a_2 \in A_2(a_1)$, with $\tilde{A}_2(a_1)$ being the set of solutions to this problem.[24] Then, he solves $Max\ u_1\left(a_1, \mu\left(\tilde{A}_2(a_1)\right)\right) + u_2\left(\tilde{A}_2(a_1)\right)$ s.t. $a_1 \in A_1$, with $\tilde{A}_1$ being the corresponding set of solutions. An optimal solution (equivalent to a Strotz equilibrium) is then defined as a pair $(\tilde{a}_1, \tilde{a}_2)$ such that $\tilde{a}_1 \in \tilde{A}_1$ and $\tilde{a}_2 = \tilde{A}_2(\tilde{a}_1)$. Note that Caplin and Leahy assume that DMs solve a SDP: they internalize the effect of their actions on their level of anxiety. Alternatively, if the DM was behavioural, he would solve the following maximization problem: $Max\ u_1(a_1, p) + u_2\left(\tilde{A}_2(a_1)\right)$ s.t. $a_1 \in A_1$. Defining $\hat{A}_1(p)$ as the set of solutions of the preceding maximization problem, the set of BDP outcomes (equivalent to the Nash equilibrium studied by Peleg and Yaari, 1973) would consist of a triple (a_1^*, a_2^*, p^*) such that $a_1^* \in \hat{A}_1(p^*)$, $a_2^* = \tilde{A}_2(a_1^*)$ and $p^* = \mu(a_2^*)$.[25]

1.3.4 *Reference-dependent Consumption (Shalev, 2000; Kőszegi and Rabin, 2006)*

In Kőszegi and Rabin's (2006) model (see also Shalev, 2000), preferences not only depend on the consumption bundle chosen, but also on what the DM expects to consume in equilibrium.

[22] Considering the possibility of partial prediction of psychological states has an interesting normative implication though, which we discuss in the welfare section of the paper.

[23] We are aware that Caplin and Leahy (2001) is essentially a model of uncertainty. However, we chose a deterministic version only to avoid introducing new notation to the paper. By redefining actions and psychological states appropriately, it is possible to show that our framework is a reduced form representation of their model with uncertainty.

[24] For simplicity assume that $\tilde{A}_2(a_1)$ is non-empty and single valued for each a_1.

[25] Caplin and Leahy (2001) provide a set of axioms underpinning the representation of preferences with anticipatory feelings in an expected utility setting. In this sense, the choice-theoretic, axiomatic characterization we provide in this paper complements their work.

These (endogenous) expectations correspond to psychological states in our framework. Preferences are modeled as $u(c|r) = m(c) + n(c|r)$, where $m(c)$ is the intrinsic "consumption utility" that depends on a K-dimensional consumption bundle c, and $n(c|r)$, is the gain-loss utility relative to endogenous reference bundles, r. Both consumption utility and gain-loss utility are separable across dimensions, so that $m(c) = \sum_k m_k(c_k)$ and $n(c|r) = \sum_k n_k(c_k|r_k)$. They assume that $n_k(c_k|r_k) = \mu\left(m_k(c_k) - m_k(r_k)\right)$, where $\mu(.)$ satisfies the properties of Kahneman and Tversky's (1979) value function. The reference bundles are determined in a Personal Equilibrium (PE) (Kőszegi, 2010) by the requirement that they must be consistent with the optimal c computed conditionally on rational forecasts of r i.e., the consumption bundle c^* is a PE iff $u(c^*|c^*) \geq u(c'|c^*)$ for any other feasible consumption bundle c'. Thus, setting A and P to be the set of feasible consumption bundles (where a psychological state corresponds to the expected consumption) and π to be the identity map, in Kőszegi and Rabin's (2006), a PE is equivalent to a BDP equilibrium and SDP is equivalent to a Choice Acclimating Personal Equilibrium (CAPE) (where $u(c|c) \geq u(c'|c')$ for any other feasible consumption bundle c'). Let C^* denote the set of PE consumption bundles. A Preferred Personal Equilibrium (PPE) corresponds to $\max_{c^* \in C^*} u(c^*|c^*)$. Provided the set of feasible consumption bundles is compact and $u(c|c)$ is continuous in c, then a CAPE exists. The existence of a PE requires the solution to a fix-point problem. The existence of a PPE follows if the set of feasible consumption bundles is compact and $u(c|c)$ is continuous in c, as the set of PE is a closed subset of the set of feasible consumption bundles and hence, compact whenever the set of feasible consumption bundles is compact.

1.3.5 *Aspirations Failure (Dalton et al., 2016)*

One phenomenon that cannot be accommodated by existing models, but yet can be accommodated by our framework is the notion of aspiration failures, defined as the failure to aspire to one's own potential. Aspirations failures have been documented elsewhere in the literature of anthropology and sociology (e.g. Appadurai, 2004) and recently introduced to economics by Dalton's et al. (2016) application of the concepts of SDP and BDP.

In their model, the utility the individual derives from choosing effort e depends not only on the cost of effort $c(e)$ and the benefit $b(\theta)$ of achieving final wealth θ, but also on aspirations g. More specifically, $u(e, g, \theta) = b(\theta) + v(x) - c(e)$, where $x = \frac{\theta - g}{\theta}$ and $v(.)$ is a reference-dependent value function that captures gains and losses relative to the reference point g. When final wealth is equal to the aspired level of wealth $\theta = g$, there is no gain or loss relative to the reference point. Final wealth depends on effort e and initial wealth θ_0 via the production function $\theta = f(e, \theta_0)$. At any solution to the decision problem, aspirations are set equal to final wealth given effort i.e., $g = f(e, \theta_0)$, and $(e, g = f(e, \theta_0))$ is a consistent effort-aspirations pair. They consider two types of DMs: a *standard* and a *behavioural*. A standard DM chooses among consistent effort-aspirations pairs $(e, f(e, \theta_0))$, i.e., the feedback between effort and aspirations is fully internalized. Formally, a *standard* solution is then a pair $(\hat{e}, \hat{g})$ such that

$$\hat{e} \in \arg\max_{e\in[0,1]} s(e, \theta_0) = u(e, f(e, \theta_0), f(e, \theta_0)) = b(f(e, \theta_0)) + v\,(0) - c(e)$$

and $\hat{g} = f(\widehat{e}, \theta_0)$. As $v(0)$ enters the above expression as an additive constant, the induced utility at a SDP is reference-independent so that it is assumed without loss of generality that $v(0) = 0$. A behavioural decision maker, instead, does not fully internalize how their aspirations are determined by their effort choices. While choosing effort e, a behavioural decision-maker takes g as fixed, although at a *behavioural* solution, effort-aspirations pair is required to be mutually consistent (self-fulfilling). Formally, a pair (e^*, g^*) is a BDP if (i) $e^* \in \arg\max_{e\in[0,1]} u(e, g^*, \theta)$, and (ii) $g^* = f(e^*, \theta_0)$. They use this set-up to distinguish "standard poverty traps", in which external constraints limit poor people's productivity of effort, from "behavioural poverty traps", in which an "aspirations failure" causes poor people to exert less effort and fall even farther behind.

1.3.6 *Nash vs. Stackelberg in an Intra-self Game*

In a formal sense, we can interpret the distinction between a SDP and a BDP as corresponding to the *Stackelberg* and, respectively, the *Nash* equilibrium of dual-self intra-personal game where one self chooses actions a and the other self chooses the psychological state p and $\pi(a)$ describes the best-response of the latter self for each $a \in A$. In a Stackelberg equilibrium, the self choosing actions anticipates that the other self chooses a psychological state according to the function $\pi(.)$. In a Nash equilibrium, both selves take the choices of the other self as given when making its own choices. Consistent with the dynamic interpretation of the general framework introduced above, in the definition of an SDP, internalization (i.e. rationally anticipating the actual effects of one's actions) is equivalent to the DM anticipating the equilibrium (e.g. one's own actions is what one expects it to be, or what others expect it to be) and behaving accordingly.[26]

2 Characterization of BDPs and SDPs

2.1 *Axiomatic Restrictions on Choice Data*

We now proceed to provide a choice theoretic axiomatic characterization of SDPs and BDPs in order to examine the normative implications of models with endogenous psychological states.[27] We ask under what conditions choice data can be rationalized as the outcome of a SDP or a BDP.

[26]For example, consider the model of cognitive dissonance (e.g. Akerlof and Dickens, 1982) where the psychological states are (endogenous) beliefs about the state of the world. In Akerlof and Dickens (1982), DMs manipulate their own beliefs to conform to their desired beliefs under a rational expectations assumption.

[27]Note that in this setting, Proposition 1 continues to apply.

In what follows, we show that both decision procedures are fully characterized by three axioms of choice.

To fix ideas, in this section, we focus on the case where A and P are assumed to be finite,[28] universal sets. Fix $\succeq$, $\pi : A \to P$ and a family $\mathcal{A}$ of non-empty subsets of A. Define two choice correspondences, $\mathfrak{S}$ and $\mathfrak{B}$, from $\mathcal{A}$ to A as

$$\mathfrak{S}(A') = \left\{a \in A' : (a, \pi(a)) \succeq \left(a', \pi(a')\right) \text{ for all } a' \in A'\right\}$$

and

$$\mathfrak{B}(A') = \{a \in A' : a \succeq_{\pi(a)} a' \text{ for all } a' \in A'\},$$

as the choices corresponding to a standard and behavioural decision procedure, respectively. Note that $\mathfrak{S}(A') \subseteq A'$ and $\mathfrak{B}(A') \subseteq A'$ for each $A' \in \mathcal{A}$.[29]

We say that $\mathfrak{S}(.)$ is *admissible* if the preference relation $\succeq$ is transitive over the set of consistent decision states. We say that $\mathfrak{B}(.)$ is *admissible* if for each $a \in A$, the preference relation $\succeq_{\pi(a)}$is transitive over the set of actions.

Suppose we observe a non-empty correspondence C from $\mathcal{A}$ to A such that $C(A') \subseteq A'$ for all $A' \in \mathcal{A}$. We say that a SDP (respectively, a BDP) rationalizes C if there exist P, π and $\succeq$ such that $C(A') = \mathfrak{S}(A')$ (respectively, $C(A') = \mathfrak{B}(A')$) for all $A' \in \mathcal{A}$.

Next, consider the following axioms introduced by Sen (1971):

Sen's axiom α. For all $A', A'' \subseteq A$, if $A'' \subseteq A'$ and $C(A') \cap A''$ is non-empty, then $C(A') \cap A'' \subseteq C(A'')$.

Sen's axiom β. For all $A', A'' \subseteq A$, if $A'' \subseteq A'$ and $a, a' \in C(A'')$, then $a \in C(A')$ if and only if $a' \in C(A')$.

Sen's axiom γ. Let M be any class of sets $\{A'_k \subseteq A : k \geq 1\}$ and let V be the union of all sets in M. Then any a that belongs to $C(A')$ for all A' in M must belong to $C(V)$.

Axiom α requires that the choice correspondence be (weakly) increasing as the choice set shrinks when all alternatives chosen in the larger set are also present in the smaller set. Axiom β requires that when two actions are both chosen in a given set, and one of them is chosen in a larger set that includes the first set, then both are chosen in the larger set. Axiom γ requires that if an action is chosen in each set in a class of sets, it it must be also be chosen in their union.

We are now in a position to fully characterize choice data compatible with a SDP and a BDP. We begin with SDPs.

[28]Given the absence of convexity assumptions in (A1)-(A5), Proposition 1 and Corollary 1 continue to guarantee the existence of a SDP and a BDP when A and P are both finite sets.

[29]In contrast, $S \subseteq A \times P$ and $B \subseteq A \times P$.

PROPOSITION 2. *Choice data C is rationalizable as the outcome of an admissible SDP if and only if both Sen's axioms α and β are satisfied.*

Proof. See Appendix. ∎

Proposition 2 has two implications. First, choice data are compatible with an admissible SDP if and only if they are compatible with rational choice theory. This is because rational choice theory is falsifiable if Arrow's (1959) axiom holds (and hence, WARP and menu independence) which is in turn satisfied if and only if both Sen's axioms α and β are satisfied (Sen, 1971: Theorems 3 and 7).[30] This provides an axiomatic justification for a SDP to be the welfare benchmark that should be used in the models that are encompassed in our framework. In Section 3.2 we expand this point in further detail.

The second implication of Proposition 2 has to do with the identification of psychological states. Suppose that we are interested in identifying P. Inasmuch as the choice data are rationalized as the outcome of an admissible SDP, all we need is to identify one psychological state (note that we can prove part (ii) of Proposition 2 by setting $\#P = 1$).

Now we move on and characterize choice data compatible with a BDP.

PROPOSITION 3. *Choice data C is rationalizable as the outcome of an BDP if and only if both Sen's axioms α and γ are satisfied.*

Proof. See Appendix. ∎

One implication of Proposition 3 is that the outcomes of a BDP violates IIA.[31] Deleting an irrelevant alternative may expand the choice correspondence in the smaller set. Sen (1971) has shown that choice data that satisfies axioms α and γ (but violates axiom β) can be represented by a revealed preference relation over actions that violates transitivity. However, such a failure of transitivity is a necessary but not sufficient condition for such choice to be rationalized as the outcome of a BDP (and hence, in our setting, consistent with revealed welfare dominated choice). For example, suppose $A = \{a, b, c\}$ and $C(A) = \{a\}$, $C(\{a, b\}) = \{a\}$, $C(\{b, c\}) = \{b\}$ but $C(\{c, a\}) = \{c\}$. Although such pattern of choice, exhibiting pairwise cycles, violates transitivity, it cannot be rationalized as the outcome of a BDP (as axiom α would be violated). Axiom γ implies that if a common alternative is chosen in two different sets of actions, then it would also be chosen in their union; however, an alternative chosen in one of the two smaller sets (along with

[30] Arrow (1959)'s axiom (Independence of Irrelevant Alternatives (IIA)): If $A' \subseteq A$ and $C(A) \cap A'$ is non-empty, then $C(A') = C(A) \cap A'$. In words, when the set of feasible alternatives shrinks, the choice from the smaller set *consists precisely* of those alternatives chosen in the larger set and remain feasible, if there is any. WARP requires that for all non-empty $A', A'' \subseteq A$ and for all $a', a'' \in A' \cap A''$, if $a' \in C(A')$ and $a'' \in C(A'')$, then $a' \in C(A'')$. Richter (1966) carries out a revealed preferences analysis over the domain of linear budget sets. Thus, his analysis cannot be directly applied to the choice scenario studied here as we want to allow for finite actions sets. A menu is a non-empty subset A' of A. A menu-specific revealed preference for any $a, a' \in A'$, $aR_{A'}a' \Leftrightarrow a \in C(A')$. Menu independent choice requires the existence of a binary relation R_o over A such that for all non-empty $A' \subseteq A$ and for all $a, a' \in A'$, $aR_{A'}a' \Leftrightarrow aR_oa'$.

[31] Masatlioglu and Ok (2005)'s axiomatic characterization of rational choice with status quo bias (exogenous to the actions chosen by the DM) satisfies *Arrow's axiom* among other axioms.

the common alternative) need to be chosen in their union (violating axiom β).

The following result puts a lower bound on the number of psychological states required to rationalize choice data as the outcome of an admissible BDP:

PROPOSITION 4. *Suppose $\#A \geq 3$. Then, choice data C satisfying Sen's axioms α and γ (but not axiom β) can be rationalized as the outcome of an admissible BDP only if $\#P \geq 2$.*

Proof. See Appendix. ■

Proposition 4 shows that choice data satisfying Sen's axioms α and γ (but not β) can be rationalized as the outcome of a BDP only if $\#P > 1$. Note that without the additional requirement that choice data satisfying axioms α and γ be rationalized as the outcome of an admissible BDP, it is without loss of generality to set $\#P = 1$ in part (ii) of the proof of Proposition 3. If choice data can be rationalized as the outcome of a BDP with $\#P = 1$, then, by definition, a SDP outcome is always a BDP outcome and vice versa so that it is not possible to show the existence of dominated choice. It is only when the rationalisation of choice data as the outcome of a BDP requires that $\#P > 1$ that there is the possibility of dominated choice, a point dealt with explicitly in Proposition 5 and 6 below.

There remains the issue of whether we can further refine the lower bound on the number of psychological states required to rationalise choice data satisfying Sen's axioms α and γ (but not β) as the outcome of an admissible BDP. Although a completely general answer is not possible, we provide two examples where the minimum number of psychological states required to rationalise choice data C is explicitly computed.

Example 1. Consider $A = \{a, b, c\}$, $C(\{a, b, c\}) = \{a, b\}$, $C(\{a, b\}) = \{a, b\}$, $C(\{a, c\}) = \{a, c\}$, $C(\{b, c\}) = \{b\}$ which satisfy axioms α and γ (but not β). By Proposition 4, we must have that $\#P \geq 2$. Suppose $\#P = 2$, with $P = \{p, q\}$ and $\pi(a) = \pi(b) = p$ and $\pi(c) = q$. Then, we have that $a \succeq_p b$, $b \succeq_p a$, $a \succeq_p c$, $b \succeq_p c$, $c \succeq_q a$, $b \succeq_q c$ so that (by transitivity of $\succeq_q$), $b \succeq_q a$: in this case with $\#P = 2$ it is possible to rationalize the choice data as the outcome of an admissible BDP. ■

Example 2. Consider $A = \{a, b, c, d\}$, $C(\{a, b, c, d\}) = \{a\}$, $C(\{a, b, c\}) = \{a\}$, $C(\{a, b\}) = \{a\}$, $C(\{a, c\}) = \{a, c\}$, $C(\{b, c\}) = \{b\}$, $C(\{b, c, d\}) = \{b\}$, $C(\{b, d\}) = \{b, d\}$, $C(\{c, d\}) = \{c\}$, $C(\{a, c, d\}) = \{a, c\}$, $C(\{a, d\}) = \{a, d\}$, $C(\{a, b, d\}) = \{a, d\}$; this choice data satisfies axioms α and γ (but not β). By Proposition 4, we must have that $\#P \geq 2$. We want to show that $\#P \geq 3$. Suppose $\#P = 2$, with $P = \{p, q\}$. First note that $\pi(a) \neq \pi(c)$. If $\pi(a) = \pi(c)$, $a \succeq_{\pi(a)} b$, $c \succeq_{\pi(a)} a$ so that by transitivity, $c \succeq_{\pi(a)} b$: therefore, $c \in C(\{a, b, c\})$, a contradiction. So without loss of generality, set $\pi(a) = p$ and $\pi(c) = q$. Suppose now $\pi(b) = q$, then $a \succeq_p b$, $a \succeq_p c$, $b \succeq_q c$, $c \succeq_q a$, $b \succeq_q a$ (by transitivity) so that $b \in C(\{a, b, c\})$, a contradiction. Suppose $\pi(b) = p$. Then $a \succeq_p b$, $a \succeq_p c$, $b \succeq_p c$, $c \succeq_q a$, $b \succeq_q c$, $b \succeq_q a$ (by transitivity). If $\pi(d) = p$, we must have that $d \succeq_p a$, $d \succeq_p b$, so that as $a \succeq_p c$, $b \succeq_p c$, by transitivity, $d \succeq_p c$: therefore, $d \in C(\{b, c, d\})$, a contradiction. If $\pi(d) = q$, then $d \succeq_q b$ and as $b \succeq_q c$, by transitivity, $d \succeq_q c$ so that $d \in C(\{c, d\})$, a contradiction.

Therefore, $\pi(d) \neq p$ and $\pi(d) \neq q$: hence, $\#P \geq 3$. In this case, let $P = \{p, q, r\}$ with $\pi(a) = p$, $\pi(b) = \pi(c) = q$ and $\pi(d) = r$, it is straightforward to check that the above choice data can be rationalized as the outcome of an admissible BDP. ■

The preceding two examples show that the minimal $\#P$ required to rationalize choices as the outcome of an admissible BDP could be $\#A - 1$.[32]

2.2 *Axiomatic Characterization: Related Literature*

There is an emerging literature that provides axiomatic characterizations of decision-making models with some specific behavioural flavor.[33] Relevant contributions to this literature are Manzini and Mariotti (2007, 2012), Cherepanov et al. (2013) and Masatlioglu et al. (2012). A BDP is observationally distinguishable from each of these models on the basis of choice data alone. To start with, choice data consistent with the different procedures of choice proposed by each of these papers can account for pairwise cycles, while choice data consistent with BDP cannot: pairwise cycles of choice are inconsistent with Sen's axiom α and γ. For example, pair wise cycles of choice can be rationalized, for example, by Manzini and Mariotti's (2012) Categorize then Choose (CTC) procedure of choice, but is not consistent with a BDP. Moreover, the Rationalized Shortlist Method (RSM) proposed by Manzini and Mariotti (2007) cannot accommodate menu dependence, whereas a BDP can.

Like us, Masatlioglu et al.'s (2012) model of limited attention allows for violations of menu independence, but in a form very different from (and incompatible with) our characterization of BDP. They define a consideration set (a subset of the set of feasible alternatives) and assume that the DM only pays attention to elements in the consideration set. In their paper, revealed preferences are defined as follows: an alternative x is revealed preferred to y if x is chosen whenever y is present and x is not chosen when y is deleted. That is, the choice of an alternative from a set should be unaffected if an element which is not in the consideration set is deleted. If choice changes when an alternative is deleted, then the latter alternative was in the consideration set and

[32] It is possible to explicitly characterize the maximal number of psychological states required to rationalize choice data satisfying Sen's axioms α and γ as an outcome of a BDP. Given two decision scenarios $D = (A, P, \pi, \succeq)$ and $\tilde{D} = (A, \tilde{P}, \tilde{\pi}, \tilde{\succeq})$, we say D is *equivalent* to $\tilde{D}$ if and only if the following two conditions are satisfied: (i) $(a, \pi(a)) \succeq (a', \pi(a')) \Leftrightarrow (a, \tilde{\pi}(a)) \tilde{\succeq} (a', \tilde{\pi}(a'))$ for all $a, a' \in A$; (ii) $(a, \pi(a)) \succeq (a', \pi(a)) \Leftrightarrow (a, \tilde{\pi}(a)) \tilde{\succeq} (a', \tilde{\pi}(a))$ for all $a, a' \in A$. Now, consider a fixed decision scenario $D = (A, P, \pi, \succeq)$ and the decision scenario $D_{Id.} = (A, P = A, Id., \tilde{\succeq})$ (where $Id.$ denotes the identity function from A to itself) constructed as follows: (i) $(a, a) \tilde{\succeq} (a', a') \Leftrightarrow (a, \pi(a)) \succeq (a', \pi(a'))$ for all $a, a' \in A$, (ii) $(a, a) \tilde{\succeq} (a', a) \Leftrightarrow (a, \pi(a)) \succeq (a', \pi(a))$ for all $a, a' \in A$, with $\tilde{\succeq}$ arbitrarily defined otherwise. Then, $D_{Id.} = (A, P = A, Id., \tilde{\succeq})$ is, by construction, equivalent to $D = (A, P, \pi, \succeq)$. It follows that given any decision scenario, there is an equivalent (both from a normative and behavioural perspective) decision scenario where the set of psychological states is the set of actions and the function π is the identity function. Therefore, the maximal number of psychological states required to rationalize choice data C satisfying axioms α and γ is $\#A$.

[33] Manzini and Mariotti (2014) contrast the two existing approaches for welfare economics when agents are bounded rational and make the case for these type of model-based approaches.

clearly the chosen alternative was revealed preferred to it. This is a violation of independence of irrelevant alternatives, but in a form that is incompatible with Sen's axiom α. Such data cannot be rationalized as an outcome of a BDP, precisely because in a BDP (and also in a SDP), if x is chosen whenever y is present, x must be chosen when y is deleted.

3 Welfare Implications

3.1 *Revealing a (Partial) Welfare Ranking of Choices*

In this section, we look at the welfare implications of bounded rationality as defined by our framework. For simplicty of exposition, as in the preceeding section, we will assume that A and P are universal, finite sets. We begin by defining the welfare ranking $\widehat{W}$ as the induced ranking over actions implied by the ranking of consistent decision states. Formally, given a fixed decision scenario $D = (A, P, \pi, \succeq)$ and $a, b \in A$, define $a\widehat{W}b$ only if $(a, \pi(a)) \succeq (b, \pi(b))$. We use $\widehat{W}$ as our welfare benchmark as it would correspond to preferences of a rational decision-maker who maximizes a fixed preference relation over a set of feasible actions. By Proposition 2, whenever choice data C satisfies Sen's axioms α and β, (i) $\widehat{W}$ coincides with revealed preferences in the sense of Samuelson (1938), and (ii) all admissible standard decision scenarios that rationalize C imply the same welfare ranking $\widehat{W}$ over actions. Therefore, $\widehat{W}$ can be inferred from choice data when the DM solves a SDP. Now we ask the following two questions: Is it possible to reconstruct $\widehat{W}$ from choice data when the DM solves a BDP? If so, can we infer the existence of welfare dominated choice?

The recent influential work by BR on welfare analysis of non-rational choice addresses similar questions. BR adopt the normative position that what matters for welfare is a binary relation constructed solely on actions using choice data (that could violate axioms α and β) to derive a partial preference ordering based on pairwise coherence (BR, Salant and Rubinstein, 2008 (SR) and earlier Sen, 1971). BR define aW^*b iff for all $A' \subseteq A$ such that $\{a, b\} \subseteq A'$, $b \notin C(A')$. They interpret W^* (a strict preference relation) as pairwise coherence and show that W^* is acyclic. They define a as a weak welfare optimum whenever there exists no $b \in A$ such that bW^*a.

The advantage of W^* as a welfare criterion is that it never fails to pick welfare optima from any choice situation. However: (i) the set of welfare optima identified using W^*can be very large and, as shown by Dalton and Ghosal (2012), for a fixed decision scenario $D = (A, P, \pi, \succeq)$, W^* may not coincide with $\widehat{W}$, which is problematic if W^* is used as the welfare ranking over actions derived from choice data; (ii) BR assume a fixed set of psychological states, but psychological states are difficult to observe in practice; (iii) by construction, pairwise coherence does not allow for the possibility of dominated choice: if an alternative is chosen in a set, there is no other alternative in that set which welfare dominates it.

In what follows, we derive a welfare ranking from choice data that addresses all these points, reconstructing $\widehat{W}$ for a DM who solves a BDP in a context in which P and π are not observed.

Suppose we observe choice data that satisfy Sen's axiom's α and γ (and not β). By Proposition 4, these data can be rationalized as the outcome of an admissible BDP only if $\#P \geq 2$: this is clearly a necessary but not sufficient condition for establishing the normative significance of psychological states.

Consider the following domain restriction over which such observed choice data have to be rationalised as the outcome of a BDP: a decision scenario $D = (A, P, \pi, \succeq)$ satisfies domain restriction R if: (i) $\succeq$ is transitive over the set of consistent decision states, (ii) for each $a \in A$, the preference relation $\succeq_{\pi(a)}$is transitive over A and (iii) for each $a, a' \in A$, $(a, \pi(a)) \succeq (a', \pi(a'))$ if and only if $(a, \pi(a)) \succeq (a', \pi(a))$ and $(a, \pi(a')) \succeq (a', \pi(a'))$. Conditions (i) and (ii) are the two admissibility restrictions already imposed on a SDP and a BDP. Heuristically, condition (iii) states that the ranking of actions should be neutral with respect to psychological states.[34]

Now define the partial welfare ordering W as follows:

Partial Welfare Ordering. Fix a choice correspondence C and consider $a, b \in A$. Then, aWb if (1) there exists $D = (A, P, \pi, \succeq)$ that rationalizes C as the outcome of a BDP and satisfies R, (2) $(a, \pi(a)) \succeq (b, \pi(b))$ under all decision scenarios $D = (A, P, \pi, \succeq)$ that satisfy (1).

Evidently, by construction, W agrees with $\widehat{W}$ where both are well-defined.

In the spirit of Masatlioglu et al. (2012), we want to show that a vacuous satisfaction of the partial welfare ordering does not occur, i.e. whenever choice data are rationalized by a BDP, there exists at least one decision scenario that rationalizes the same choice data as the outcome of a BDP and is consistent with the domain restriction R. To this end, we begin by pointing out that when $\#A$ is finite, a preference relation satisfying domain restriction R would also ensure the existence of a solution to a BDP and a SDP.

RESULT 1. *Suppose $\#A$ is finite. When $\succeq$ satisfies domain restriction R, both a solution to a BDP and a solution to a SDP exist.*[35]

Proof. See Appendix. ■

Result 1 has the implication that when preferences satisfy domain restriction R, a solution to a SDP is also a solution to a BDP. Therefore, under the domain restriction R, $\mathfrak{S}(A') \subseteq \mathfrak{B}(A')$ for all non-empty $A' \subseteq A$. As the proof makes clear, Result 1 relies in an essential way on conditions (i) and (iii) in the definition of R (i.e. that the ranking of actions be neutral with respect of psychological states) but not on condition (ii). As we shall show below, the reverse implication, that a solution to a BDP is also a solution to a SDP, need not hold. This is a key feature to

[34]Munro and Sugden (2003), in their reformulation of Tversky and Kahneman, define a concept of reference-neutral preferences. Their definition of loss aversion implies condition (iii) i.e., if a is preferred to b in the reference neutral sense, then a is preferred to b when the reference point is a. For a fixed decision scenario $D = (A, P, \pi, \succeq)$, Dalton and Ghosal (2012) show that when this condition is satisfied, $\widehat{W}$ and W^* coincide where both are defined.

[35]Result 1 can be extended in a straightforward way to the case where A is a compact subset of some finite dimensional Euclidian space provided preferences satisfying the domain restriction R also satisfy a continuity restriction such as A1 (iii).

inferring the existence of welfare dominated choice in specific settings.

The next proposition shows that whenever choice data are rationalized by a BDP, there exists at least one decision scenario that rationalizes the same choice data as the outcome of a BDP and is consistent with the domain restriction R.[36]

PROPOSITION 5. *Suppose choice data C satisfy axioms α and γ. Then, there exists at least one decision scenario $D = (A, P, \pi, \succeq)$ that rationalizes C as the outcome of a BDP and is consistent with the domain restriction R.*

Proof. See Appendix. ■

A key implication of Proposition 5, taken together with Proposition 1, Corollary 1 and Result 1, is that any choice data that can be rationalised as the outcome of a BDP with an acyclic $\succeq_p$ for a fixed $p \in P$ can also be rationalised as the outcome of a BDP over a preference domain that satisfies condition R. This implies observational equivalence between the different assumptions made on individual preferences required, on the one hand, for existence of a BDP and, on the other hand, for the welfare analysis carried out in this subsection.

In the next proposition, we show that W can be revealed using choice data. Specifically, we show that W is a transitive extension of W^* and as a consequence, it is possible to infer the existence of welfare dominated choices using W.

PROPOSITION 6. *Suppose choice data C satisfy axioms α and γ. Suppose for $a, b, c \in A$, aW^*b and bW^*c. Then, aWb, bWc and aWc.*

Proof. See Appendix. ■

Some remarks:

1. As choice data exhibiting pairwise cycles of choice are inconsistent with Sen's axiom α, the welfare benchmark $\widehat{W}$ cannot be applied to all choice data exhibiting intransitivity.

2. Proposition 6 shows that W is a transitive extension of W^*, key to showing the existence of dominated choice. By way of example, consider the following scenario where choice data satisfy Sen's axioms α and γ (but not β) and W^*fails to be transitive i.e. $\{a\} = C(\{a, b, c\})$, $\{a\} = C(\{a, b\})$, $\{b\} = C(\{b, c\})$and $\{a, c\} = C(\{a, c\})$. Then, clearly, $aW^*b$and bW^*cbut $\sim aW^*c$. Therefore, using W^*we would not be able to infer the existence of welfare dominated choice. However, by Proposition 6, as W is a transitive extension of W^*, we are able to show that aWc.

3. The transitivity of the *welfare benchmark* $\widehat{W}$ is a consequence of working under the domain restriction R. Condition (i) in the domain restriction R (transitivity of preferences over the set of consistent decision states) is equivalent to the requirement that $\mathfrak{S}(.)$ is *admissible* which in turn is equivalent to requiring that choice data satisfy Sen's axioms α and β (Proposition 2). Condition

[36] Masatlioglu's et al. (2012) representation involves a utility function u and an attention mapping ψ. Their welfare criterion is such that, provided that a choice correspondence C admits a representation with limited attention, one alternative a is a welfare improvement over another b if, for any representation (u, ψ) of C, $u(a)$ gives higher utility value than $u(b)$.

(i), together with condition (iii) in the domain restriction R, is used to show (in Propositions 5 and 6) that the *welfare benchmark* $\widehat{W}$ *is transitive*: showing the transitivity of $\widehat{W}$ does not necessarily require the transitivity of $\succeq_p$ for a fixed $p \in P$. The transitivity of $\succeq_p$ for a fixed $p \in P$ is required to show (in Proposition 4) that choice data satisfying Sen's axioms α and γ (but not β) can be rationalized as the outcome of a BDP only if $\#P > 1$. This latter point matters for the following reason. If choice data can be rationalized as the outcome of a BDP with $\#P = 1$, then, by definition, a SDP outcome is always a BDP outcome and vice versa so that choice data can always be rationalised as the outcome of a BDP satisfying condition (iii) in the domain restriction R. However, in this case, it is not possible to show the existence of dominated choice. It is only when the rationalisation of choice data as the outcome of a BDP requires that $\#P > 1$ that there is the possibility of dominated choice. Hence, condition (ii) in the domain restriction R (the transitivity of $\succeq_p$ for a fixed $p \in P$) is a necessary (but not sufficient condition) condition for dominated choice.

4. Finally, we note that BR's welfare criterion is carried out for a fixed set of psychological states. In this sense, our results on the partial welfare ordering W can be interpreted as providing a characterization of the domain restrictions under which the use of BR's pairwise coherence is justified when psychological states have to be inferred from choice data in order to be rationalised as the outcome of a BDP.

3.2 *Welfare Benchmark for Existing Behavioural Economics Models*

The welfare benchmark defined in this paper is the ranking over actions implied by the ranking of consistent decision states. In this subsection, we illustrate how this welfare benchmark can be applied to the behavioural economics models discussed throughout the paper.

In models of dynamic inconsistent preferences, the (induced) preferences of the initial self (once the unique best-response of the future selves is taken onto account) provides the welfare benchmark. In models with endogenous reference points, the induced preferences over actions (internalizing the impact of actions on reference points) are the relevant welfare benchmark. In a decision problem with anticipatory feelings, assuming that the best-response of the future self is unique, the optimal solution of Caplin and Leahy (2001) provides the relevant benchmark. In a dual-self game, given the interpretation of a SDP and a BDP as corresponding, respectively, to a Stackelberg and Nash equilibrium of the dual-self game, the induced preferences of the self choosing actions at the Stackelberg equilibrium provides the relevant normative benchmark.

In general, the best response the DMs self at $t = 1$ can be multi-valued. In this case, in our model, $\pi(.)$, the feedback from actions to psychological states, will be a correspondence. The formal definition of a SDP and a BDP when $\pi(.)$ is multi-valued is presented in the online Appendix. Heuristically, when $\pi(.)$ is multi-valued, in a SDP we require that the DM chooses the consistent action-psychological state pair that maximizes his preferences from the set of consistent decision states. In models with time-inconsistent preferences, a SDP corresponds to a Strotz equilibrium where (a) the DM at $t = 1$ has multiple-best responses to the action chosen at $t = 0$, but (ii) the DM at $t = 0$ is able to convince his future self at $t = 1$to choose the best-response action that maximizes his payoff at $t = 0$. Thus, a SDP outcome is invariant to the selection of $\pi(.)$ and reduces to the definition of a SDP for the case a single-valued $\pi(.)$.

Notice that our welfare benchmark contrasts with other alternative welfare rankings adopted elsewhere in the literature. Some scholars have proposed to solve the model with one set of preference assumptions (e.g. hyperbolic discounting) and then to evaluate welfare using another set of assumptions (e.g. geometric discounting) (see, for example, O'Donoghue and Rabin, 2006). In contrast to this approach, the difference between a SDP outcome and a BDP outcome reflects a difference in decision-making procedures and not a shift in the preferences used to evaluate welfare. Another approach applied in the literature of time-inconsistent preferences is the multiself Pareto criterion, where the preferences of all the different selves in a time-inconsistent decision problem or the preferences of both selves in a dual-self game are explicitly taken into account. In our framework, in contrast, all that matters for welfare are the induced preferences of the initial self once the best-response of the future self is taken into account.

3.3 *Welfare Implications of Partial Prediction*

In settings where the DM partially internalizes the impact of actions on psychological states, the welfare impact of a greater degree of internalisation may be ambiguous. Below, we construct an example where a DM who is able to partially predict how psychological states evolve with actions may be worse-off than a DM who never predicts how psychological states evolve with actions.

We consider a decision scenario where the psychological state is multi-dimensional and the DM internalizes the effect of his action on a subset of dimensions of such a vector and believes that he doesn't affect the complement. Let $A \times P \subseteq \Re^K \times \Re^N$ and $\pi(a)$ be a non-empty and single-valued function for each $a \in A$, with $\pi(a) = (\pi_1(a), ..., \pi_N(a))$, and for clarity of exposition, assume that the binary relation $\succeq$ has a (expected) utility representation $u : A \times P \to \Re$. We will assume that the DM is able to internalize the impact of choices on a subset of psychological states. As before, we write $\pi(a) = (\pi_1(a), ..., \pi_N(a))$. Suppose the DM is able to internalize the first M psychological states, $1 \leq M \leq N$. Let $\tilde{P}$ denote the projection of P onto $P \cap \Re^{N-M}$ with $\tilde{p}$ denoting a representative element of $\tilde{P}$. Let $\tilde{v}(a, \tilde{p}) = u(a, (\pi_1(a), ..., \pi_M(a), p_{M+1}, ..., p_N))$. Let $\tilde{h}(p) = \{a \in A : a \in \arg\max_{a \in A} \tilde{v}(a, \tilde{p})\}$. In what follows, we will assume that $\tilde{h}(p)$ is unique. Fix a $p_0 \in P$. A sequence of *short-run* outcomes is determined by the relations $a_t \in \tilde{h}(p_{t-1})$ and $p_t = \pi(a_t)$, $t = 1, 2, ...$: at each step, the DM chooses a myopic best-response. *Long-run* outcomes are denoted by a pair a, p with $p = \pi(a)$ and a is defined to be the steady-state solution to the short-run outcome functions i.e. $a = \tilde{h}(\pi(a))$. It follows that long-run behavior corresponds to the outcome of a BDP where the preferences are represented by a utility function $\tilde{v}(a, \tilde{p}) = u(a, (\pi_1(a), ..., \pi_M(a), p_{M+1}, ..., p_N))$.

Next, we construct an example to show that a DM who is able to partially predict how psychological states evolve with actions may be *worse-off* than a DM who never predicts how psychological states evolve with actions.

Example 3. Consider the following decision scenario where there are two payoff relevant dimensions of choice with outcome denoted $x_1$and x_2 and preferences $u(x) = x_1 + v_1(x_1 - r_1) + x_2 + v_2(x_2 - r_2)$ where $v(\cdot)$is a Kahneman-Tversky value function with $v_i(z) = z$ if $z \geq 0$, $v(z) = \alpha_i z$, $\alpha_i > 1$if $z < 0$ and $v(0) = 0$. There are two options. Option 1 is defined by $(x_1 = 3, x_2 = 2)$and option 2 is $(x_1 = 6, x_2 = 0)$. Assume that $\pi$is the identity map so that in a consistent decision state the reference point corresponds to current choice of the DM.

Suppose the DM does not predict that the reference point shifts in both dimensions 1 and 2. The payoff table below provides a quick summary of the decision problem in this case:

	reference point 1	reference point 2
option 1	5	$7 - 3\alpha_1$
option 2	$9 - 2\alpha_2$	6

A straightforward computation establishes that (option 2, reference point 2) is the unique BDP outcome whenever $\alpha_1 > \frac{1}{3}$ and $\alpha_2 < 2$.

Now suppose the DM is able to predict that the reference point will shift in the first dimension but not in the second dimension. The payoff table below provides a quick summary of the decision problem in this case:

	reference point 1	reference point 2
option 1	5	7
option 2	$6 - 2\alpha_2$	6

A straightforward computation shows that whenever $\alpha_2 \geq 1$, (option 1, reference point 1) is the unique BDP outcome.

As (option 2, reference point 2) always payoff dominates (option 1, reference point 1), partial prediction makes the DM worse-off.

4 Concluding Remarks

All of the welfare economics we know is based on the assumption that people choose what is best for them, and that we can accordingly use these choices as a guide to welfare policy. Once we build realistic behavioural features into our models, this foundation is lost. Can we still extract some normatively relevant information from choices in a context in which DMs may not be utility maximizers? Arguably, this is an ongoing puzzle of key importance and we believe this paper contributes with some ammunition towards a better understanding of the normative implications of behavioural economics.

The first contribution of this paper is to offer a simple, yet unifying platform that encompasses different existing work in the literature on behavioural economics, and allow for self-fulfilling mistakes. This platform is not meant to explain a new behavioural procedure of choice, but it constitutes a necessary initial step to address the general question of how to do welfare economics with agents who make (self-fulfilling) mistakes.

Second, we offer a full axiomatic characterization of behavioural decisions based on choice data alone. If observed behavior is consistent with Sen's axioms α and γ (but not β), it is consistent with a decision-maker who doesn't fully internalize all the consequences of his actions.

Third, we propose a unified welfare benchmark for behavioural economics that is justified in standard axioms of choice (Sen's axioms α and β) and can be applied to existing seminal behavioural economics models. The benchmark proposed here has the same characterization of rational choice theory, which has been used since Samuelson's (1938) as the standard welfare benchmark in economics. We illustrate the welfare benchmark implied from our framework with examples on time-inconsistent preferences, endogenous reference-dependent preferences, anticipatory feelings and dual-self games.

Fourth, we propose a (partial) welfare ranking which, under a domain restriction, can be revealed using choice data and be used to infer welfare dominated (i.e. mistaken) choice. In this regard, we are able to deal, at least partially, with a key concern raised by behavioral welfare economics: how to perform welfare analysis if choices do not always reveal decision-makers' best interest. Note that Bernheim and Rangel (2009), by the very nature of their formalisation, cannot identify welfare dominated choices.

Fifth, we show that in settings where the DM partially internalizes the impact of actions on psychological states, the welfare impact of a greater degree of internalisation maybe ambiguous.

All in all, this paper demonstrates that it is still possible to extract normatively relevant information from observed choices, even when we relax the full rationality assumption. In future work, we plan to extend this work to examine behavioral decision-making under uncertainty and explore, in greater detail, the policy implications of behavioral decision-making.

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Appendix

Proof of Proposition 1

(i) Propositions 1 and 2 in Ghosal (2011) show that assumptions *(A1)-(A5)*, taken together, are sufficient to ensure that $\Psi(p)$ is non-empty and compact and for each $p \in P$, $\Psi(p)$ is a sublattice of A where both the maximal and minimal elements, denoted by $\bar{a}(p)$ and $\underline{a}(p)$ respectively, are increasing functions on P. To complete the proof of Proposition 1, define a map $\Psi : A \times P \to A \times P$, $\Psi(a,p) = (\Psi_1(p), \Psi_2(a))$ as follows: for each (a,p), $\Psi_1(p) = \{a' \in A : \succ_p (a') = \phi\}$ and $\Psi_2(a) = \pi(a)$. It follows that $\Psi_1(p)$ is a compact (and consequently, complete) sublattice of A and has a maximal and minimal element (in the usual component wise vector ordering) denoted by $\bar{a}(p)$ and $\underline{a}(p)$ respectively. Therefore, the map $(\bar{a}(p), \pi(a))$ is an increasing function from $A \times P$ to itself and as $A \times P$ is a compact (and hence, complete) lattice, by applying Tarski's fix-point theorem, it follows that $(\bar{a}, \bar{p}) = (\bar{a}(\bar{p}), \pi(\bar{a}))$ is a fix-point of Ψ and by a symmetric argument, $(\underline{a}(p), \pi(a))$ is an increasing function from $A \times P$ to itself and $\left(\underline{a}, \underline{p}\right) = \left(\underline{a}(\underline{p}), \pi(\underline{a})\right)$ is also a fix-point of Ψ; moreover, $(\bar{a}, \bar{p})$ and $\left(\underline{a}, \underline{p}\right)$ are respectively the largest and smallest fix-points of Ψ.

(ii) By assumption A is compact and π is a continuous function so that the set

$$\{p \in P : p = \pi(a) \text{ for some } a \in A\}$$

is compact and therefore, the set of consistent decision states is compact. Then, under the assumption that $\succ$ is acyclic and has an open lower section, it follows that S is non-empty from Bergstrom (1975).■

Proof of Proposition 2

(i) We show that if choice data are rationalizable as the outcome of an admissible SDP, then, both Sen's axiom α and β hold. Fix $\succeq$, $\pi : A \to P$. For $A'' \subseteq A' \subseteq A$, if

$$a \in \mathfrak{S}(A') = \left\{ \begin{array}{c} a : (a,p) \succeq (a',p') \text{ for all } a' \in A',\, p' = \pi(a') \\ \text{and } p = \pi(a) \end{array} \right\}$$

then

$$a \in \mathfrak{S}(A'') = \left\{ \begin{array}{c} a : (a,p) \succeq (a',p') \text{ for all } a' \in A'',\, p' = \pi(a') \\ \text{and } p = \pi(a) \end{array} \right\}.$$

Therefore, $\mathfrak{S}(A') = C(A') \cap A'' \subseteq C(A'') = \mathfrak{S}(A'')$ so that Sen's axiom α is satisfied. Next, given $A'' \subseteq A'$, suppose $a', a'' \in C(A'') = \mathfrak{S}(A'')$ but $a' \in \mathfrak{S}(A')$ and $a'' \notin \mathfrak{S}(A')$. By construction, both $(a',p') \succeq (a'',p'')$ and $(a',p') \preceq (a'',p'')$ for $p' = \pi(a')$ and $p'' = \pi(a'')$. Therefore, by transitivity of $\succeq$ over consistent decision states, $a'' \in \mathfrak{S}(A')$, a contradiction so that Sen's axiom β is satisfied.

(ii) We show that if choice data satisfy Sen's axioms α and β, they are rationalizable as the outcome of an admissible SDP. To this end, we specify $\pi : A \to P$, $\#P \geq 1$ so that π is onto. Next we specify preferences $\succeq$: for each non-empty $A' \subseteq A$ and $a \in C(A')$, $\succeq$ satisfies the condition that $(a, p) \succeq (a', p')$ for all $a' \in A'$, $p = \pi(a)$ and $p' = \pi(a')$, $p, p' \in P$. Consider $C(A')$ for some non-empty $A' \subseteq A$. By construction if $a \in C(A') \Rightarrow \mathfrak{S}(A')$ and therefore, $C(A') \subseteq \mathfrak{S}(A')$. We need to check that for the above specification of $\succeq$, $\pi : A \to P$, $\mathfrak{S}(A') \subseteq C(A')$. Suppose to the contrary, there exists $a' \in \mathfrak{S}(A')$ but $a' \notin C(A')$. It follows that $(a', \pi(a')) \succeq (b, \pi(b))$ for all $b \in A'$. Since $a' \notin C(A')$, by construction this is only possible if for each $b \in A'$, $a' \in C(A''_b)$ with $\{a, b\} \subseteq A''_b$. By Sen's axiom α, as $a' \in C(\{a, b\})$ and as $\{a, b\} \subseteq A'$, again by Sen's axiom α, $b \in C(\{a, b\})$ for $b \in C(A')$. Now, by construction, $A' = \cup_{b \in A'} \{a, b\}$. By Sen's axiom β, $a' \in C(A')$. Therefore, $\mathfrak{S}(A') = C(A')$. Finally, note that when choice data satisfy axioms α and β, $\succeq$ is transitive (Sen, 1971: Theorem 1) and therefore, $\mathfrak{S}(A')$ is admissible. ■

Proof of Proposition 3

(i) We show that if choice data are rationalizable as the outcome of a BDP, then both *Sen's* α and γ hold. Fix $\succeq$, $\pi : A \to P$. For $A'' \subseteq A' \subseteq A$, if

$$a \in \mathfrak{B}(A') = \left\{a : a \succeq_{\pi(a)} a' \text{ for all } a' \in A'\right\}$$

then

$$a \in \mathfrak{B}(A'') = \left\{a : a \succeq_{\pi(a)} a' \text{ for all } a' \in A''\right\}.$$

Therefore, $C(A') \cap A'' \subseteq C(A'')$ as required so that Sen's axiom α is satisfied. Next, let M denote a class of sets $\{A'_k \subseteq A : k \geq 1\}$. If

$$a \in \mathfrak{B}(A'_k) = \left\{a : a \succeq_{\pi(a)} a' \text{ for all } a' \in A'_k\right\}$$

and $V = \cup_{k \geq 1} A'_k$, it follows that

$$a \in \mathfrak{B}(V) = \left\{a : a \succeq_{\pi(a)} a' \text{ for all } a' \in V\right\}$$

so that Sen's axiom γ is satisfied.

(ii) We show that if choice data satisfy both Sen's α and γ, they are rationalizable as the outcome of a BDP. To this end, we specify $\pi : A \to P$ so that $\#P \geq 1$ and π is onto. Next we specify preferences $\succeq$: for each non-empty $A' \subseteq A$ and $a \in C(A')$, $\succeq$ satisfies the condition that $a \succeq_p a'$ for all $a' \in A'$ and $p = \pi(a)$. Consider $C(A')$ for some non-empty $A' \subseteq A$. By construction if $a \in C(A')$, then $a \in \mathfrak{B}(A')$ and therefore, $C(A') \subseteq \mathfrak{B}(A')$. We need to check that for the above specification of $\succeq$, $\pi : A \to P$, $\mathfrak{B}(A') \subseteq C(A')$. Suppose to the contrary, there exists $a' \in \mathfrak{B}(A')$ but $a' \notin C(A')$. It follows that $a' \succeq_{p'} b$ for each $b \in A'$ and $p' = \pi(a')$. Since $a' \notin C(A')$, by construction this is only possible only if $a' \in C(A''_b)$ for some A''_b with $\{a', b\} \subseteq A''_b$. Let $A'' = \cup_{b \in A'} A''_b$. It follows that $a' \in A''$ and by Sen's axiom γ, $a' \in C(A'')$. As $A' \subseteq A''$ and $a' \in C(A'')$, by Sen's axiom α, $a' \in C(A')$ a contradiction. Therefore, $\mathfrak{B}(A') = C(A')$. ■

Proof of Proposition 4

If choice data satisfy Sen's axioms α and γ (but not axiom β) and $\#A \geq 3$, then there exists two non-empty sets A' and A'' with $A' \subseteq A$, $A'' \subseteq A$ and $A'' \subseteq A'$ such that $(C(A') \cap A'') \subset C(A'')$. Assume that $\#P = 1$ with $P = \{p\}$. Consider the preference relation defined over actions $\succeq_p$ where $P = \{p\}$ and for each non-empty $A' \subseteq A$ and $a \in C(A')$, $\succeq_p$ satisfies the condition that $a \succeq_p a'$ for all $a' \in A'$ and $p = \pi(a)$. We require that this choice data be rationalized as the outcome of a BDP (i.e. $\mathfrak{B}(A') = C(A')$, $A' \subseteq A$) with $\#P = 1$ and $P = \{p\}$, $p = \pi(a)$ for all $a \in A$ and $\succeq_p$ is transitive. Then, there exists $b, c \in A''$ s.t. $b \in (C(A') \cap A'')$, $c \in C(A'')$ but $c \notin C(A')$. Therefore, it follows that both $b \succeq_p c$ and $c \succeq_p b$ so that as $\succeq_p$ is transitive, $c \succeq_p a$ whenever $b \succeq_p a$ for any $a \in A$; therefore, $c \in C(A')$, a contradiction. It follows that $\#P > 1$ and so in part (ii) of the proof of Proposition 3, we must have that $\#P \geq 2$. ■

Proof of Result 1

When preferences satisfy domain restriction R, $\succeq$ is transitive over the set of consistent decision states. Given that $\#A$ is finite, the existence of a SDP immediately follows. Therefore, there exists $a \in A$ such that $(a, \pi(a)) \succeq (a', \pi(a'))$ for all $a' \in A$. As preferences satisfy domain restriction R, $(a, \pi(a)) \succeq (a', \pi(a))$ for all $a' \in A$ so that a solution to a BDP exists as well. ■

Proof of Proposition 5

By Proposition 3(i), we already know that when C satisfies axioms α and γ the decision scenario $D = (A, P, \pi, \succeq)$ specified so that $\pi : A \to P$, $\#P \geq 1$ and π is onto, and the preference relation $\succeq$ such that for each non-empty $A' \subseteq A$ and $a \in C(A')$, $a \succeq_p a'$, $p = \pi(a)$, for all $a' \in A'$, already rationalizes C the outcome of a BDP. By inspection, it is straightforward to check that the proof of Proposition 3(ii) continues to hold when $D = (A, P, \pi, \succeq)$ is such that $A = P$ and $\pi : A \to A$ is the identity map. Suppose, in addition, we require that $D = (A, P, \pi, \succeq)$ is consistent with the domain restriction R. If $\{a\} = C(\{a, a'\})$, then it follows that $(a, a) \succeq (a', a)$ and $(a, a') \succeq (a', a')$ so that $(a, a) \succeq (a', a')$; if $\{a'\} = C(\{a', a''\})$, it follows that $(a', a') \succeq (a'', a')$ and $(a', a'') \succeq (a'', a'')$ so that $(a', a') \succeq (a'', a'')$. It follows that $(a, a) \succeq (a'', a'') \Leftrightarrow (a, a) \succeq (a'', a)$ and $(a, a'') \succeq (a'', a'')$ so that $a \in \mathfrak{B}(\{a, a''\})$. We check that $a \in C(\{a, a''\})$. So suppose $\{a''\} = C(\{a, a''\})$. Then, $\{a''\} = C(\{a, a''\}) \Rightarrow a \notin C(\{a, a', a''\})$ while $a' = C(\{a', a''\}) \Rightarrow a'' \notin C(\{a, a', a''\})$. Therefore, $\{a'\} = C(\{a, a', a''\})$ and by axiom α, $a' \in C(\{a, a'\})$, a contradiction. Therefore, $a \in C(\{a, a''\})$ so that it is without loss of generality to assume that the preference relation $\succeq_a$ is transitive over A. It follows that if choice data satisfy both Sen's α and γ, they are rationalizable as the outcome of a BDP where there exists at least one decision scenario $D = (A, P, \pi, \succeq)$ that also rationalizes C and is consistent with the domain restriction R. ■

Proof of Proposition 6

As aW^*b and bW^*c, it follows that $\{a\} = C(\{a,b\})$ and $\{b\} = C(\{b,c\})$ (both $C(\{a,b\})$ and $C(\{b,c\})$ must be non-empty, b is never chosen in the presence of a and c is never chosen in the presence of b); hence, $\{a\} = C(\{a,b,c\})$. As choice data satisfies axiom α, $a \in C(\{a,c\})$. Suppose we require that this choice data to be rationalized as the outcome of an admissible BDP satisfying the domain restriction R. From the proof of Proposition 5, it also follows that aWb, bWc. If $\{a\} = C(\{a,c\})$, then $(a,\pi(a)) \succeq (c,\pi(a))$ and $(a,\pi(c)) \succeq (c,\pi(c))$ so that under the domain restriction R it necessarily follows that $(a,\pi(a)) \succeq (c,\pi(c))$ and hence, aWc. Next, suppose that $\{a,c\} = C(\{a,c\})$. First, note that the choice data satisfy Sen's axioms α and γ (but not β). Therefore, by Proposition 4, $\#P \geq 2$. Suppose $\#P = 2$ with $P = \{p_1,p_2\}$, $p_1 \neq p_2$ and $\pi(a) = p_1$ and $\pi(b) = \pi(c) = p_2$. Consider the preference relation defined over actions $\succeq_p$ where for each non-empty $A' \subseteq A$ and $a \in C(A')$, $\succeq_p$ satisfies the condition that $a \succeq_p a'$ for all $a' \in A'$ and $p = \pi(a)$. Then, $a \succeq_{p_1} b$, $a \succeq_{p_1} c$, $b \succeq_{p_2} c$, $c \succeq_{p_2} a$ and by transitivity of $\succeq_{p_2}$, $b \succeq_{p_2} a$ which implies that $C(\{a,b\}) = \{a,b\}$ a contradiction. Next, suppose that $\#P = 2$ with $P = \{p_1,p_2\}$, and $\pi(a) = \pi(c) = p_1$ and $\pi(b) = p_2$. Then, $a \succeq_{p_1} b$, $a \succeq_{p_1} c$, $b \succeq_{p_2} c$, $b \succeq_{p_2} c$, $c \succeq_{p_2} a$ and $a \succeq_{p_2} b$ and by transitivity of $\succeq_{p_1}$, $c \succeq_{p_1} b$ which implies that $C(\{a,b,c\}) = \{a,c\}$ a contradiction. So suppose $\#P = 2$ with $P = \{p_1,p_2\}$, and $\pi(a) = \pi(b) = p_1$ and $\pi(c) = p_2$. Then, $a \succeq_{p_1} b$, $a \succeq_{p_1} c$, $b \succeq_{p_1} c$, $b \succeq_{p_2} c$, $c \succeq_{p_2} a$ and $b \succeq_{p_2} a$. It follows by domain restriction R that $(b,p_1) \succeq (c,p_2)$ and as $(a,p_1) \succeq (b,p_1)$, $(a,p_1) \succeq (c,p_2)$. Therefore, aWc and $C(\{a,c\}) = \{a,c\}$ contains the dominated action c. By Proposition 5, it remains to check the case when $p_1,p_2,p_3 \in P$ $p_1 \neq p_2 \neq p_3$ with $\pi(a) = p_1$, $\pi(b) = p_2$ and $\pi(c) = p_3$. Then, $a \succeq_{p_1} b$, $a \succeq_{p_1} c$, $a \succeq_{p_2} b$, $b \succeq_{p_2} c$, $b \succeq_{p_3} c$, $c \succeq_{p_3} a$ and $b \succeq_{p_3} a$. It follows that $(a,p_1) \succeq (b,p_2)$ and $(b,p_2) \succeq (c,p_3)$ so that $(a,p_1) \succeq (c,p_2)$ so that aWc and again, $C(\{a,c\}) = \{a,c\}$ contains the dominated action c.■