Environmental public good provision under robust decision making*

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Abstract

We study public good provision in a two-country dynamic setup with environmental externalities. In this framework, we examine robust decision making under potential misspecification of the process that describes the evolution of the environmental public good. Robust policies, arising from fear of model misspecification, help to correct for the inefficiencies associated with free riding and thus increase the provision of the public good. As a result, there can be welfare gains from robust policies even when the fear of model misspecification proves to be unfounded.

Keywords: model uncertainty; environmental externalities; robust policy
JEL codes: H41, D81, H23

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1 Introduction

Public goods are under-provided in the absence of coordination. This arises because of the incentive to free ride. In this paper, we revisit this classic issue in a dynamic context where there is uncertainty about the correct model specification regarding the public good. Environmental quality exemplifies this kind of public good, since the true model that generates environmental damage is poorly known. Examples of such environmental public goods (and their converse) include pollution, climate change, biodiversity, prevention and containment of disease outbreaks, and the production of scientific knowledge with respect to green technologies (see e.g. the discussion in Barrett (2008), Bramoullé and Treich (2009), Boucher and Bramoullé (2010) and Vardas and Xepapadeas (2010)).

There is a significant literature that examines individual voluntary contributions to the provision of a public good in a setting with uncertainty (see e.g. Cornes and Sandler (1996, ch. 6) for an exposition and Keenan *et al.* (2006) for a more recent contribution). Depending on properties of the utility function, the literature shows that a mean-preserving increase in the variance of the contribution by others to the public good can increase voluntary contributions and hence equilibrium public good provision. This result finds a parallel in work on dynamic models exhibiting precautionary behaviour, where the latter takes the form of increased asset accumulation, an outcome that typically requires a convex marginal utility function (see e.g. Ljungqvist and Sargent (2012, ch. 18)). The findings on increased public good provision under uncertainty naturally extend to the provision of international environmental public goods. For instance, in a static model with uncertainty about environmental damage and strategic interactions, Bramoullé and Treich (2009) show that higher uncertainty, in the form of increased variance in the stochastic processes that affect the quantity of the public good, can decrease emissions.[1] However, in this literature, it is assumed that agents know the model and uncertainty typically takes the form of random draws from known exogenous processes.

We contribute to this literature by focusing instead on model uncertainty. Under model uncertainty, the decision maker worries about potential model misspecification that is endogenous and may feed back on the state of the system, implying that shocks assigned to model uncertainty might arise as an

[1]The literature has also studied different forms of uncertainty and mechanisms for alleviating the free-riding problem in environmental public good provision (see e.g. Na and Shin (1998), Boucher and Bramoullé (2010) and Finus and Pintassilgo (2012) for the effect of uncertainty on international environmental agreements and Bogmans (2015) and Markusen (2014) for terms-of-trade incentives). See below for further details.

unknown function of the actions of the decision maker. Model uncertainty is a more general form of uncertainty and, we believe, a more appropriate one in the context of environmental public goods, where uncertainty typically relates to lack of knowledge about the true environmental mechanism and where disturbances to the environmental process are believed to be related to human activity in ways that are not fully known. Moreover, given that both the environmental process and the impact of economic activity on it are dynamic, we formally study their dynamic interaction.

To examine decision making under model uncertainty, we follow the literature on robust decision-making that provides a formalisation of a conservative or precautionary approach to choices under model uncertainty. A robust decision maker makes choices assuming that the unknown processes of the model, capturing potential misspecification, follow an appropriately defined worst-case scenario. This is achieved formally by focusing on the implementation of a *max-min* method (see e.g. Hansen and Sargent (2008)), resulting in a general recommendation to adopt precautionary policies that ensure a minimum level of welfare.[2]

In this paper, building on the above two literatures, we analyse the implications of robust decision-making for the provision of a public good in the absence of a coordination mechanism. In line with the examples in the opening paragraph, we work with a model in which the public good is the environment. In particular, we use a dynamic two-country model where environmental quality in one country affects the welfare of the other country. Cross-border externalities of this type lead to the standard incentive to free ride on one another's contribution to the public good, where free-riding takes the form of over-production and over-pollution. To this well-recognised setup, we add model uncertainty in the form of uncertainty about the evolution of environmental quality. We solve for a non-cooperative Nash equilibrium in which robust policymakers in one country do not internalise the effects of their economic and environmental policies on the welfare of the other country.

Our results show that, under model uncertainty, robust policies increase the provision of the public good. In other words, robust decision-making works as a substitute for cooperation. In particular, fear of model misspecification about the process driving the environmental public good creates a precautionary incentive that decreases the depletion of the environmental good and works in the opposite direction from the standard incentive to

[2] See e.g. the applications in Hansen and Sargent (2008 and 2010), Dennis (2010), Athanassoglou and Xepapadeas (2012) and Svec (2012) for examples of precautionary behaviour resulting from robust decision making in asset pricing, as well as in monetary, fiscal and environmental policy.

free ride on the contribution of others. Precautionary behaviour in general, and the increase in public good provision in particular, are associated with robust decision-making that arises from the fear of having misspecified the conditional mean of the stochastic state variables and so do not rely on the assumption of a convex marginal utility function.

The above incentive effect means that precautious policy can play a welfare-enhancing role, even when fear of model misspecification proves to be unfounded. When this corrective role outweighs the misallocation of resources that result in case fears of unfavourable scenarios prove to be unfounded, robust policy is welfare superior even though its *raison d'être* proves to be unjustified. What is critical is that conservatism of this type helps to correct for the underlying market imperfection of externalities that lead to free riding. The intuition here is consistent with that of Dennis (2010), who shows that the existence of a policy imperfection (in his setup, lack of commitment on the part of the central bank) implies that robust policy plays an additional corrective role, improving outcomes even when fear of model misspecification is unfounded. In our setup, the additional corrective channel leads to a trade-off between the unnecessary costs of precaution and the benefits from the correction of free riding. We find that these benefits dominate quantitatively for a large range of parameter values, hence robust policy is welfare-improving even when the fear of model misspecification proves to be unfounded. This result is different from the conventional wisdom regarding robust policy-making in which welfare comparisons between robust and non-robust policies are typically inconclusive at the point when policy is decided.

At the policy level, it is this inconclusiveness that has led to considerable debates about the desirability of precautious policies. This is evident, for instance, in the discussions about whether, or not, to adopt precautionary policies relating to environmental protection and climate change, health risk and disease spread, defence systems, financial regulation, etc. Proponents of precautionary measures highlight the potentially huge costs of model misspecification if societies are not well prepared for a "bad scenario". On the other hand, opponents highlight the large, potentially unnecessary costs of adopting such measures in the case where fear of the "bad scenario" proves unfounded. Here, we show that robust policy-making, driven by uncertainty about the model determining the provision of the public good, can be welfare-enhancing even if the fear of model misspecification proves to be unfounded.

We also consider, as an extension, uncertainty about the economic model, in addition to environmental model uncertainty. Economic model uncertainty refers to potential model misspecification that may, for instance, relate to developments in technology and institutions that affect the productive structure of the economy. If fears about misspecification of the economic model outweigh those about the environmental model, then precaution primarily takes the form of increased production, in turn intensifying the incentive to free ride and reducing the provision of the environmental public good. We also solve for asymmetric Nash equilibria, where asymmetry relates to which type of model uncertainty is more important for the two countries. For instance, consider the case in which the home country enjoys a more certain economic climate relative to the environmental one, and vice versa in the foreign country (corresponding to the typical north-south paradigm). Under precautious policies, this type of asymmetry leads to more intense free riding on the part of the foreign country compared to the symmetric case: the foreign country finds it optimal to reduce its environmental quality in order to take advantage of robust environmental policy adopted by the home country.

The rest of the paper is organised as follows. In section 2, we present a simple static model solved analytically to show some key results. The dynamic model is presented and solved in sections 3 and 4. The main results are presented and discussed in section 5. In section 6 we extend the model to add economic model uncertainty. Section 7 closes the paper. Further details on the solution and the robustness of the quantitative findings are in an Appendix.

2 A simple static model

We start out the paper by demonstrating analytically the main idea (namely, that free-riding decreases with robust policy) using a simple static model with public good type externalities. The underlying setup is the standard model of public good provision and voluntary contributions from individual agents. To this, we add model uncertainty and robust policy, following the relevant static models in Hansen and Sargent (2008, ch. 6). At this stage, we present model uncertainty and robust decision-making intuitively, leaving a more formal presentation and technical details for the following sections.

There are N identical agents indexed by the superscript $h = 1, 2, ..., N$. The agents are linked via a public good, which is obtained by aggregating agents' voluntary contributions. The budget constraint of each h is

$$c^h + \tau^h = y^h, \tag{1}$$

where c^h is h's private consumption, τ^h is h's voluntary contribution to a public good and the parameter y^h is an exogenous endowment.

The true model of public good provision is unknown to the agents. Each agent knows that the total amount of the public good provided, G, is determined by aggregating individual contributions but he also suspects that the process of transforming these contributions to effective public good units is misspecified. Assume, in particular, that for each contribution τ^h, the effective amount augmenting the public good is given by $\tau^h + \sigma w^h$, where w^h denotes an unknown distortion and the parameter σ scales the effect of w^h. The latter, namely w^h, can have many interpretations like agent-specific shocks with unspecified properties or other unknown individual characteristics and circumstances that may affect effective contributions to the public good. Therefore, G is given by

$$G = \frac{\sum_h \left[\tau^h + \sigma w^h\right]}{N} = \frac{\sum_h \tau^h}{N} + \frac{\sigma \sum_h w^h}{N}, \tag{2}$$

where we use per capita terms to avoid scale effects in equilibrium.

The basic idea of robust decision making is that a robust decision maker makes his optimal choices assuming that the unknown w^h is determined in a worst-case scenario. The latter means that w^h is chosen by a "malevolent" agent with the objective to minimise h's welfare function. Formally, a robust decision-maker faces the following problem

$$\max_{c^h,\tau^h} \min_{w^h} U = -\frac{\mu}{2}(c^h - c^*)^2 - \frac{(1-\mu)}{2}(G - G^*)^2,$$

subject to (1), (2) and a constraint on the size of w^h, namely $(w^h)^2 \leq \eta^S$, where $\eta^S > 0$ is a constant that bounds the choice of w^h (see subsection 3.2 below for details). In this specification, c^* and G^* denote exogenous utility bliss points for private consumption and public good provision, respectively, while $0 < \mu < 1$ is the weight given to private consumption relative to the public good. The constraint $(w^h)^2 \leq \eta^S$ captures the extent of fear of potential model misspecification in the sense that higher values of η^S indicate an increased fear of model misspecification and hence more precautious or robust policies. The linear-quadratic specification is chosen to match the main model in the next sections and, as in the literature, to facilitate analytical results.

Following Hansen and Sargent (2008, ch. 6), we solve the equivalent "multiplier" version of this problem, namely

$$\max_{c^h,\tau^h} \min_{w^h} -\frac{\mu}{2}(c^h - c^*)^2 - \frac{(1-\mu)}{2}(G - G^*)^2 + \frac{\theta}{2}(w^h)^2,$$

where $\theta > 0$ is a penalty parameter restraining the choice of w^h and which is inversely related to η^S (i.e. lower values of θ indicate an increased fear of model misspecification and hence more precautious or robust policies).

In a non-cooperative Nash setup, each agent takes the actions of other agents as given. Using the budget constraints to replace c^h and G, in a symmetric Nash equilibrium (we omit the superscript h in the symmetric solution below), the first-order conditions for τ^h and w^h are respectively

$$\tau = \frac{\mu(y - c^*) - \frac{(1-\mu)}{N}(\sigma w - G^*)}{\mu + \frac{(1-\mu)}{N}}$$

$$w = \frac{\frac{(1-\mu)\sigma}{N}(\tau - G^*)}{\theta - \frac{(1-\mu)\sigma^2}{N}},$$

where the second-order condition for w^h requires $\theta - \frac{(1-\mu)\sigma^2}{N} > 0$.

Combining the above two conditions, we have the reduced-form solution for τ (the solution for w is omitted)

$$\tau = \frac{\left(\theta - \frac{(1-\mu)\sigma^2}{N}\right)\mu(y - c^*) + \frac{(1-\mu)}{N}\theta G^*}{\left(\theta - \frac{(1-\mu)\sigma^2}{N}\right)\left(\mu + \frac{(1-\mu)}{N}\right) + \frac{(1-\mu)\sigma^2}{N^2}}. \tag{3}$$

Straightforward comparative statics in the above solution for τ imply that the sign of $\frac{\partial\tau}{\partial\theta}$ is the sign of $(y - c^* - G^*)$. Hence, in the economically interesting parameter region in which $(y - c^* - G^*)$ is negative,[3] $\frac{\partial\tau}{\partial\theta}$ is also negative. In other words, as θ falls (meaning that agents become more conservative), τ rises (meaning that the free riding problem becomes milder). Therefore, robustness works as a substitute to cooperation.

It is interesting to study the welfare implications of robust behaviour when the fear of model misspecification proves to be unfounded, i.e. when $w = 0$ ex post. In a setup without market imperfections, this typically leads to welfare losses, as a result of unnecessary precautious behaviour, which proves to be non-optimal ex post. To see what happens in our setup with public good type externalities, we substitute (3) back into the objective function, set $w = 0$, and obtain the partial derivative of utility with respect to θ

$$\frac{\partial U}{\partial \theta} = \mu(c^h - c^*)\frac{\partial\tau}{\partial\theta} - (1-\mu)(G - G^*)\frac{\partial\tau}{\partial\theta}, \tag{4}$$

so that, if the second term on the right-hand side of (4) dominates the first term, this partial is negative implying that more precautious behaviour (i.e. lower θ) increases welfare, even when the fear of model misspecification proves

[3] In other words, we assume that the bliss or satiation levels of private and public consumption are large relative to the typical value of income. Notice that $y = c + G$, so that the condition $(y - c^* - G^*) < 0$, is equivalent to $c + G < c^* + G^*$ (when $w = 0$).

to be unfounded.[4] Thus, regarding welfare, the effect of θ is ambiguous depending on the magnitude of the two terms on the right-hand side. Below, in a dynamic and richer model, we will investigate the quantitative trade-off implied in (4) in more detail. However, it is useful to note that the final result depends on the extent of under-provision of the public good. In particular, the worse the social problem of public good under-provision, and thus the bigger the gap $(G - G^*)$ is in absolute value, the bigger the quantitative importance of the second term, and thus of the welfare-enhancing role of precaution.

3 A dynamic model with externalities and robust decision making

There are two countries called home (h) and foreign (f) which, for simplicity, are assumed to be identical. Each country is populated by one representative agent, who consumes, produces and pollutes the environment. Pollution occurs as a by-product of production. The two countries are linked via environmental quality, which is an international public good. In particular, the agent in each country values economy-wide, or world, environmental quality, defined as the weighted average of environmental quality in each country. Since world environmental quality is treated as a public good, there are standard free-riding incentives. At the same time, the agent in each country is uncertain about the true process that generates pollution, or equivalently environmental quality (for model uncertainty in models with environmental public goods, see e.g. Athanassoglou and Xepapadeas (2012)).

3.1 Setup

We present the problem for the home country in more detail and just summarise afterwards the symmetric problem for the foreign country. The agent in the home country derives utility from consumption, c_t^h, where the superscript h denotes outcomes in the home country, and environmental quality. The latter is a weighted sum of environmental quality at home, $Q_t^{h,h}$, and abroad, $Q_t^{h,f}$ (the meaning of the double superscript will be discussed below).

[4] Recall that we work in the economically interesting area with relatively large bliss points so that $(c^h - c^*)$, $(g^h - G^*)$ and hence $\frac{\partial \tau}{\partial \theta}$ are all negative.

Formally, the within-period utility of the agent in h is given by[5]

$$U_t^h = -\left(\mu(c_t^h - c^*)^2 + (1-\mu)\left(Q_t^{h,h} + \xi Q_t^{h,f} - (1+\xi)\, Q^*\right)^2\right) \tag{5}$$

where c^* and Q^* are utility bliss points for consumption and environmental quality respectively, the parameter $0 < \xi < 1$ measures the extent of environmental externalities from one country to the other and, as in the simple model above, the parameter $0 < \mu < 1$ measures the relative weight given to consumption.

The agent produces output by using a linear AK-type technology. Substituting the resource constraint and the production function in the capital evolution equation, the "economic model" in country h is given by

$$k_{t+1}^h - (1-\delta)k_t^h + c_t^h = Ak_t^h$$

where k_{t+1}^h and k_t^h denote respectively the end-of-period and the beginning-of-period capital stock, $0 < \delta < 1$ is a depreciation rate and $A > 0$ is a technology scale factor.

The agent is uncertain about the environmental model, that is, about the process of environmental quality. In particular, the "true" model for the motion of environmental quality is assumed to be

$$Q_{t+1}^{h,h} = (1-\rho^Q)Q^h + \rho^Q Q_t^{h,h} - \varphi k_t^h + \sigma^Q(\varepsilon_{t+1}^{h,Q} + w_{t+1}^{h,h}) \tag{6}$$

$$Q_{t+1}^{h,f} = (1-\rho^Q)Q^f + \rho^Q Q_t^{h,f} - \varphi k_t^f + \sigma^Q(\varepsilon_{t+1}^{f,Q} + w_{t+1}^{h,f}), \tag{7}$$

where $0 < \rho^Q < 1$ measures the persistence of environmental quality; Q^h and Q^f are constant terms to be further explained below; $\varphi > 0$ measures the extent to which economic activity, as captured by k_t^h and k_t^f, damages environmental quality; $\varepsilon_{t+1}^{h,Q}$ and $\varepsilon_{t+1}^{f,Q}$ are *i.i.d.* Gaussian variables distributed with zero mean and unit variance; $w_{t+1}^{h,h}$ and $w_{t+1}^{h,f}$ are unknown processes capturing potential misspecification of the environmental model; and σ^Q scales the size of $\varepsilon_{t+1}^{h,Q}$ and $\varepsilon_{t+1}^{f,Q}$ and the unknown $w_{t+1}^{h,h}$ and $w_{t+1}^{h,f}$. Hence Q^h and Q^f give the expected long-run values of environmental quality in the two countries in the absence of economic activity and model misspecification.

The presence of the unknown processes $w_{t+1}^{h,h}$ and $w_{t+1}^{h,f}$, which may be functions of the model's state variables, implies that the true environmental model is not known to the agent. Note that the agent in the home country is uncertain about environmental processes in both countries. Thus, $w_{t+1}^{h,h}$

[5]This functional forms for utility and production are chosen so that the problem can be written in linear-quadratic form (see e.g. Hansen and Sargent (2008, ch. 10) for utility and Economides and Philippopoulos (2008) for the production function).

captures model misspecification, as feared by country h, for the environmental process of country h. Similarly, $w_{t+1}^{h,f}$, captures model misspecification, as feared by country h, for the environmental process of country f. This representation allows the decision maker to choose allocations under potentially different perceptions about the worst-case processes in the two countries.

3.2 Robust decision making

A robust decision maker is an agent who makes decisions that are robust to model misspecification, in the sense that they give him good results even in the presence of unfavourable w shocks, where w is defined to include the unknown perturbations that are relevant for the agent (i.e. in country h, w includes $w_{t+1}^{h,h}$ and $w_{t+1}^{h,f}$). In order for his decision rule to assure him a lower bound on utility in an unfavourable environment, the agent makes his choices (for consumption, production and investment) as if the unknown w process follows a worst-case scenario. In particular, he pretends that w is chosen by a fictional malevolent agent, whose objective is to minimise his (the agent's) objective. By planning against such a worst-case process, he designs a decision rule that performs well under a set of perturbed models.[6] In other words, the maximising agent uses the malevolent agent as a device to achieve robustness. This implies that the agent/country solves a *maxmin* problem. A robust decision maker makes his choices under a constraint imposed on the malevolent agent which measures the degree of fear of model misspecification that characterises the robust decision maker. The tighter the constraint, the less conservative policy-making is.

To formalise this approach, we need to measure the distance between the true model and a benchmark model that the decision maker would trust to be correct in the absence of model misspecification. This benchmark or "approximating" model of the motion for environmental quality is

$$Q_{t+1}^{h,h} = (1-\rho^Q)Q^h + \rho^Q Q_t^{h,h} - \varphi k_t^h + \sigma^Q \widehat{\varepsilon}_{t+1}^{h,Q} \tag{8}$$

$$Q_{t+1}^{h,f} = (1-\rho^Q)Q^f + \rho^Q Q_t^{h,f} - \varphi k_t^f + \sigma^Q \widehat{\varepsilon}_{t+1}^{f,Q}, \tag{9}$$

where $\widehat{\varepsilon}_{t+1}^{h,Q}$ and $\widehat{\varepsilon}_{t+1}^{f,Q}$ are Gaussian variables distributed identically and independently through time with zero mean and unit variance. Hence, the true model is a "distorted" or "perturbed" version of the approximating model.

To measure the difference between the approximating environmental model and the distorted environmental model for the home country, let x_t^h denote

[6] By defintion, robust choices will be optimal under the worst-case scenario.

the vector $\left(Q_t^{h,h}, Q_t^{h,f}\right)$, f_0^h denote the one-step transition density associated with the approximating environmental model and f^h denote the one-step transition density associated with the true, or distorted, environmental model. Following Hansen and Sargent (2008), we use *conditional relative entropy*, defined as the expected log-likelihood ratio of the two models, evaluated with respect to the true model, to measure the statistical discrepancy between the two models in the transition from x_t^h to x_{t+1}^h

$$I(f_0^h, f^h)(x^h) = \int \log\left(\frac{f^h(x_{t+1}^h \mid x_t^h)}{f_0^h(x_{t+1}^h \mid x_t^h)}\right) f^h(x_{t+1}^h \mid x_t^h) dx_{t+1}^h, \tag{10}$$

where it can be shown that

$$I(f_0^h, f^h)(x^h) = 0.5\left[\left(w_{t+1}^{h,h}\right)^2 + \left(w_{t+1}^{h,f}\right)^2\right].$$

In turn, the quantity $2E_0 \sum_{t=0}^{\infty} \beta^{t+1} I(f_0^h, f^h)(x^h)$ can be considered as an intertemporal measure of feared model misspecification for the home country, where $0 < \beta < 1$ is the subjective discount factor and the mathematical expectation is evaluated with respect to the distorted model.

The restriction that the robust decision maker imposes on the malevolent agent is thus given by

$$E_0 \sum_{t=0}^{\infty} \beta^{t+1}\left[\left(w_{t+1}^{h,h}\right)^2 + \left(w_{t+1}^{h,f}\right)^2\right] \leq \eta,$$

where η measures the fear of model misspecification. When $\eta = 0$, the problem collapses to the case where the agent believes that the approximating model is also the true model. In this case, the agent/country makes choices without fearing model uncertainty. On the other hand, the higher is η, the more conservative behavior is incorporated in decision making, as the entropy constraint allows for bigger (and thus more harmful) model misspecification.

3.3 Summarising the countries' problems

To summarise, given the extent of fear of model misspecification η, the home country solves the following problem

$$\max_{\{c_t^h\}_{t=0}^{\infty}} \min_{\{w_{t+1}^{h,h}, w_{t+1}^{h,f}\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} -\beta^t \left(\mu(c_t^h - c^*)^2 + (1-\mu)\left(Q_t^{h,h} + \xi Q_t^{h,f} - (1+\xi) Q^*\right)^2\right), \tag{11}$$

where the motions of the related state variables are

$$k_{t+1}^{h} - (1-\delta)k_t^h + c_t^h = Ak_t^h$$

$$Q_{t+1}^{h,h} = (1-\rho^Q)Q^h + \rho^Q Q_t^{h,h} - \varphi k_t^h + \sigma^Q(\varepsilon_{t+1}^{h,Q} + w_{t+1}^{h,h})$$
$$Q_{t+1}^{h,f} = (1-\rho^Q)Q^f + \rho^Q Q_t^{h,f} - \varphi k_t^f + \sigma^Q(\varepsilon_{t+1}^{f,Q} + w_{t+1}^{h,f}),$$

$w_{t+1}^{h,h}$ and $w_{t+1}^{h,f}$ satisfy the constraint

$$E_0 \sum_{t=0}^{\infty} \beta^{t+1} \left[\left(w_{t+1}^{h,h} \right)^2 + \left(w_{t+1}^{h,f} \right)^2 \right] \leq \eta \tag{12}$$

and the actions of the foreign country are taken as given, and, in particular, capital in the foreign country follows

$$k_{t+1}^{f} - (1-\delta)k_t^f + c_t^f = Ak_t^f.$$

Note that the entropy constraint in (12) is a constraint that is used for decision-making purposes only and does not apply to the real, unknown $w_{t+1}^{h,h}$ and $w_{t+1}^{h,f}$ processes. In other words, a specific value of η matters for the perceived $w_{t+1}^{h,h}$ and $w_{t+1}^{h,f}$ processes, but not for the actual or realised $w_{t+1}^{h,h}$ and $w_{t+1}^{h,f}$ processes.

The problem of the foreign country is symmetric. We present it for completeness.

$$\max_{\{c_t^f\}_{t=0}^{\infty}} \min_{\{w_{t+1}^{f,f}, w_{t+1}^{f,h}\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} -\beta^t \left(\mu(c_t^f - c^*)^2 + (1-\mu)\left(Q_t^{f,f} + \xi Q_t^{f,h} - (1+\xi)\, Q^*\right)^2 \right), \tag{13}$$

where the motions of the related state variables are

$$k_{t+1}^{f} - (1-\delta)k_t^f + c_t^f = Ak_t^f$$

$$Q_{t+1}^{f,f} = (1-\rho^Q)Q^f + \rho^Q Q_t^{f,f} - \varphi k_t^f + \sigma^Q(\varepsilon_{t+1}^{f,Q} + w_{t+1}^{f,f})$$
$$Q_{t+1}^{f,h} = (1-\rho^Q)Q^h + \rho^Q Q_t^{f,h} - \varphi k_t^h + \sigma^Q(\varepsilon_{t+1}^{h,Q} + w_{t+1}^{f,h}),$$

$w_{t+1}^{f,f}$ and $w_{t+1}^{f,h}$ are defined analogously to $w_{t+1}^{h,h}$ and $w_{t+1}^{h,f}$ and satisfy the constraint

$$E_0 \sum_{t=0}^{\infty} \beta^{t+1} \left[\left(w_{t+1}^{f,f} \right)^2 + \left(w_{t+1}^{f,h} \right)^2 \right] \leq \eta$$

and

$$k_{t+1}^{h} - (1-\delta)k_t^h + c_t^h = Ak_t^h.$$

Our modeling allows the decision makers in the two countries to choose economic and environmental allocations under potentially different perceptions about the worst-case processes in the two countries. Thus, for the purposes of robust decision making, we do not impose $w_{t+1}^{h,h} = w_{t+1}^{f,h}$ and $w_{t+1}^{f,f} = w_{t+1}^{h,f}$ and thus we do not impose $Q_{t+1}^{h,h} = Q_{t+1}^{f,h}$ and $Q_{t+1}^{f,f} = Q_{t+1}^{h,f}$. Of course, given that all w represent a fictional device in the robust problem used only to determine the choices for the variables under the control of the maximising agents, the actual process for environmental quality must satisfy $Q_{t+1}^{h,h} = Q_{t+1}^{f,h} \equiv Q_{t+1}^{h}$ and $Q_{t+1}^{f,f} = Q_{t+1}^{h,f} \equiv Q_{t+1}^{f}$ ex post. In the special case in which $w_{t+1}^{h,h} = w_{t+1}^{f,h} \equiv w_{h,t+1}^{h} = 0$ and $w_{t+1}^{f,f} = w_{t+1}^{h,f} \equiv w_{f,t+1}^{f} = 0$, the problem collapses to the standard case without model uncertainty, where $Q_{t+1}^{h,h} = Q_{t+1}^{f,h} \equiv Q_{t+1}^{h}$ and $Q_{t+1}^{f,f} = Q_{t+1}^{h,f} \equiv Q_{t+1}^{f}$. For the more general case with robust policy making, when we simulate the solution to the model, there will be one process for $Q_{t+1}^{h,h} = Q_{t+1}^{f,h} \equiv Q_{t+1}^{h}$ and one process for $Q_{t+1}^{f,f} = Q_{t+1}^{h,f} \equiv Q_{t+1}^{f}$ ex post, as there is a single exogenous realisation of $w_{h,t+1}^{h}$ and $w_{f,t+1}^{f}$ respectively.

4 Nash equilibrium with robust policies

In this section, we solve a Nash game between the two countries when, at the same time, there is a *maxmin* (robust control) problem within each country. Thus, each country chooses its robust policies (i.e. it solves its *maxmin* problem) by taking the robust policies of the other country (i.e. the *maxmin* problem of the other country) as given.

4.1 Linear-quadratic representation of the problem

4.1.1 Home country

We first present the problem of the home country. By defining $\widetilde{c}_t^h \equiv c_t^h - c^*$, $\widetilde{c}_t^f \equiv c_t^f - c^*$, $\widetilde{Q}_t^{h,h} \equiv Q_t^{h,h} - Q^*$, $\widetilde{Q}_t^{h,f} \equiv Q_t^{h,f} - Q^*$, $\widetilde{Q}_t^{f,f} \equiv Q_t^{f,f} - Q^*$ and $\widetilde{Q}_t^{f,h} \equiv Q_t^{f,h} - Q^*$, we can rewrite the problem as

$$\max_{\{\widetilde{c}_t^h\}_{t=0}^{\infty}} \min_{\{w_{t+1}^{h,h}, w_{t+1}^{h,f}\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} -\beta^t \left(\mu (\widetilde{c}_t^h)^2 + (1-\mu) (\widetilde{Q}_t^{h,h} + \xi \widetilde{Q}_t^{h,f})^2 \right), \tag{14}$$

where the motions of the related state variables are

$$k_{t+1}^h = -c^* + (A + 1 - \delta) k_t^h - \widetilde{c}_t^h \tag{15}$$

$$k_{t+1}^f = -c^* + (A + 1 - \delta) k_t^f - \widetilde{c}_t^f \tag{16}$$

$$\widetilde{Q}^{h,h}_{t+1} = (1-\rho^Q)Q^h + \left(\rho^Q - 1\right) Q^* + \rho^Q \widetilde{Q}^{h,h}_t - \varphi k^h_t + \sigma^Q(\varepsilon^{h,Q}_{t+1} + w^{h,h}_{t+1}) \quad (17)$$
$$\widetilde{Q}^{h,f}_{t+1} = (1-\rho^Q)Q^f + \left(\rho^Q - 1\right) Q^* + \rho^Q \widetilde{Q}^{h,f}_t - \varphi k^f_t + \sigma^Q(\varepsilon^{f,Q}_{t+1} + w^{h,f}_{t+1}) \quad (18)$$
$$\widetilde{Q}^{f,f}_{t+1} = (1-\rho^Q)Q^f + \left(\rho^Q - 1\right) Q^* + \rho^Q \widetilde{Q}^{f,f}_t - \varphi k^f_t + \sigma^Q(\varepsilon^{f,Q}_{t+1} + w^{f,f}_{t+1}) \quad (19)$$
$$\widetilde{Q}^{f,h}_{t+1} = (1-\rho^Q)Q^h + \left(\rho^Q - 1\right) Q^* + \rho^Q \widetilde{Q}^{f,h}_t - \varphi k^h_t + \sigma^Q(\varepsilon^{h,Q}_{t+1} + w^{f,h}_{t+1}), \quad (20)$$

and

$$E_0 \sum_{t=0}^{\infty} \beta^{t+1} \left[\left(w^{h,h}_{t+1} \right)^2 + \left(w^{h,f}_{t+1} \right)^2 \right] \leq \eta. \quad (21)$$

Let $\widetilde{h}$ equal the number of states $\left(1, k^h_t, k^f_t, \widetilde{Q}^{h,h}_t, \widetilde{Q}^{h,f}_t, \widetilde{Q}^{f,f}_t, \widetilde{Q}^{f,h}_t\right)$; $n^{h,\max}$ equal the number of controls $(\widetilde{c}^h_t)$ for the maximising agent in the home country; $n^{h,\min}$ equal the number of controls $\left(w^{h,h}_{t+1}, w^{h,f}_{t+1}\right)$ for the minimising agent in the home country; $n^{f,\max}$ equal the number of controls $(\widetilde{c}^f_t)$ for the maximising agent in the foreign country; $n^{f,\min}$ equal the number of controls $\left(w^{f,f}_{t+1}, w^{f,h}_{t+1}\right)$ for the minimising agent in the foreign country; and n^{ex} equal the number of exogenous shocks $\left(\varepsilon^{h,Q}_{t+1}, \varepsilon^{f,Q}_{t+1}\right)$. We can now write the linear constraints in (15)-(20) above in matrix form as

$$x_{t+1} = Ax_t + B^h u^h_t + B^f u^f_t + C\varepsilon_{t+1} + D^h w^h_{t+1} + D^f w^f_{t+1},$$

where

$$x_t = \left[\begin{array}{ccccccc} 1 & k^h_t & k^f_t & \widetilde{Q}^{h,h}_t & \widetilde{Q}^{h,f}_t & \widetilde{Q}^{f,f}_t & \widetilde{Q}^{f,h}_t \end{array} \right]';$$
$$u^h_t = \left[\widetilde{c}^h_t\right]'; \quad u^f_t = \left[\widetilde{c}^f_t\right]';$$
$$w^h_{t+1} = \left[\begin{array}{cc} w^{h,h}_{t+1} & w^{h,f}_{t+1} \end{array} \right]'; \quad w^f_{t+1} = \left[\begin{array}{cc} w^{f,f}_{t+1} & w^{f,h}_{t+1} \end{array} \right]';$$
$$\varepsilon_{t+1} = \left[\begin{array}{cc} \varepsilon^{h,Q}_{t+1} & \varepsilon^{f,Q}_{t+1} \end{array} \right]';$$

$$A_{(\widetilde{h}x\widetilde{h})} = \left[\begin{array}{lllllll} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -c^* & A+1-\delta & 0 & 0 & 0 & 0 & 0 \\ -c^* & 0 & A+1-\delta & 0 & 0 & 0 & 0 \\ (1-\rho^Q)Q^h + \left(\rho^Q-1\right)Q^* & -\varphi & 0 & \rho^Q & 0 & 0 & 0 \\ (1-\rho^Q)Q^f + \left(\rho^Q-1\right)Q^* & 0 & -\varphi & 0 & \rho^Q & 0 & 0 \\ (1-\rho^Q)Q^f + \left(\rho^Q-1\right)Q^* & 0 & -\varphi & 0 & 0 & \rho^Q & 0 \\ (1-\rho^Q)Q^h + \left(\rho^Q-1\right)Q^* & -\varphi & 0 & 0 & 0 & 0 & \rho^Q \end{array} \right];$$

$$B^h_{(\widetilde{h}xn^{h,\max})} = \begin{bmatrix} 0 \\ -1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}; \quad B^f_{(\widetilde{h}xn^{f,\max})} = \begin{bmatrix} 0 \\ 0 \\ -1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix};$$

$$C_{(\widetilde{h}xn^{ex})} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \sigma^Q & 0 \\ 0 & \sigma^Q \\ 0 & \sigma^Q \\ \sigma^Q & 0 \end{bmatrix}; \quad D^h_{(\widetilde{h}xn^{h,\min})} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \sigma^Q & 0 \\ 0 & \sigma^Q \\ 0 & 0 \\ 0 & 0 \end{bmatrix}; \quad D^f_{(\widetilde{h}xn^{f,\min})} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \sigma^Q & 0 \\ 0 & \sigma^Q \end{bmatrix}$$

Following Hansen and Sargent (2008), we solve the multiplier version of this problem, by adding the entropy constraint, (21), to the objective function with a time-invariant multiplier, denoted as θ, so that we rewrite the problem of the home country as

$$\max_{\{u^h_t\}^\infty_{t=0}} \min_{\{w^h_{t+1}\}^\infty_{t=0}} E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \left(u^h_t\right)' R^h_u u^h_t + \left(w^h_{t+1}\right)' R^h_w w^h_{t+1} + x'_t Q^h x_t \right\}, \qquad (22)$$

subject to

$$x_{t+1} = Ax_t + B^h u^h_t + B^f u^f_t + C\varepsilon_{t+1} + D^h w^h_{t+1} + D^f w^f_{t+1}, \qquad (23)$$

where

$$R^h_{u(n^{h,\max}xn^{h,\max})} = [-\mu]; \quad R^h_{w(n^{f,\min}xn^{f,\min})} = \begin{bmatrix} \beta\theta & 0 \\ 0 & \beta\theta \end{bmatrix};$$

$$Q^h_{(\widetilde{h}x\widetilde{h})} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -(1-\mu) & -((1-\mu)\xi) & 0 & 0 \\ 0 & 0 & 0 & -((1-\mu)\xi) & -(1-\mu)\xi^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

As said above, the nonnegative multiplier, θ, is a penalty on the minimising agent for choosing policies that reduce welfare for the maximising agent and can be used as a measure of the degree of robustness. The value of θ is inversely related to the value of η, so that the lower is θ, the higher the degree of robustness. We require R_u^h to be negative definite, Q^h to be negative semi-definite and R_w^h to be positive definite (see also Anderson *et al.* (1996), and Hansen and Sargent (2008), for assumptions regarding the coefficient matrices for linear-quadratic problems).

To solve each country's *maxmin* problem, we solve for Markov strategies in a Nash game between the maximising and the minimising agent. To implement this solution, we use the observation that in (22)-(23), the first-order conditions of the maximising agent with respect to u_t^h and of the minimising agent with respect to w_{t+1}^h, are the same as the first-order conditions of an ordinary (i.e. non-robust) optimal linear regulator (OLR) who chooses $\widetilde{u}_t^h$, where $\widetilde{u}_{t(\widetilde{n}x1)}^h = \begin{pmatrix} u_t^{h\prime} & w_{t+1}^{h\prime} \end{pmatrix}'$ (see also Hansen and Sargent (2008, ch. 2)). Hence, we write the *extremisation*[7] problem in (22)-(23) as

$$\underset{\{\widetilde{u}_t^h\}_{t=0}^{\infty}}{ext} E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \left(\widetilde{u}_t^h\right)' R^h \widetilde{u}_t^h + x_t' Q^h x_t \right\} \tag{24}$$

$$x_{t+1} = Ax_t + \widetilde{B}^h \widetilde{u}_t^h + \widetilde{B}^f \widetilde{u}_t^f + C\epsilon_{t+1}, \tag{25}$$

where

$$R_{(\widetilde{n}x\widetilde{n})}^h = \begin{bmatrix} [R_u^h] & 0_{(n^{h,\max}xn^{h,\min})} \\ 0_{(n^{h,\min}xn^{h,\max})} & [R_w^h] \end{bmatrix}$$

and $\widetilde{B}_{(\widetilde{h}x\widetilde{n})}^h = \begin{bmatrix} B^h & D^h \end{bmatrix}$, $\widetilde{u}_{t(\widetilde{n}x1)}^f = \begin{pmatrix} u_t^{f\prime} & w_{t+1}^{f\prime} \end{pmatrix}'$, $\widetilde{B}_{(\widetilde{h}x\widetilde{n})}^f = \begin{bmatrix} B^f & D^f \end{bmatrix}$ and $\widetilde{n} = n^{h,\max} + n^{h,\min} = n^{f,\max} + n^{f,\min}$.

4.1.2 Foreign country

Working similarly, the extremisation problem in the foreign country is given by

$$\underset{\{\widetilde{u}_t^f\}_{t=0}^{\infty}}{ext} E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \left(\widetilde{u}_t^f\right)' R^f \widetilde{u}_t^f + x_t' Q^f x_t \right\}$$

$$x_{t+1} = Ax_t + \widetilde{B}^h \widetilde{u}_t^h + \widetilde{B}^f \widetilde{u}_t^f + C\epsilon_{t+1},$$

where the matrices A, $\widetilde{B}^h$, $\widetilde{B}^f$and C and the vectors x_t, $\widetilde{u}_t^h$, $\widetilde{u}_t^f$ and ϵ_{t+1} are as above and

$$R_{(\widetilde{n}x\widetilde{n})}^f = \begin{bmatrix} [R_u^f] & 0_{(n^{f,\max}xn^{f,\min})} \\ 0_{(n^{f,\min}xn^{f,\max})} & [R_w^f] \end{bmatrix},$$

$$R_{u(n^{f,\max}xn^{f,\max})}^f = [-\mu]; \quad R_{w(n^{f,\min}xn^{f,,\min})}^f = \begin{bmatrix} \beta\theta & 0 \\ 0 & \beta\theta \end{bmatrix} \quad ;$$

[7] Following Whittle (1990), extremisation denotes joint maximisation and minimisation.

$$Q^f_{(\widetilde{h}x\widetilde{h})} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -(1-\mu) & -((1-\mu)\xi) \\ 0 & 0 & 0 & 0 & 0 & -((1-\mu)\xi) & -(1-\mu)\xi^2 \end{bmatrix}.$$

4.2 Nash game in Markov strategies between countries

We are now ready to solve the Nash game between countries and between the benevolent and the malevolent agent within each country. We solve for Markov strategies in this game. The Nash equilibrium will then be obtained by combining the first-order conditions of both countries.

4.2.1 Bellman equations

We make use of a type of *certainty equivalence*, which applies to the class of linear-quadratic games relevant here (see e.g. Hansen and Sargent, 2008, ch. 2). In particular, the decision rules for both the maximising and the minimising agents in the *maxmin* game are the same in a particular non-stochastic version of the problem, where $\varepsilon_{t+1} = 0$. Therefore, we focus on the problem

$$\underset{\{\widetilde{u}^h_t\}^{\infty}_{t=0}}{ext} \sum_{t=0}^{\infty} \beta^t \left\{ \left(\widetilde{u}^h_t\right)' R^h \widetilde{u}^h_t + x_t' Q^h x_t \right\} \tag{26}$$

$$x_{t+1} = Ax_t + \widetilde{B}^h \widetilde{u}^h_t + \widetilde{B}^f \widetilde{u}^f_t. \tag{27}$$

The Bellman equation for the extremisation problem is

$$v^h(x) = \underset{\widetilde{u}^h}{ext} \left\{ \left(\widetilde{u}^h\right)' R^h \widetilde{u}^h + x' Q^h x + \beta v^h(\widetilde{x}) \right\}, \tag{28}$$

where $v^h(x)$ is the value function, x is current period's state, $\widetilde{x}$ is next period's state. We guess that the value function for the extremisation problem is quadratic

$$v^h(x) = x' P^h x, \tag{29}$$

where P^h is a negative semidefinite symmetric $(\widetilde{h} \times \widetilde{h})$ matrix. Substituting the guess function in (28) the Bellman equation gives

$$x' P^h x = \underset{\widetilde{u}^h}{ext} \left\{ \left(\widetilde{u}^h\right)' R^h \widetilde{u}^h + x' Q^h x + \beta \widetilde{x}' P^h \widetilde{x} \right\}. \tag{30}$$

The Bellman equation for the foreign country is symmetric to the above, obtained by substituting the superscript h with f and f with h.

4.2.2 Recursive solution

We describe in detail in Appendix A how to obtain the solution for this problem, first, by combining the first-order conditions for the two countries and solving the system to obtain the Nash equilibrium given the guesses for P^h and P^f, and, then, by obtaining P^h and P^f and verifying the guessed form for the solution and iterating on the resulting system of Riccati equations. The solution is summarised by the following recursive, state-space form

$$\begin{aligned} \widetilde{u}_t^h &= F x_t, \\ \widetilde{u}_t^f &= K x_t \text{ and} \\ x_{t+1} &= A x_t + \widetilde{B}^h \widetilde{u}_t^h + \widetilde{B}^f \widetilde{u}_t^f + C\epsilon_{t+1}, \text{ or} \\ x_{t+1} &= \left(A + \widetilde{B}^h F + \widetilde{B}^f K\right) x_t + C\epsilon_{t+1}. \end{aligned} \tag{31}$$

Working backwards, an appropriate partitioning of the matrix F will give us the policy function for u_t^h and the worst case scenario for w_{t+1}^h

$$\begin{aligned} u_t^h &= F^u x_t \\ w_{t+1}^h &= F^w x_t, \end{aligned} \tag{32}$$

and, similarly, an appropriate partitioning of the matrix K will give us the policy function for u_t^f and the worst case scenario for w_{t+1}^f

$$\begin{aligned} u_t^f &= K^u x_t \\ w_{t+1}^f &= K^w x_t. \end{aligned} \tag{33}$$

We can therefore summarise the solution as

$$u_t^h = F^u x_t \tag{34}$$
$$w_{t+1}^h = F^w x_t \tag{35}$$
$$u_t^f = K^u x_t \tag{36}$$
$$w_{t+1}^f = K^w x_t \tag{37}$$
$$\begin{aligned} x_{t+1} &= \left(A + B^h F^u + B^f K^u\right) x_t + \\ &\quad + D^h w_{t+1}^h + D^f w_{t+1}^f + C\epsilon_{t+1}. \end{aligned} \tag{38}$$

5 Implications of robust policy in a Nash equilibrium

As discussed earlier, the malevolent agent, or else the worst-case scenario, is only used as a fictional device by the conservative decision-maker in order to achieve robustness. The actual model will be given in general by

$$u_t^h = F^u x_t \tag{39}$$

$$u_t^f = K^u x_t \tag{40}$$

$$\begin{aligned} x_{t+1} = &\left(A + B^h F^u + B^f K^u\right) x_t + \\ &+ D^h w_{t+1}^h + D^f w_{t+1}^f, \end{aligned} \tag{41}$$

where w_{t+1}^h and w_{t+1}^f follow unknown processes, and not necessarily the worst-case processes in (32) and (33).[8] Note that the matrices F and K (and thus F^u and K^u) depend on the model's parameters and thus on θ as well. Hence, fear of model misspecification, i.e. the value of θ, affects the robust policies u_t^h and u_t^f chosen by the countries, which in turn matter for the evolution of the state equation, although the actual w_{t+1}^h and w_{t+1}^f are independent of θ. For instance, if the countries ignore model uncertainty in their choices, so that $\theta \to \infty$ in (39)-(41), the unknown processes w_{t+1}^h and w_{t+1}^f do not disappear.

To calculate equilibrium outcomes and welfare for different values of θ, we assume that, at time period 0, the equilibrium is given by the steady state of the Nash equilibrium under model certainty[9] and then simulate the solution following (39)-(41), conditional on specific ex post scenarios regarding the processes $\left\{w_{t+1}^h, w_{t+1}^f\right\}_{t=0}^T$ that will be discussed below. The time horizon for the simulations is $T = 300$ years.

The parameter values used for the numerical solutions of the model are given in Table 1. The economic and environmental parameters are similar to those commonly used in the literature (see e.g. Angelopoulos *et al.* (2013) for references). The target values for consumption and environmental quality and the productivity parameters are chosen to ensure interior, well-defined solutions for the economic and environmental variables in the model. A sensitivity analysis for the quantitative effect of the parameters is conducted and summarised below.

Table 1: Parameter values

β	μ	c^*	Q^*	ξ	δ	A	φ	Q^h	Q^f	ρ^Q	σ^Q
0.97	0.7	1.5	2	0.8	0.1	2	0.05	1	1	0.95	0.01

[8] The representation in (39)-(41) encompasses (34)-(38) as a special case, in particular when the unknown processes in (39)-(41) follow the feared worst-case scenario in (34)-(38). For simplicity, we turn off all random shocks associated with the known distribution ε_{t+1}.

[9] Note that this is equivalent to assuming that at time 0 the equilibrium is given by the steady state of the Nash equilibrium without fear of model misspecification in policies (i.e. when $\theta \to \infty$) and fear of model misspecification proves to be unfounded ex post, i.e. $w_{t+1}^h = w_{t+1}^f = 0$, ∇t, in (39)-(41).

5.1 Scenarios studied

We examine equilibrium outcomes under different scenarios for $\left\{w_{t+1}^{h}, w_{t+1}^{f}\right\}_{t=0}^{T}$. In particular, we first examine outcomes under two examples of "bad scenarios", i.e. under negative realisations of model uncertainty. At the other extreme, we examine outcomes in the limiting case where the fear of model misspecification proves to be unfounded. These scenarios demonstrate the two key results of the model, namely, the increased public good provision under robust policies and the ex ante welfare superiority of robust policies.

In the case of bad scenarios, we present examples when the unknown distributions associated with model uncertainty always take negative values. In particular, for the results presented in the Tables below, we assume that the values for the model misspecification variables do not follow the worst-case scenario, as given by (35) and (37), but are equal to the absolute of values drawn from a standard normal, multiplied by $-2 * \sigma^{Q}$ to obtain a "very bad scenario" and by $-\sigma^{Q}$ to obtain a "bad scenario". We thus simulate the model as given in (39)-(41), where w_{t+1}^{h} and w_{t+1}^{f} take these negative values. We examine cases of such "bad scenarios" and not the "worst-case" scenario, as, under the worst-case scenario, robust policy is optimal by definition. These examples serve to capture the potential benefits of robust, relative to non-robust, policy-making in adverse outcomes of model misspecification.

In the case of unfounded fear of model misspecification, we present results by simulating robust and non-robust policies under the approximating model. This is obtained by setting $w_{t+1}^{h} = w_{t+1}^{f} \equiv 0$ in equations (39)-(41). The latter implies that the approximating model is the correct one; however, the agents, being precautious, have solved for robust decision rules (see e.g. Hansen and Sargent (2008, ch. 2)). This serves to capture the potential cost of robust policy-making when the fear of model misspecification proves to be unfounded.

5.2 Main results

Table 2 presents results for the home country (results for the foreign country are identical since here we solve for a symmetric equilibrium). Robust policies are numerically implemented by a relatively low value of θ, i.e. $\theta = 0.02$, while a high value of θ, i.e. $\theta = 10^{6}$, approximates non-robust policies reflecting the absence of fear of model misspecification. We report outcomes for consumption ($\overline{c}$), capital ($\overline{k}$), output ($\overline{y}$), environmental quantity ($\overline{Q}$) and utility ($\overline{u}$) in the long run, under all cases. We also report discounted lifetime welfare (U).[10]

Table 2: Outcomes for robust and non-robust policies

	Fear is founded (very bad scenario)		Fear is founded (bad scenario)		Fear is unfounded	
	$\theta = 0.02$	$\theta = 10^6$	$\theta = 0.02$	$\theta = 10^6$	$\theta = 0.02$	$\theta = 10^6$
$\overline{c}$	1.006	1.136	1.026	1.214	1.047	1.170
$\overline{k}$	0.530	0.598	0.540	0.607	0.551	0.616
$\overline{y}$	1.059	1.196	1.081	1.141	1.102	1.231
$\overline{Q}$	0.164	0.096	0.307	0.240	0.449	0.385
$\overline{u}$	-3.447	-3.618	-2.944	-3.095	-2.482	-2.613
U	-105.954	-108.393	-95.235	-97.384	-85.232	-87.102

Starting with outcomes under bad scenarios, we see that there are gains from following robust policies, both in the long run and over lifetime. As can be seen, the preference for robustness triggers a form of precautionary behaviour, in the form of better environmental protection at the cost of output, and this acts as a buffer against the bad environmental realisations (see also e.g. Vardas and Xepapadeas (2010) and Athanassoglou and Xepapadeas (2012) on precautionary environmental behaviour resulting from robust decision making).

We then evaluate outcomes and welfare when the fear is unfounded ex post. The results in Table 2 confirm that environmental quality is improved under precautious policies and that there are welfare gains from following robust policies even when the fear of model misspecification proves to be unfounded. Thus, in a second-best environment, robust policy-making is not redundant and can outperform non-robust policy, even when its *raison d'être*, namely, model misspecification, is not fulfilled. This is because the precautionary principle corrects for the under-provision of the public good. Environmental protection is inefficiently low in a Nash equilibrium and robust policy-making helps to remedy this, irrespective of whether fears of model misspecification are founded or not. To put it differently, robust behaviour works as a substitute for cooperation. This intuition is consistent with the analysis in Dennis (2010), where the cost of robustness, even when the fear

[10] All results presented here are obtained, as discussed above, by excluding cooperation between the countries. We have also solved the model under the assumption that the countries can cooperate on economic and environmental policies, so that the externalities are internalised. This cooperative solution, as expected, results in higher welfare compared to the Nash equilibrium. Moreover, it reproduces the standard results of the literature on robust control, i.e. that there are welfare gains from following robust policies under bad outcomes of nature and that robustness premia emerge when fears of model misspecification are unfounded ex post.

of model misspecification is unfounded, is eliminated because robust policy-making serves to correct for a policy failure, namely, the lack of commitment on the part of monetary authorities. Similarly, in our model, robust policy-making serves to correct for a market failure, namely, externalities that result in free-riding and under-provision of public goods.

Our first result, namely that robust policies increase the provision of public good and hence mitigate the free-riding problem, is the outcome of the incentives embedded into robust decision-making under model uncertainty (see also the simple model in section 2). This finding complements and extends the related literature that has focused on the role of uncertainty, in the form of exogenous stochastic processes within a known model for the decision maker, in correcting free-riding problems. As said in the Introduction, the public economics literature has studied extensively whether extrinsic uncertainty can lead atomistic risk-averse individuals to increase their voluntary contribution to the public good (see e.g. Cornes and Sandler (1996, ch. 6) and Keenan *et al.* (2006)). Such uncertainty, in the form of exogenous stochastic processes with known statistical properties can, for instance, refer to the provision of the public good supplied by others, to productivity processes, etc. In the context of environmental public good-type situations, the same literature has also shown that global environmental quality can be improved, if the environmental model admits bigger exogenous disturbances (see e.g. Bramoullé and Treich (2009)). Typically in these studies, the conditions under which an alleviation of the commons problem is possible under uncertainty involve assumptions about the risk-aversion and prudence embodied in the utility function, e.g. assumptions about the convexity of the marginal utility function.

Our findings contribute to this literature in several ways. First, we study a different and more general form of uncertainty, i.e. model uncertainty, which refers to the potential (in)correct specification of the model that determines the quantity of the public good and which is allowed to feed back to the state of the system, therefore it is not necessarily exogenous to the decision maker. This type of uncertainty is particularly relevant for environmental processes (see e.g. the examples of environmental uncertainty in Barrett (2008) and Bramoullé and Treich (2009)). Second, the result that there is an increase in the quantity of the public good provided does not require further assumptions on the utility function.[11] Third, we study explicitly the dynamic

[11] For instance, typically, a positive third derivative of the utility function is required to generate precautionary behaviour, in the form of increases in stock variables in dynamic models, when uncertainty takes the form of mean-preserving increases in the variance of exogenous processes (see e.g. the review and analysis in Ljungqvist and Sargent (2012)). By contrast, the third derivative of the quadratic utility function employed here is zero.

interaction between the environmental process and economic activity, which is also relevant for environmental problems.

Our work therefore contributes to the growing literature on mechanisms that can alleviate the free-riding problem in environmental public good provision. A related literature has also studied the effect of extrinsic uncertainty on international environmental agreements (see e.g. Na and Shin (1998), Boucher and Bramoullé (2010) and Finus and Pintassilgo (2012)) and suggests that uncertainty can increase public good provision by increasing the incentives for cooperation. An alternative channel that can contribute to improving environmental standards in a non-cooperative setup is one that works through terms of trade incentives (see e.g. Bogmans (2015) and Markusen (2014)). In particular, when countries trade in differentiated commodities, environmental policies can improve terms-of-trade and this creates an incentive in favour of environmental policies, which works against the standard free-riding incentive. This consideration is strengthened quantitatively when vertical linkages are strong, that is, when production is increasingly organised by means of global supply chains.

Regarding our second result, namely that there are welfare gains from precautious policy even when the fear of model misspecification is unfounded, as already shown in the simple static model of section 2, the welfare effects from robust policymaking depend on the quantitative strength of the channels embedded in the model determining the costs and benefits of precautious policies. In a second-best environment with market failures like externalities, there is a trade-off regarding the effects of model uncertainty and robust policy-making on welfare. On one hand, robust policy-making increases public good provision for the reasons discussed above. The beneficial impact of this channel on welfare depends on the distortions implied by the underlying strategic interactions and externalities. In particular, the more under-provided the public good is in a decentralised equilibrium, the higher the benefit from robust policies. On the other hand, robust policies may imply costs, in the form of misallocation of resources, when the actual outcomes from model misspecification are not very different from the approximating model that can be used for non-robust policy. This trade-off results from introducing the market failure leading to under-provision of public goods into an equilibrium with robust policies. In the absence of this extra channel, robust policy is unambiguously associated with robustness premia when the fear of model misspecification proves to be unfounded.

Therefore, in general, the net welfare effect of robust policy-making depends on a quantitative evaluation of the above trade-off and, as such, it depends on the parameterisation of the model and, in particular, on the strength of externalities, the valuation of the public good and the calibration of the assumed process for pollution and environmental quality. In terms of our model, the parameterisation used in Table 1 implies that there are welfare gains from robust policies, even when the fear of model misspecification proves to be unfounded. This welfare result is robust to a large range of parameter values encompassing those reported in Table 1. An extensive sensitivity analysis of the welfare gains, or losses, arising from robust policy when the fear of model misspecification is unfounded is summarised in Appendix B and offers two main results. First, as expected, changes in parameters that imply, directly or indirectly, a reduction in the under-provision of the environmental public good in the Nash equilibrium, and thus a reduction in the corrective role of precautious environmental policy, lead to a reduction in the welfare gains from robust policy when the fear of model misspecification proves to be unfounded. Second, the finding that there are welfare gains from precautious environmental policy, that is, in terms of our solution, by choosing policies for $\theta = 0.02$ compared with $\theta = 10^6$, when the fear of model misspecification is unfounded, is robust to sizeable changes in the baseline parameter values reported in Table 1.

To further evaluate the effect of the precautionary principle on economic and environmental outcomes, Figure 1 plots the transition path of the system from time 0, when robust policies start to be implemented, until the system converges to the new steady state under the assumption that the fear of model misspecification is unfounded.

[Figure 1 here]

As shown in Figure 1, the economy starts from the non-robust equilibrium under model certainty, as given by the last column in Table 2, and converges towards the steady state captured by the immediately preceding column in Table 2. The incentive for robust policies is manifested by increases over time in environmental quality, which reflect the desire to create a buffer stock of environmental capital, as discussed above. This is achieved at the expense of lower production.

6 Economic and environmental uncertainty

In our analysis so far we have allowed for environmental model uncertainty only. However, country-decision makers may also be uncertain about the "economic" model. For example, they may worry about potential misspecification of the process that determines the evolution of economic output, as captured in this model by the process for physical capital. This may reflect, for instance, fears about technology shocks, social tensions or external threats that can impact negatively on infrastructure and output. We generalise the model presented in Section 3 by also allowing for "economic" model uncertainty, in addition to "environmental" model uncertainty, and by examining outcomes for different types (and sizes) of model uncertainty. Moreover, we will also study the case of asymmetric equilibria, in the sense that different countries, linked by environmental externalities, may have different preoccupations with different types of model uncertainty; for instance, one country is particularly preoccupied with un-modelled shocks to the economic structure relative to the environmental one, and vice versa for the other country.

6.1 Extended set up

We present the problem for the home country (the foreign country's problem is symmetric). The "environmental" model remains as in Section 3. The "economic model" in country h is given by

$$k_{t+1}^h - (1-\delta)k_t^h + c_t^h = Ak_t^h + \sigma^k(\varepsilon_{t+1}^{h,k} + z_{t+1}^h), \tag{42}$$

where $\varepsilon_{t+1}^{h,k}$ is an *i.i.d.* Gaussian variable distributed with zero mean and unit variance as above, z_{t+1}^h is an unknown process that may affect the economic model and σ^k scales the size of $\varepsilon_{t+1}^{h,k}$ and z_{t+1}^h. The approximating model in this case is given by

$$k_{t+1}^h - (1-\delta)k_t^h + c_t^h = Ak_t^h + \sigma^k(\widehat{\varepsilon}_{t+1}^{h,k}), \tag{43}$$

where $\widehat{\varepsilon}_{t+1}^{h,k}$ is an *i.i.d.* Gaussian variable distributed with zero mean and unit variance.

Let $f_0^{k,h}$ denote the one-step transition density associated with the approximating economic model and $f^{k,h}$ the one-step transition density associated with the true, or distorted, economic model, so that

$$I(f_0^{k,h}, f^{k,h})(k^h) = \int \log\left(\frac{f^{k,h}(k_{t+1}^h \mid k_t^h)}{f_0^{k,h}(k_{t+1}^h \mid k_t^h)}\right) f^{k,h}(k_{t+1}^h \mid k_t^h) dk_{t+1}^h$$

and again

$$I(f_0^{k,h}, f^{k,h})(k^h) = 0.5\left(z_{t+1}^h\right)^2.$$

To summarise, given the extent of fear of model misspecification with respect to the economic model, η^k, the home country solves the following problem

$$\max_{\{c_t^h\}_{t=0}^\infty} \min_{\{w_{t+1}^{h,h}, w_{t+1}^{h,f}, z_{t+1}^h\}_{t=0}^\infty} E_0 \sum_{t=0}^{\infty} -\beta^t \left(\mu(c_t^h - c^*)^2 + (1-\mu)\left(Q_t^{h,h} + \xi Q_t^{h,f} - (1+\xi)\,Q^*\right)^2\right) \tag{44}$$

where the motion of the related state variables is given by

$$k_{t+1}^{h,h} - (1-\delta)k_t^{h,h} + c_t^h = Ak_t^{h,h} + \sigma^k(\varepsilon_{t+1}^{h,k} + z_{t+1}^{h,h})$$

$$k_{t+1}^{h,f} - (1-\delta)k_t^{h,f} + c_t^f = Ak_t^{h,f} + \sigma^k(\varepsilon_{t+1}^{f,k} + z_{t+1}^{h,f})$$

$$Q_{t+1}^{h,h} = (1-\rho^Q)Q^h + \rho^Q Q_t^{h,h} - \varphi k_t^{h,h} + \sigma^Q(\varepsilon_{t+1}^{h,Q} + w_{t+1}^{h,h})$$

$$Q_{t+1}^{h,f} = (1-\rho^Q)Q^f + \rho^Q Q_t^{h,f} - \varphi k_t^{h,f} + \sigma^Q(\varepsilon_{t+1}^{f,Q} + w_{t+1}^{h,f}),$$

$w_{t+1}^{h,h}$ and $w_{t+1}^{h,f}$ satisfy the constraint

$$E_0 \sum_{t=0}^{\infty} \beta^{t+1} \left[\left(w_{t+1}^{h,h} \right)^2 + \left(w_{t+1}^{h,f} \right)^2 \right] \leq \eta, \tag{45}$$

and $z_{t+1}^{h,h}$ satisfies the constraint

$$E_0 \sum_{t=0}^{\infty} \beta^{t+1} \left(z_{t+1}^{h} \right)^2 \leq \eta^k. \tag{46}$$

Using the appropriate definitions for deviations from targets as previously, we can write the model in linear-quadratic form and then proceed to solve this more general model in the same way as the model solved above with environmental uncertainty only. When analysing results below in this set up, we focus on the situation where the fear of model misspecification proves to be unfounded.

6.2 Symmetric Nash equilibria with both types of uncertainty

To illustrate the effect of economic model uncertainty, and its importance relative to environmental model uncertainty, Table 3 presents outcomes of symmetric Nash equilibria, with the same parameterisation as in Table 1, under different assumptions regarding which type of model uncertainty and which fear of model misspecification prevails in decision making (even though both types of fear of model misspecification prove to be unfounded). We denote by θ the multiplier associated with (45), i.e. measuring the extent of fear of misspecification of the environmental model, and by θ^k the multiplier associated with (46), i.e. measuring the extent of fear of misspecification of the economic model. The first column in Table 3 presents the benchmark case where policy is non-robust and the fear of model misspecification proves to be unfounded, which is obtained by setting both θ and θ^k to 10^6 and the processes capturing economic and environmental model uncertainty to be zero ex post, i.e. when simulating the model. As expected, these results are identical to those in the last column in Table 2.[12]

Table 3: Outcomes when fear is unfounded in a symmetric equilibrium

	$\theta, \theta^k = 10^6$	$\theta = 10^6, \theta^k = 0.02$	$\theta = 0.02, \theta^k = 0.02$	$\theta = 0.5, \theta^k = 0.02$
$\overline{c}$	1.170	1.173	1.052	1.170
$\overline{k}$	0.616	0.618	0.554	0.616
$\overline{y}$	1.231	1.235	1.107	1.231
$\overline{Q}$	0.385	0.383	0.446	0.384
$\overline{u}$	-2.613	-2.618	-2.487	-2.614
U	-87.102	-87.170	-85.290	-87.111

The second column of results in Table 3 considers the implications of robust policies when there is fear of misspecification of the economic model only, and this fear proves to be unfounded (this is captured by setting $\theta = 10^6$ and $\theta^k = 0.02$). The results confirm the intuition that, in this case, outcomes are worse relative to those in the first column; namely, when precautious policy does not correct a market failure, and its *raison d'être* proves to be unfounded, it represents an unnecessary diversion of resources. Precautious policy, in this case, implies an increase in economic activity (higher capital stock and thus higher production and consumption in the long run) which is, however, at the cost of lower environmental quality. Hence, the outcome is in stark contrast to the results under robust policies incorporating a fear of misspecifying the environmental model only, shown in Table 2.

Obviously, when both types of model uncertainty influence decision making, the final result depends on the quantitative strength of each type. Two cases that show that either outcome is indeed possible are shown in the last two columns of Table 3. In the first case (see second column from the end), when the fear of model misspecification is the same for the two types of model uncertainty, the correction to the underlying market failure prevails, the provision of the environmental public good is increased and welfare is improved. In other words, under the parameterisation used here, the positive

[12] As previously in Table 2, the lifetime welfare numbers in column 1 are obtained by initialising the simulations from the long-run of this scenario, i.e. the values of the state variables in this column. All remaining experiments in Tables 3 and 4 will be initialised from this set of initial conditions, so that results are comparable. Moreover, in all experiments in Tables 3 and 4, all processes relating to model uncertainty are set to zero when simulating the model.

effects associated with environmental model uncertainty are stronger than the negative effects arising from economic model uncertainty. We need to consider sizeable differences between θ and θ^k to over-turn this result and obtain "robustness premia". The last column in Table 3 provides such an example when we set $\theta = 0.5$ and $\theta^k = 0.02$.

6.3 Asymmetric Nash equilibria with both types of uncertainty

An interesting result emerges when we examine asymmetric equilibria, where asymmetry refers to differences in the degree of fear of misspecifying the economic model, relative to the environmental model, in the two countries. Imagine a situation where the home country is characterised by a relatively stable economic environment, but there exists uncertainty about the process of the environmental public good. On the contrary, imagine that the foreign country is unstable in economic (and perhaps sociopolitical) terms, so that, in that country, uncertainty about the economic model prevails relative to uncertainty about the environmental model. To capture this situation, we consider two experiments and summarise the findings in Table 4.

Table 4: Outcomes when fear is unfounded in an asymmetric equilibrium

	Case 1		Case 2	
	Home	Foreign	Home	Foreign
	$\theta = 0.02$	$\theta = 10^6$	$\theta = 0.02$	$\theta = 0.02$
	$\theta^k = 10^6$	$\theta^k = 0.02$	$\theta^k = 10^6$	$\theta^k = 0.02$
$\overline{c}$	1.030	1.184	1.047	1.052
$\overline{k}$	0.542	0.623	0.551	0.554
$\overline{y}$	1.084	1.247	1.102	1.107
$\overline{Q}$	0.458	0.377	0.449	0.446
$\overline{u}$	-2.575	-2.518	-2.486	-2.483
U	-87.260	-85.098	-85.306	-85.216

First, we set $\theta = 0.02$ and $\theta^k = 10^6$ in the home country and $\theta = 10^6$ and $\theta^k = 0.02$ in the foreign country, meaning that the home country is concerned more about environmental type uncertainty than economic one, and vice versa for the foreign country. The Nash equilibrium from this scenario is reported as case 1 in Table 4. Second, we present results when both countries have the same fear of misspecifying the environmental model, i.e. $\theta = 0.02$ in both countries, but we also set $\theta^k = 10^6$ in the home country and $\theta^k = 0.02$ in the foreign country, meaning that the home country enjoys a higher degree of economic certainty than the foreign country but they both share the same degree of fear of misspecifying the environmental model. This is reported as case 2 in Table 4. In both cases, we focus on the situation in which the fears of model misspecification are unfounded.

As can be seen, this type of asymmetry leads to a more intense free riding on the part of the country which is, in relative terms, preoccupied with economic model uncertainty, the foreign country in these examples. In particular, in the first example in Case 1, the foreign country finds it optimal to increase its economic activity and reduce its environmental quality even more as compared to the symmetric case (compare Case 1 in Table 4 to the first two columns in Table 3) by optimally taking advantage of the environmental precaution in the home country. On the contrary, although the home country increases the quantity of its environmental public good, it suffers welfare losses when the fear of model misspecification is unfounded. Therefore, in such a non-symmetric world, one-sided precautious environmental policy will not improve welfare ex ante for the country that implements it, so that coordination with respect to the degree of precaution is required.

In the second example, Case 2 in Table 4, although both countries follow precautious environmental policies, the additional precaution expressed in terms of increased production and capital accumulation in the foreign country, stemming from its concern about economic model uncertainty, implies that this country ends up with a higher capital stock and lower environmental quality than the home country. Therefore, although both countries benefit from precautious environmental policies which corrects the under-provision of the environmental public good (compare Case 2 in Table 4 to the first column in Table 3), it is the foreign country that gains more.

7 Conclusion

In this paper, we have studied the benefits of robust policy in a two-country setup with environmental externalities and fear of model misspecification. Solving a dynamic noncooperative Nash game, we showed that robust policies can out-perform non-robust policies, even when the fear of model misspecification proves to be unfounded; this happens when the predominant fear of model misspecification is about the environmental process. The key mechanism behind this result is that the precautionary principle, associated with robustness, corrects for the inefficiencies caused by the standard incentive for free riding. We found that, for a wide range of parameters capturing environmental preferences and externalities in our model, the benefits of precautionary policy, the form of increased public good provision, outweigh the costs even when the fear of model misspecification is unfounded.

These findings demonstrate the potential role of robust policies in improving economic outcomes even when the common motivation for implementing them (i.e. protection against bad scenarios associated with model uncertainty) is not justified ex post. In turn, this also suggests that it would be valuable to conduct further research on the conditions under which robust policies constitute the equilibrium outcome. This is particularly important given the result in this paper that there are gains for decision makers that do not follow precautious policies when others do so, in an environment where externalities create spill-over effects.

Finally, it would be useful to consider additional or alternative mechanisms to improve environmental quality in the context of model uncertainty and robust policy, by focusing on different factors contributing to under-provision of the environmental public good. For instance, to the extent that environmental deterioration is also due to the lobbying activities of specific groups within a country, it would be useful to evaluate under what conditions model uncertainty, and the implied precautionary behaviour, can intensify or reduce this lobbying and resulting pollution. When model uncertainty relates to the cost of environmental protection for the interest group concerned, robust lobbyists would be expected to increase lobbying to avoid pro-environmental policies, whereas, when model uncertainty relates to the cost of lobbying, robust lobbyists should reduce lobbying activity.

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8 Appendix A

To obtain the solution in (31), we first use (30) for the home country and the transition law in (27) to eliminate next period's state

$$x'P^h x = \underset{\widetilde{u}^h}{ext}\{\left(\widetilde{u}^h\right)' R^h \widetilde{u}^h + x'Q^h x +$$
$$\beta\left(Ax + \widetilde{B}^h \widetilde{u}^h + \widetilde{B}^f \widetilde{u}^f\right)' P^h \left(Ax + \widetilde{B}^h \widetilde{u}^h + \widetilde{B}^f \widetilde{u}^f\right)\}.$$

The above equation implies

$$\begin{aligned} x'P^h x = \underset{\widetilde{u}^h}{ext}\{&\left(\widetilde{u}^h\right)' R^h \widetilde{u}^h + x'Q^h x + \beta x'A'P^h A x + \beta x'A'P^h \widetilde{B}^h \widetilde{u}^h + \\ &\beta x'A'P^h \widetilde{B}^f \widetilde{u}^f + \beta\left(\widetilde{u}^h\right)'\left(\widetilde{B}^h\right)' P^h A x + \beta\left(\widetilde{u}^h\right)'\left(\widetilde{B}^h\right)' P^h \widetilde{B}^h \widetilde{u}^h + \\ &\beta\left(\widetilde{u}^h\right)'\left(\widetilde{B}^h\right)' P^h \widetilde{B}^f \widetilde{u}^f + \beta\left(\widetilde{u}^f\right)'\left(\widetilde{B}^f\right)' P^h A x + \\ &\beta\left(\widetilde{u}^f\right)'\left(\widetilde{B}^f\right)' P^h \widetilde{B}^h \widetilde{u}^h + \beta\left(\widetilde{u}^f\right)'\left(\widetilde{B}^f\right)' P^h \widetilde{B}^f \widetilde{u}^f\}. \end{aligned} \tag{47}$$

The Bellman equation for the foreign country is symmetric to the above, where it suffices to substitute the superscript h with f and f with h.

8.1 First-order conditions

The first order condition with respect to $\widetilde{u}^h$ that is necessary for the maximisation problem on the right-hand side of (47) implies

$$\begin{aligned} \widetilde{u}^h = &-\beta\left(R^h + \beta\left(\widetilde{B}^h\right)' P^h \widetilde{B}^h\right)^{-1}\left(\widetilde{B}^h\right)' P^h A x - \\ &\beta\left(R^h + \beta\left(\widetilde{B}^h\right)' P^h \widetilde{B}^h\right)^{-1}\left(\widetilde{B}^h\right)' P^h \widetilde{B}^f \widetilde{u}^f. \end{aligned} \tag{48}$$

In the solution, $\widetilde{u}^f = Kx$, where K is an undetermined matrix. Thus, (48) can be written as

$$\begin{aligned} \widetilde{u}^h = &-\beta\left(R^h + \beta\left(\widetilde{B}^h\right)' P^h \widetilde{B}^h\right)^{-1}\left(\widetilde{B}^h\right)' P^h A x - \\ &\beta\left(R^h + \beta\left(\widetilde{B}^h\right)' P^h \widetilde{B}^h\right)^{-1}\left(\widetilde{B}^h\right)' P^h \widetilde{B}^f K x, \text{ or} \end{aligned}$$

$$\widetilde{u}^h = Fx, \text{ where} \tag{49}$$

$$F = -\beta\left(R^h + \beta\left(\widetilde{B}^h\right)' P^h \widetilde{B}^h\right)^{-1}\left(\widetilde{B}^h\right)' P^h A -$$
$$\beta\left(R^h + \beta\left(\widetilde{B}^h\right)' P^h \widetilde{B}^h\right)^{-1}\left(\widetilde{B}^h\right)' P^h \widetilde{B}^f K. \tag{50}$$

8.2 Nash equilibrium

Note that the solution for the problem of the foreign country is exactly symmetric. Hence, we obtain

$$\widetilde{u}^f = Kx, \text{ where} \tag{51}$$

$$K = -\beta\left(R^f + \beta\left(\widetilde{B}^f\right)' P^f \widetilde{B}^f\right)^{-1}\left(\widetilde{B}^f\right)' P^f A -$$
$$\beta\left(R^f + \beta\left(\widetilde{B}^f\right)' P^f \widetilde{B}^f\right)^{-1}\left(\widetilde{B}^f\right)' P^f \widetilde{B}^h F. \tag{52}$$

The Nash equilibrium (for given matrices P^h and P^f) is obtained by solving the system in (50) and (52). This gives

$$F = -\begin{pmatrix} I - \beta^2\left(R^h + \beta\left(\widetilde{B}^h\right)' P^h \widetilde{B}^h\right)^{-1}\left(\widetilde{B}^h\right)' \times \\ P^h \widetilde{B}^f \left(R^f + \beta\left(\widetilde{B}^f\right)' P^f \widetilde{B}^f\right)^{-1}\left(\widetilde{B}^f\right)' P^f \widetilde{B}^h \end{pmatrix}^{-1} \times$$
$$\beta\left(R^h + \beta\left(\widetilde{B}^h\right)' P^h \widetilde{B}^h\right)^{-1}\left(\widetilde{B}^h\right)' P^h A +$$
$$\begin{pmatrix} I - \beta^2\left(R^h + \beta\left(\widetilde{B}^h\right)' P^h \widetilde{B}^h\right)^{-1}\left(\widetilde{B}^h\right)' P^h \widetilde{B}^f \times \\ \left(R^f + \beta\left(\widetilde{B}^f\right)' P^f \widetilde{B}^f\right)^{-1}\left(\widetilde{B}^f\right)' P^f \widetilde{B}^h \end{pmatrix}^{-1} \times \tag{53}$$
$$\beta^2\left(R^h + \beta\left(\widetilde{B}^h\right)' P^h \widetilde{B}^h\right)^{-1}\left(\widetilde{B}^h\right)' P^h \widetilde{B}^f \times$$
$$\left(R^f + \beta\left(\widetilde{B}^f\right)' P^f \widetilde{B}^f\right)^{-1}\left(\widetilde{B}^f\right)' P^f A$$

and

$$K = -\begin{pmatrix} I - \beta^2 \left(R^f + \beta \left(\widetilde{B}^f \right)' P^f \widetilde{B}^f \right)^{-1} \left(\widetilde{B}^f \right)' P^f \widetilde{B}^h \times \\ \left(R^h + \beta \left(\widetilde{B}^h \right)' P^h \widetilde{B}^h \right)^{-1} \left(\widetilde{B}^h \right)' P^h \widetilde{B}^f \end{pmatrix}^{-1} \times$$

$$\beta \left(R^f + \beta \left(\widetilde{B}^f \right)' P^f \widetilde{B}^f \right)^{-1} \left(\widetilde{B}^f \right)' P^f A +$$

$$\begin{pmatrix} I - \beta^2 \left(R^f + \beta \left(\widetilde{B}^f \right)' P^f \widetilde{B}^f \right)^{-1} \left(\widetilde{B}^f \right)' P^f \widetilde{B}^h \times \\ \left(R^h + \beta \left(\widetilde{B}^h \right)' P^h \widetilde{B}^h \right)^{-1} \left(\widetilde{B}^h \right)' P^h \widetilde{B}^f \end{pmatrix}^{-1} \times \tag{54}$$

$$\beta^2 \left(R^f + \beta \left(\widetilde{B}^f \right)' P^f \widetilde{B}^f \right)^{-1} \left(\widetilde{B}^f \right)' P^f \widetilde{B}^h \times$$

$$\left(R^h + \beta \left(\widetilde{B}^h \right)' P^h \widetilde{B}^h \right)^{-1} \left(\widetilde{B}^h \right)' P^h A$$

8.3 Verifying the guesses

If the guesses are correct, then the solution must satisfy the Bellman equations for the two countries. We thus first substitute (49) and (51) in (47) and solve for the matrices P^h and P^f that satisfy the resulting equation. This gives

$$\begin{aligned} P^h = {} & Q^h + F'R^hF + \beta A'P^hA + \beta A'P^h\widetilde{B}^hF + \beta A'P^h\widetilde{B}^fK + \\ & \beta F' \left(\widetilde{B}^h \right)' P^hA + \beta F' \left(\widetilde{B}^h \right)' P^h\widetilde{B}^hF + \beta F' \left(\widetilde{B}^h \right)' P^h\widetilde{B}^fK \\ & + \beta K' \left(\widetilde{B}^f \right)' P^hA + \beta K' \left(\widetilde{B}^f \right)' P^h\widetilde{B}^hF + \beta K' \left(\widetilde{B}^f \right)' P^h\widetilde{B}^fK. \end{aligned} \tag{55}$$

Working similarly, we obtain for the foreign country

$$\begin{aligned} P^f = {} & Q^f + K'R^fK + \beta A'P^fA + \beta A'P^f\widetilde{B}^fK + \beta A'P^f\widetilde{B}^hF + \\ & \beta K' \left(\widetilde{B}^f \right)' P^fA + \beta K' \left(\widetilde{B}^f \right)' P^f\widetilde{B}^fK + \beta K' \left(\widetilde{B}^f \right)' P^f\widetilde{B}^hF \\ & + \beta F' \left(\widetilde{B}^h \right)' P^fA + \beta F' \left(\widetilde{B}^h \right)' P^f\widetilde{B}^fK + \beta F' \left(\widetilde{B}^h \right)' P^f\widetilde{B}^hF. \end{aligned} \tag{56}$$

9 Appendix B: Sensitivity analysis

Here, we examine the sensitivity of the results in Table 2 to changes in some key parameters. We will focus on the strength of environmental externalities, the extent of environmental damage caused by economic activity, how abundant environmental resources are in nature, and how much the decision maker values the environment relative to consumption in the utility function. In each case reported below, all changes in parameters are implemented by maintaining the remaining parameters at their baseline values in Table 1. Moreover, in each case, and for the new parameter value, we first identify the long-run solution of the model under non-robust policies ($\theta = 10^6$) when the fear of model misspecification proves to be unfounded. This solution then provides the initial values for the new dynamic simulation by obtaining policies for $\theta = 0.02$, compared with $\theta = 10^6$. This ensures that the effects can be comparable to those obtained from the experiments presented in Table 2.

Starting with ξ, which is the parameter measuring the strength of environmental externalities, we find that, as expected, reductions in the extent of externalities reduce the gains arising from the implementation of precautious policies when the fear of model misspecification is unfounded. This is because, under lower externalities, the Nash equilibrium is less distorted, so that there is relatively little benefit from precautious policy and the associated increase in the environmental public good. However, welfare gains from robust policy do remain even with small externalities (e.g. when $\xi = 0.1$). Similar effects are obtained by reductions in the adverse environmental impact of economic activity, i.e. by reductions in ϕ, since this weakens the link between countries and thus makes the problem of public good under-provision less acute. However, as above, positive welfare effects of precautious policy, when the fear of model misspecification is unfounded, do remain even when the environmental damage caused by the capital stock is very small (e.g. when ϕ is reduced to 0.005).

Regarding μ, which measures the valuation of consumption versus environmental quality, we find that increases in μ decrease the welfare gains of precautious policies when the fear of model misspecification is unfounded. This happens because when the decision maker values consumption more relative to environment, the distortions under the Nash equilibrium carry a smaller weight for the decision maker's welfare, so that the corrective role of precaution becomes less important. However, the welfare gains do remain even when we increase μ as high as 0.9. We then consider the effect of increases in $Q^h = Q^f$, which measures the expected long-run value of environmental quality, in the absence of economic activity and of model misspecification. Lower values of this lead to higher welfare gains from precautious policy when the fear of model misspecification is unfounded, because they increase the distance of actual environmental quality from its target value, thus acting to amplify the welfare effect of public good under-provision and thus amplify the corrective role of robust policy. On the contrary, in a more (environmentally) resource abundant world economy, the effects are reversed. However, the welfare gains from precautious policy when the fear of model misspecification is unfounded do remain in this setup even for relatively high values of $Q^h = Q^f$ approaching Q^*, e.g. when $Q^h = Q^f = 1.8$ (recall that $Q^* = 2$).

Finally, we examine the importance of the targets of consumption and environmental quality in the utility function. Given the previous analysis, one would expect that increases in Q^* relative to c^* would increase the welfare gains from precautious policy when the fear of model misspecification is unfounded. This is because this would increase the distance of actual environmental quality from its target value, relative to the distance of private goods consumption, thus acting to amplify the welfare effect of under-provision of the public good and thus amplify the corrective role of robust policy. This is indeed confirmed from the numerical solutions. However, the welfare gains from precautious policy when the fear of model misspecification is unfounded do remain even when the ratio of Q^*/c^* is reduced to 0.8 from 1.33 which was its value in the benchmark calibration in Table 1.

We focused on the above parameters for two reasons. First, these parameters are related to the environmental process so that their magnitudes are less known, at least in comparison to parameters relating to economic quantities, for which there is relatively more empirical evidence or, at least, consensus. For instance, depreciation rates and discount factors are commonly used in calibration of economic models. Second, the parameters related to the environmental process have a direct effect on the main incentives that drive our results, namely (under)provision of environmental quality.

Figure 1: The effects of robust policy