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A STUDY on BUCKLING RESPONSE of FML MEMBERS of ‘CLASSIC’ VERSUS THIN-PLY DESIGN
Subject of consideration

- 3 approaches of FML profiles buckling analysis.
- mechanical properties of components.
- eigen-buckling and non-linear post-buckling.
- experimental buckling.
- thin-ply design.
- conclusions.

This study is supported by the Ministry of Science and Higher Education in Poland – National Science Centre Grant No UMO-2012/07/B/ST8/04093.
**Thin-walled open cross-section stringers**

Subject of consideration

<table>
<thead>
<tr>
<th>Sequence</th>
<th>Lay-up</th>
</tr>
</thead>
<tbody>
<tr>
<td>1&lt;sup&gt;a&lt;/sup&gt;</td>
<td>Al/0/90/Al/90/0/Al</td>
</tr>
<tr>
<td>2</td>
<td>Al/90/0/Al/0/90/Al</td>
</tr>
<tr>
<td>3</td>
<td>Al/45/0/Al/0/45/Al</td>
</tr>
<tr>
<td>4</td>
<td>Al/0/45/Al/45/0/Al</td>
</tr>
<tr>
<td>5&lt;sup&gt;b&lt;/sup&gt;</td>
<td>Al/0/0/Al/0/0/Al</td>
</tr>
<tr>
<td>6</td>
<td>Al/25/0/Al/0/25/Al</td>
</tr>
<tr>
<td>7</td>
<td>Al/0/25/Al/25/0/Al</td>
</tr>
<tr>
<td>8</td>
<td>Al/Al/Al/Al/Al/Al/Al/Al</td>
</tr>
<tr>
<td>9</td>
<td>Al/Iso/Iso/Al/Iso/Iso/Al/Al</td>
</tr>
<tr>
<td>10&lt;sup&gt;c&lt;/sup&gt;</td>
<td>Al/45/-45/Al/-45/45/Al</td>
</tr>
</tbody>
</table>

- **a** – GLARE 3;
- **b** – GLARE 2A;
- **c** – GLARE 6A.

\[ t_{Al} = 0.3 \text{ mm.} \quad t_{C} = 0.25 \text{ mm} \]
Material properties of FML components

<table>
<thead>
<tr>
<th>FML component property</th>
<th>Aluminium</th>
<th>TVR380</th>
<th>120EP-513/CF</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>[ N/mm² ] E</td>
<td>72×10³</td>
<td>77×10³</td>
<td>46.4×10³</td>
</tr>
<tr>
<td>[ ] ν</td>
<td>0.33</td>
<td>0.33</td>
<td>14.9×10³</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>[ ] G₁₂</td>
<td>5.2×10³</td>
<td>4.661×10³</td>
<td></td>
</tr>
<tr>
<td>[ ] ν₁₂</td>
<td>0.269</td>
<td>0.274</td>
<td></td>
</tr>
<tr>
<td>[ N/mm² ] Rₑ *)</td>
<td>360</td>
<td>309</td>
<td>1534</td>
</tr>
<tr>
<td>Rₘ</td>
<td>448</td>
<td>408</td>
<td>74.5</td>
</tr>
<tr>
<td>Sₗ</td>
<td>1046</td>
<td>88.26</td>
<td></td>
</tr>
<tr>
<td>Cₗ</td>
<td>115</td>
<td>869</td>
<td></td>
</tr>
</tbody>
</table>

*) very small orthotropy of yield limit
Test stand
Buckling modes

SHELL

EKSPERYMENT
## Buckling force as a function of GFR lay-up

<table>
<thead>
<tr>
<th>Lay-up</th>
<th>exp [kN]</th>
<th>FEM [kN]</th>
<th>ANM Koiter [kN]</th>
</tr>
</thead>
<tbody>
<tr>
<td>AL/0/90/AL/90/0/AL</td>
<td>31.434</td>
<td>30.189</td>
<td>28.568</td>
</tr>
<tr>
<td>AL/90/0/AL/0/90/AL</td>
<td>nd</td>
<td>29.871</td>
<td>28.408</td>
</tr>
<tr>
<td>AL/45/0/AL/0/45/AL</td>
<td>32.634</td>
<td>31.399</td>
<td>29.876</td>
</tr>
<tr>
<td>AL/0/45/AL/45/0/AL</td>
<td>nd</td>
<td>30.588</td>
<td>29.015</td>
</tr>
<tr>
<td>AL/0/0/AL/0/0/AL</td>
<td>29.836</td>
<td>30.310</td>
<td>28.630</td>
</tr>
<tr>
<td>AL/25/0/AL/0/25/AL</td>
<td>nd</td>
<td>30.745</td>
<td>29.334</td>
</tr>
<tr>
<td>AL/0/25/AL/25/0/AL</td>
<td>29.856</td>
<td>30.977</td>
<td>28.859</td>
</tr>
<tr>
<td>Al/Al/Al/Al/Al/Al/Al</td>
<td>nd</td>
<td>40.472</td>
<td>38.510</td>
</tr>
<tr>
<td>Al/Iso/Iso/Al/Iso/Iso/Al</td>
<td>nd</td>
<td>30.805</td>
<td>29.311</td>
</tr>
<tr>
<td>Al/45/-45/Al/-45/45/Al</td>
<td>nd</td>
<td>31.752</td>
<td>30.208</td>
</tr>
</tbody>
</table>

The table shows the buckling force as a function of different lay-ups, with experimental (exp), finite element method (FEM), and Koiter's ANM results. The last column indicates the standard deviation.
GLARE 3 [Al/0/90/Al/90/0/Al]_T
A_S B_0 D_S

GLARE 6A [Al/45/-45/Al/-45/45/Al]_T
A_S B_0 D_F

Standard FML designs with Aluminum and E-Glass/Epoxy
Governing ABD matrix of CLPT

\[
\begin{align*}
\begin{bmatrix} N_x \\ N_y \\ N_{xy} \end{bmatrix} &= \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{22} & A_{26} & \text{sym} \\ \text{sym} & A_{66} & \text{sym} \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix} + \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{22} & B_{26} & \text{sym} \\ \text{sym} & B_{66} & \text{sym} \end{bmatrix} \begin{bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{bmatrix} \\
\begin{bmatrix} M_x \\ M_y \\ M_{xy} \end{bmatrix} &= \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{22} & B_{26} & \text{sym} \\ \text{sym} & B_{66} & \text{sym} \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix} + \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{22} & D_{26} & \text{sym} \\ \text{sym} & D_{66} & \text{sym} \end{bmatrix} \begin{bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{bmatrix}
\end{align*}
\]

For **Extensionally Isotropic Laminates:**
\[A_{11} = A_{22} \quad \text{and} \quad A_{66} = (A_{11} - A_{12})/2\]

For **Fully Isotropic Laminates:**
\[D_{ij} = A_{ij} \frac{H^2}{12}\]

For **FML:** Properties may be Extensionally Isotropic, but:
\[D_{ij} \propto A_{ij} \frac{H^2}{12}\]
Modulus invariants

\[ U_1 = \frac{(3Q_{11} + 3Q_{22} + 2Q_{12} + 4Q_{66})}{8} \]
\[ U_2 = \frac{(Q_{11} - Q_{22})}{2} \]
\[ U_3 = \frac{(Q_{11} + Q_{22} - 2Q_{12} - 4Q_{66})}{8} \]
\[ U_4 = \frac{(Q_{11} + Q_{22} + 6Q_{12} - 4Q_{66})}{8} \]
\[ U_5 = \frac{(Q_{11} + Q_{22} - 2Q_{12} + 4Q_{66})}{8} \]

\( Q_{ij} \) - the reduced stiffness matrix elements

For Equivalent Fully Isotropic Laminate:
\[ E_{iso} = 2(1 + \nu_{iso})G_{iso} = U_1(1 - \nu_{iso}^2) \]
\[ \nu_{iso} = \frac{U_4}{U_1} \]
\[ G_{iso} = U_5 \]
\[ A_{iso} = A_{11} = A_{22} = \frac{E_{iso}H}{(1 - \nu_{iso}^2)} = U_1H \]
\[ A_{12} = \nu_{iso}A_{11} \]
\[ A_{66} = U_5H \]
\[ D_{iso} = \frac{E_{iso}H^3}{(1 - \nu_{iso}^2)/12} = U_1H^3/12 \]

FML 8 - \( D_{iso} \) for Aluminum = 49,391 N.mm
FML 9 - \( D_{iso} \) for FML = 44,014 N.mm, but \( D_{ij} \neq A_{ij}H^2/12 \)
## Buckling force - GFRP versus CFRP

<table>
<thead>
<tr>
<th>Lay-up</th>
<th>GFRP (kN)</th>
<th>Alu reduction</th>
<th>CFRP (kN)</th>
<th>Alu reduction</th>
</tr>
</thead>
<tbody>
<tr>
<td>AL/0/90/AL/90/0/AL</td>
<td>30.189</td>
<td>0.746</td>
<td>31.722</td>
<td>0.783</td>
</tr>
<tr>
<td>AL/90/0/AL/0/90/AL</td>
<td>29.871</td>
<td>0.738</td>
<td>31.132</td>
<td>0.769</td>
</tr>
<tr>
<td>AL/45/0/AL/0/45/AL</td>
<td>31.399</td>
<td>0.776</td>
<td>35.164</td>
<td>0.868</td>
</tr>
<tr>
<td>AL/0/45/AL/45/0/AL</td>
<td>30.588</td>
<td>0.756</td>
<td>33.015</td>
<td>0.816</td>
</tr>
<tr>
<td>AL/0/0/AL/0/0/AL</td>
<td>30.310</td>
<td>0.749</td>
<td>31.979</td>
<td>0.790</td>
</tr>
<tr>
<td>AL/25/0/AL/0/25/AL</td>
<td>30.745</td>
<td>0.760</td>
<td>34.241</td>
<td>0.846</td>
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<tr>
<td>AL/0/25/AL/25/0/AL</td>
<td>30.977</td>
<td>0.765</td>
<td>32.540</td>
<td>0.804</td>
</tr>
<tr>
<td>Al/Al/Al/Al/Al/Al/Al</td>
<td>40.472</td>
<td>1.000</td>
<td>40.472</td>
<td>1.000</td>
</tr>
<tr>
<td>Al/Iso/Iso/Al/Iso/Iso/Al</td>
<td>30.805</td>
<td>0.761</td>
<td>35.928</td>
<td>0.888</td>
</tr>
<tr>
<td>Al/45/-45/Al/-45/45/Al</td>
<td>31.752</td>
<td>0.785</td>
<td>36.279</td>
<td>0.896</td>
</tr>
</tbody>
</table>
Buckling factor curves for rectangular plate

‘GLARE 3’ [Al/0/90/Al/90/0/Al]ₜ

‘GLARE 6A’ [Al/45/-45/Al/-45/45/Al]ₜ.

Standard FML designs with Aluminum and Carbon/Epoxy
Buckling factor curves – NORTH ply FML

\[ k_x = \frac{N_x b^2}{\pi^2 D_{\text{Iso}}} \]

Plate aspect ratio \( \left( \frac{a}{b} \right) \)

GLARE 6A [Al/45\text{12}/-45\text{12}/Al/-45\text{12}/45\text{12}/Al]_T

\([\text{Al/} \pm 45\text{2}/-45\text{2}/45\text{2}/\pm 45\text{2}/\text{Al/} \pm 45\text{2}/-45\text{2}/45\text{2}/\pm 45\text{2}/\text{Al}]_T\)

NORTH ply FML designs with Aluminum and Carbon/Epoxy
Lamination parameters

\[ A_{11} = \{U_1 + \xi_1 U_2 + \xi_2 U_3\} \times H \]
\[ A_{12} = A_{21} = \{-\xi_2 U_3 + U_4\} \times H \]
\[ A_{22} = \{U_1 - \xi_1 U_2 + \xi_2 U_3\} \times H \]
\[ A_{66} = \{-\xi_2 U_3 + U_5\} \times H \]

\[ D_{11} = A_{11} \times H^2/12 \]
\[ D_{12} = D_{21} = A_{12} \times H^2/12 \]
\[ D_{16} = D_{61} = \{\xi_{11} U_2/2\} \times H^3/12 \]
\[ D_{22} = A_{22} \times H^2/12 \]
\[ D_{26} = D_{62} = \{\xi_{11} U_2/2 - \xi_{12} U_3\} \times H^3/12 \]
\[ D_{66} = A_{66} \times H^2/12 \]

\[ \xi^A_{\{1,2,3,4\}} = \frac{1}{h} \int_{-h/2}^{h/2} \{\cos(2\theta(z)), \cos(4\theta(z)), \sin(2\theta(z)), \sin(4\theta(z))\} dz \]

\[ \xi^B_{\{1,2,3,4\}} = \frac{1}{h} \int_{-h/2}^{h/2} \{\cos(2\theta(z)), \cos(4\theta(z)), \sin(2\theta(z)), \sin(4\theta(z))\} z dz \]

\[ \xi^D_{\{1,2,3,4\}} = \frac{1}{h} \int_{-h/2}^{h/2} \{\cos(2\theta(z)), \cos(4\theta(z)), \sin(2\theta(z)), \sin(4\theta(z))\} z^2 dz \]
Buckling factor curves for rectangular plate (quasi-homogenous)

quasi-isotropic laminates with
\((\xi_9, \xi_{10}) = (0,0)\) and \(0 \leq \xi_{11} \leq 0.5\)

angle-ply laminates with
\((\xi_9, \xi_{10}) = (0,-1)\) and \(0.0 \leq \xi_{11} \leq 1.0\)
**Thin ply sandwich FML**

Laminates possessing **Fully Isotropic** properties are very few in number:

- 36 with \( n = 18 \) plies \( (\pi/3 \text{ isotropy}) \)
- 1 with \( n = 24 \) plies \( (\pi/4 \text{ isotropy}) \)

For GLARE 2, 3 and 6:
\[
\begin{align*}
\text{t}_{\text{FML}} &= 1.9 \text{ mm}; \\
\text{t}_{\text{Al}} &= 0.3 \text{ mm}; \\
\text{t}_C &= 0.5 \div 0.25 \text{ mm} \quad (n = 2; \ 300\text{gsm})
\end{align*}
\]

For thin ply sandwich FML:
\[
\text{t}_C = 0.5 \div 0.01 \text{ mm} \quad (n \approx 24; \ 30\text{gsm})
\]

\[\mathbf{A}_1\mathbf{B}_0\mathbf{D}_1 \text{ with } n = 24:
\begin{align*}
[-45/90/0/45/0/45/90/45/-45/90/-45/90/45/0/-45/0/45/0/45/-45/90]_T
\end{align*}\]

For buckling comparison the following 12 ply **Quasi-Homogenous Orthotropic** sub-laminate is used \([\pm 45_2/-45_2/45_2/\pm 45_2]_T\)

(with 60gsm material): \([\text{Al}/\pm 45_2/-45_2/45_2/\pm 45_2/\text{Al}/\pm 45_2/-45_2/45_2/\pm 45_2/\text{Al}]_T\)

and GLARE 6A \[\rightarrow [\text{Al}/45_{12}/-45_{12}/\text{Al}/-45_{12}/45_{12}/\text{Al}]_T.\]

Engineering Sciences Data Unit, “Stiffnesses of laminated plates”, ESDU No. 94003, 1994
Buckling mode – web deflection

AL/0/90/AL/90/0/AL

Al/45/-45/Al/-45/45/Al
Buckling mode – long plate $f(\xi_{10}, \xi_{11})$

(a) $\xi_{11} = 0.0$, $k_{x,\infty} = 4.00$ and $\lambda = b$

(b) $\xi_{11} = 0.1$, $k_{x,\infty} = 3.98$

and $\lambda = b$

(c) $\xi_{11} = 0.2$, $k_{x,\infty} = 3.98$

and $\lambda = (298/300)b$

(d) $\xi_{11} = 0.3$, $k_{x,\infty} = 3.78$

and $\lambda = (296/300)b$

(e) $\xi_{11} = 0.4$, $k_{x,\infty} = 3.61$

and $\lambda = (292/300)b$

(f) $\xi_{11} = 0.5$, $k_{x,\infty} = 3.37$

and $\lambda = (286/300)b$
References


Acknowledgments
This study is supported by the Ministry of Science and Higher Education in Poland – National Science Centre Grant No UMO-2012/07/B/ST8/04093.
Employed three analysis methods (exp, ANM, FEM) of buckling and post-buckling response of FML profiles gave results of acceptable agreement.

The buckling response of considered thin-walled FML panels is dominated by metallic aluminium component (≈46% v.f.).

Application of CFRP leads to lower critical force reduction with respect to aluminium but in a wider value range for considered stacking sequences than for GFRP.

Acknowledgments
This study is supported by the Ministry of Science and Higher Education in Poland – National Science Centre Grant No UMO-2012/07/B/ST8/04093.
Conclusions

- Thin plies allow more flexible lay-up ‘tailoring’ and greater homogeneity of a hybrid laminate

- Thin ply sub-laminates can also include C-Ply and TeXtreme architectures and provide a range of different mechanical properties, all within the design thickness constraints of standard FML

- Volume fractions of the two phases have a significant effect on the FML properties and need further investigation in the light of these new design configurations. Shear buckling may reveal additional benefits

Acknowledgments
This study is supported by the Ministry of Science and Higher Education in Poland – National Science Centre Grant No UMO-2012/07/B/ST8/04093.
Thank you for attention