Monetary union, even higher integration, or back to national currencies?*

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Abstract

This paper quantifies the welfare differences among a monetary union, flexible exchange rates (economic disintegration) and a monetary plus fiscal transfer union (higher economic integration). The vehicle of analysis is a medium-scale New Keynesian DSGE model consisting of two heterogeneous countries. The model is solved using data from Germany and Italy. Our solutions imply that a switch to flexible exchange rates and independent monetary policies would have negligible welfare implications. A similar result applies when we add interregional fiscal tranfers as insurance. By contrast, the addition of fiscal tranfers as redistribution has non-trivial implications and these depend crucially on whether such one-sided transfers trigger moral hazard behavior or not.

Keywords: Fiscal union; monetary union; New Keynesian, DSGE.

JEL classification: E6, F3, H6.

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1 Introduction

Most eurozone periphery countries are in a debt crisis and a long lasting recession. This has profound implications for the whole euro area. Since the crisis erupted in 2008, several options have been discussed in economic and policy circles regarding the re-design of the European Union (EU) in general and the eurozone in particular. At one extreme, there is the scenario of breakdown of the eurozone and a return to national currencies. At the other extreme, there is the proposal to add a fiscal union to the existing monetary union.

Although a fiscal union can mean different things to different people, it is widely believed that an essential element of a fiscal union is interregional fiscal transfers.[1] Actually, such transfers have been one of the most debated policies of the EU since the early days of the European integration process (see e.g. the programs of Structural and Regional Funds as well as Common Agricultural Policy). Moreover, since the seminal papers by Mundell (1961), McKinnon (1963) and Kenen (1969), it is believed that a single currency should be supported by a fiscal transfer scheme to help absorb asymmetric macroeconomic shocks; in other words, fiscal tranfsers are believed to be an important condition for a successful currency union (see e.g. Werning and Farhi (2012) for a recent general equilibrium study). On the other hand, one cannot deduce that adding just one element of a fiscal union to a monetary union will always lead to an improvement; as e.g. Perotti (2001) has shown, a centralized fiscal transfer policy can lead to less efficient outcomes if other imperfections are present.

In light of the above, this paper compares and welfare ranks three distinct policy regimes. First, the case of a monetary union. Second, the case in which the monetary union is replaced by national currencies and thus independent monetary policies. Third, the case of even higher economic integration in which the monetary union is enriched by a fiscal union, where the latter takes the form of interregional fiscal transfers.

Regarding interregional fiscal transfers, by mimicing policy practice in the EU (see the programs mentioned above) and by following e.g. Persson and Tabellini (1996a and 1996b) and Fatas (1998), we distinguish two types. Transfers as insurance and transfers as redistribution. The former are transfers that insure countries against temporary country-specific shocks. Resources are redistributed from countries facing positive shocks to countries facing adverse shocks. This is like interregional risk sharing. Such transfers are also known as ex post redistribution in the sense that they are not anticipated. Transfers as redistribution, by contrast,

[1] Other elements can include fiscal rules and coordination, a crisis resolution mechanism, a joint guarantee for government debt and a relatively large federal budget jointly with federal taxes (see e.g. Fuest and Peichl, 2012). See also Bordo et al. (2013) for a fiscal union within the euroarea and history lessons from other fiscal unions in the world economy. Breuss (2011) and De Grauwe (2013) discuss other policy options in the EU.

redistribute resources systematically from relatively rich to relatively poor member-countries. Now, resources are redistributed to reduce chronic country disparities in, say, GDP per capita. They are also known as ex ante redistribution in the sense that they are anticipated.

The vehicle of analysis is a medium-scale New Keynesian world economy DSGE model consisting of two countries, home and foreign. We define the home country as Germany and the foreign country as Italy. The countries are assumed to be identical except for fiscal policy and the degree of impatience (this is explained right below). An international asset allows agents in one country to borrow from, or lend to, agents in the other country. Under a monetary union, there is a single currency and a single monetary policy, but the two countries are free to follow independent or national fiscal policies. We follow a feedback policy rule type approach to (monetary and fiscal) policy.

We solve this model numerically. Fiscal policies are set as in the data in the two countries, Germany and Italy, while their degrees of patience (or their discount factors) are set so as to match real interest rates in the two countries, where the time period is the euro years since 2001. The rest of parameters are assumed to be the same across countries and are set at conventional values, subject to a sensitivity analysis. The steady state solution of this model can mimic relatively well the averages of the key data in the two countries over the euro years. In other words, although there can be many types of heterogeneity between these two countries, a model that simply allows for differences in fiscal policy and the degree of patience can account for one of the most distinct macroeconomic imbalances during the euro years, namely, current account deficits and hence accumulation of net foreign debt in a periphery country like Italy, and current account surpluses and hence accumulation of net foreign assets in a center country like Germany.[2] It is worth emphasizing that this particular macroeconomic imbalance, and the associated conflict of national interests, are at the heart of the policy debate in Europe nowadays (see e.g. Sinn (2010) and Fuest and Peichl (2012)).

In turn, we compare this to the cases of flexible exchange rates and a fiscal (transfer) union. In the case of flexible exchange rates, each country can follow its own Taylor-type independent monetary policy. In the case of fiscal union, we add interregional transfers, either as insurance or as redistribution, to the reference regime of a monetary union.

Our main results are as follows. A switch to national currencies has negligible implications

[2] See e.g. the EEAG Report on the European Economy (2012 and 2015) and EMU Public Finances (2014) published by CESifo and the European Commission respectively. Italy's net foreign debt position, although sizeable in absolute terms, is not one of the worst in the euroarea. For instance, Greece, Portugal, Spain, Ireland and Cyprus, are in a worse position (see e.g. the EEAG Report on the European Economy, 2012). However, since these countries have received financial aid from the so-called Troika (EC, ECB and IMF), we prefer to use Italy as our periphery country.

quantitatively. Specifically, welfare differences between a monetary union (with a single monetary policy) and flexible exchange rates (with national monetary policies) are practically zero in all cases studied. Thus, the merits of flexible exchange rates, as a way of allowing for an extra national policy instrument, are questionable. A similar result emerges in the case of a fiscal union with interregional transfers as insurance. Namely, any welfare benefits from adding transfers as insurance are very small. Thus, our results do not provide any strong arguments for interregional risk-sharing. On the other hand, the addition of interregional fiscal transfers as redistribution has non-trivial implications. Such transfers always hurt the donor country (Germany) as probably expected, while what happens to the recipient country (Italy) depends on whether these transfers trigger moral hazard effects or not. By the latter, we mean that ex ante transfers distort the incentives to work and save in the recipient country. If they do not trigger moral hazard effects, the recipient country benefits; actually, in our experiments, the whole union benefits since the benefit of the recipient country more than outweighs the loss of the donor country. But if they do trigger moral hazard effects, even the recipient country loses. In other words, in the case of transfers as redistribution and with moral hazard problems, transfers are self-destructive for all member countries, including those at the receiving end, and the resulting losses appear to be relatively large.

The rest of the paper is organized as follows. Section 2 reviews the literature. Section 3 presents a two-country currency union model. Section 4 discusses the solution methodology. The solution of the currency union model, using data from Germany and Italy, is in section 5. Section 6 studies flexible exchange rates. Section 7 models a fiscal (transfer) union. Section 8 closes the paper. Technical details are gathered in an online Appendix `http://www.aueb.gr/users/aphil/files/appendix_to_fiscal_union_paper.pdf`.

2 Related literature and how our work differs

Our work is related to several literatures. It is first related to the literature on currency union models (see e.g. Galí and Monacelli (2005, 2008), Beetsma and Jensen (2005), Coenen et al. (2008), Forni et al. (2010), Werning and Farhi (2012), Erceg and Lindé (2013), Okano (2013) and Evers (2015)). Second, it is related to the literature on international interdependence and strategic cooperation (see e.g. Mendoza and Tesar (2005), Beetsma and Jensen (2005) and Okano (2013)). Third, it is related to the policy literature on the interaction between monetary and fiscal policies (see e.g. Leeper (1991), Schmitt-Grohé and Uribe (2005 and 2007), Leith and Wren-Lewis (2008), Leeper et al. (2009), Kirsanova et al. (2009), Malley et al. (2009) and Philippopoulos et al. (2014)). Fourth, it is related to the literature on fiscal

reforms (see e.g. Coenen et al. (2008), Forni et al. (2010), Erceg and Lindé (2013), Cogan et al. (2013), Bi et al. (2013), Benigno and Romei (2014), Benigno et al. (2014) and Philippopoulos et al. (2015)), although most of these papers focus on debt consolidation. Finally, it is related to the recent literature on fiscal unions and its various elements (see e.g. Werning and Farhi (2012), Beetsma and Mavromatis (2014), Luque et al. (2014) and Evers (2015)).

Nevertheless, as far as we know, our paper is one of the first attempts to quantify the welfare differences among a monetary union, flexible exchange rates (economic disintegration) and a monetary plus fiscal union (higher economic integration) in a unified dynamic stochastic general equilibrium setup. We believe that two papers close to ours are Werning and Farhi (2012) and Evers (2015). As already said, Werning and Farhi (2012) have added transfers as insurance to a model of a monetary union. Evers (2015) has provided a quantitative assessment of different forms of fiscal federalism within a monetary union model. But our work differs. For instance, these papers focus on symmetric countries. By contrast, in our model countries differ (one country is a creditor and the other is a borrower) which is as in the eurozone data and, perhaps more importantly, is at the heart of the policy debate in Europe these days. Besides, we study fiscal transfers both as insurance and as redistribution, while most of the related macroeconomic literature has focused on the former only. Transfers as redistribution are also at the heart of the policy debate. Finally, we also compare unions (monetary and fiscal) to flexible exchange rates.

3 A model of a monetary union

This section sets up a New Keynesian DSGE model consisting of two heterogeneous countries forming a monetary union. To help the reader, we start with an informal description of the model.

3.1 Informal description of the model and discussion of assumptions

Two countries form a closed system in a New Keynesian setup. In each country, there are households, firms and a national fiscal authority or a government. In a currency union regime, there is a single monetary authority.

Households in each country can save in the form of physical capital, domestic government bonds and internationally traded assets. The government in each country can sell its bonds to domestic and foreign households. The latter, namely government's borrowing from abroad, takes place via the international asset market. In other words, the international asset market allows national governments to sell their bonds to foreign private agents and it also allows

private agents across countries to borrow from, or lend to, each other. We assume that international borrowing/lending takes place through a financial intermediary or bank. This financial intermediation requires a transaction, or monitoring, cost proportional to the amount of the nation's debt. This cost creates, in turn, a wedge between the borrowing and the lending interest rate. As a result, when they participate in the international asset market, agents (private and public) in the debtor country face a higher interest rate than agents (private and public) in the creditor country.[3] Also, when interest rates differ, the bank can make a profit and this profit is rebated lump-sum to agents in the creditor country.

As is well-known, systematic borrowing and lending cannot occur in an homogeneous world. Some type of heterogeneity is needed. A popular and intuitive way of producing borrowers and lenders has been to assume that agents differ in their patience to consume or, equivalently, in their discount factors. In particular, the discount factor of lenders is higher than that of borrowers or, equivalently, borrowers are more impatient than lenders.[4] It is also well-known that such differences in discount factors have to be combined with an imperfection in the capital market in order to get a well-defined solution; in our model, as said above, the capital market imperfection is the transaction, or monitoring, cost of the loan.[5]

This modelling will imply that, because of differences in discount factors, one country is a net lender and the other is a net borrower in the international asset market and that interest rates are higher in the debtor country. Given the current account data over the euro years, we will think of the lender country as Germany and the debtor country as Italy. In this case, in equilibrium, the relatively impatient Italians will finance their current account deficits by borrowing funds from the patient Germans who run current account surpluses. This scenario is also consistent with the literature on the interpretation of current accounts in the sense that systematic low saving rates and current account deficits are believed to reflect relatively low patience.[6]

[3] Two things should be clarfied here. First. instead of using the device of a financial intermediary or bank, we could just use transaction costs incurred upon borrowers. We prefer the bank-type modeling because we find it to be more intuitive (see also e.g. Curdia and Woodford (2009, 2010) and Benigno et al (2014) although in a closed economy). Second, here, differences in interest rates across countries are produced by transcation or monitoring costs incurred by the bank. As is known such differences can be produced in various ways including the probability of sovereign default (see subsection 3.5 below for further details).

[4] See also e.g. Benigno et al. (2014). Kiyotaki and Moore (1997) also use a general equilibrium model with two types of agents, creditors and borrowers, who discount the future differently. Moreover, we could enrich our model so as the discount factors are formed endogenously; see e.g. Becker and Mulligan (1997) and Doepke and Zilibotti (2008) for an endogenous formation of discount factors depending on income, education, effort, religion, etc. See also e.g. Schmitt-Grohe and Uribe (2003) and Choi et al. (2008) for calibrated models where the discount factor depends on consumption changes.

[5] See again Benigno et al. (2014). See also Doepke and Zilibotti (2008) where some financial market imperfections are necessary for getting differences in patience across different agents. .

[6] See e.g. Choi et al. (2008).

On other dimensions, the model is a rather standard New Keynesian currency union model.[7] In particular, each country produces an array of differentiated goods and, in both countries, firms act monopolistically facing Calvo-type nominal fixities. Nominal fixities can give a real role to monetary and exchange rate policy, at least in the transition path. In a monetary union regime, we assume a single monetary policy but independent national fiscal policies. Policy (both monetary and fiscal) is conducted by state-contingent policy rules.

In the home economy, there are N identical households and N firms each one of them producing a differentiated domestically produced tradable good. Similarly, in the foreign economy. For simplicity, population in both countries, N and N^*, is constant over time and the two countries are of equal size, $N = N^*$.

The rest of this section formalizes the above story. We will present the domestic country. The foreign country will be analogous except otherwise said. A star will denote the counterpart of a variable or a parameter in the foreign country.

3.2 Households

This subsection presents households in the domestic country. There are N identical households indexed by $i = 1, 2, ..., N$.

3.2.1 Consumption bundles

The quantity of each variety h produced at home by domestic firm h and consumed by each domestic household i is denoted as $c^H_{i,t}(h)$. Using a Dixit-Stiglitz aggregator, the composite of domestic goods consumed by each domestic household i, $c^H_{i,t}$, consists of h varieties and is given by:[8]

$$c^H_{i,t} = \left[\sum_{h=1}^{N} \kappa [c^H_{i,t}(h)]^{\frac{\phi-1}{\phi}} \right]^{\frac{\phi}{\phi-1}} \tag{1}$$

where $\phi > 0$ is the elasticity of substitution across goods produced in the domestic country and $\kappa = 1/N$ is a weight chosen to avoid scale effects in equilibrium.

Similarly, the quantity of each imported variety f produced abroad by foreign firm f and consumed by each domestic household i is denoted as $c^F_{i,t}(f)$. Using a Dixit-Stiglitz aggregator, the composite of imported goods consumed by each domestic household i, $c^F_{i,t}$, consists of f

[7] See Okano (2014) for a review of the related literature dating back to Galí and Monacelli (2005, 2008).
[8] As in e.g. Blanchard and Giavazzi (2003), here we work with summations rather than with integrals.

varieties and is given by:

$$c_{i,t}^F = \left[\sum_{f=1}^{N} \kappa [c_{i,t}^F(f)]^{\frac{\phi-1}{\phi}} \right]^{\frac{\phi}{\phi-1}} \tag{2}$$

In turn, having defined $c_{i,t}^H$ and $c_{i,t}^F$, domestic household i's consumption bundle, $c_{i,t}$, is:

$$c_{i,t} = \frac{\left(c_{i,t}^H\right)^{\nu} \left(c_{i,t}^F\right)^{1-\nu}}{\nu^{\nu}(1-\nu)^{1-\nu}} \tag{3}$$

where ν is the degree of preference for domestic goods (if $\nu > 1/2$, there is a home bias).

3.2.2 Consumption expenditure, prices and terms of trade

Domestic household i's total consumption expenditure is:

$$P_t c_{i,t} = P_t^H c_{i,t}^H + P_t^F c_{i,t}^F \tag{4}$$

where P_t is the consumer price index (CPI), P_t^H is the price index of home tradables, and P_t^F is the price index of foreign tradables (expressed in domestic currency).

Each domestic household's total expenditure on home goods and foreign goods are respectively:

$$P_t^H c_{i,t}^H = \sum_{h=1}^{N} \kappa P_t^H(h) c_{i,t}^H(h) \tag{5}$$

$$P_t^F c_{i,t}^F = \sum_{f=1}^{N} \kappa P_t^F(f) c_{i,t}^F(f) \tag{6}$$

where $P_t^H(h)$ is the price of each variety h produced at home and $P_t^F(f)$ is the price of each variety f produced abroad, both denominated in domestic currency.

We assume that the law of one price holds meaning that each tradable good sells at the same price at home and abroad. Thus, $P_t^F(f) = S_t P_t^{H*}(f)$, where S_t is the nominal exchange rate (where an increase in S_t implies a depreciation) and $P_t^{H*}(f)$ is the price of variety f produced abroad denominated in foreign currency. As said above, a star denotes the counterpart of a variable or a parameter in the rest-of-the world. Note that the terms of trade are defined as $\frac{P_t^F}{P_t^H}$ $(= \frac{S_t P_t^{H*}}{P_t^H})$, while the real exchange rate is defined as $\frac{S_t P_t^*}{P_t}$. In a currency union model, we will exogenously set $S_t \equiv 1$ at all t.

3.2.3 Household's optimization problem

Each domestic household i acts competitively to maximize expected discounted lifetime utility, V_0, defined as:

$$V_0 \equiv E_0 \sum_{t=0}^{\infty} \beta^t U\left(c_{i,t}, n_{i,t}, m_{i,t}, g_t\right) \tag{7}$$

where $c_{i,t}$ is i's consumption bundle as defined above, $n_{i,t}$ is i's hours of work, $m_{i,t}$ is i's real money holdings, g_t is per capita public spending, $0 < \beta < 1$ is domestic agents' discount factor, and E_0 is the rational expectations operator.

For our numerical solutions, the period utility function will be (see also e.g. Galí, 2008):

$$u_{i,t}\left(c_{i,t}, n_{i,t}, m_{i,t}, g_t\right) = \frac{c_{i,t}^{1-\sigma}}{1-\sigma} - \chi_n \frac{n_{i,t}^{1+\varphi}}{1+\varphi} + \chi_m \frac{m_{i,t}^{1-\mu}}{1-\mu} + \chi_g \frac{g_t^{1-\zeta}}{1-\zeta} \tag{8}$$

where χ_n, χ_m, χ_g, σ, φ, μ, ζ are standard preference parameters. Thus, $1/\sigma$ is the elasticity of substitution between consumption at two points in time and φ is the inverse of Frisch labour elasticity.

The period budget constraint of household i written in real terms is:

$$\begin{aligned} & (1+\tau_t^c)\left[\frac{P_t^H}{P_t} c_{i,t}^H + \frac{P_t^F}{P_t} c_{i,t}^F\right] + \frac{P_t^H}{P_t} x_{i,t} + b_{i,t} + m_{i,t} + \frac{S_t P_t^*}{P_t} f_{i,t}^h = \\ = \; & \left(1-\tau_t^k\right)\left[r_t^k \frac{P_t^H}{P_t} k_{i,t-1} + \widetilde{\omega}_{i,t}\right] + \left(1-\tau_t^n\right) w_t n_{i,t} + R_{t-1} \frac{P_{t-1}}{P_t} b_{i,t-1} + \\ & + \frac{P_{t-1}}{P_t} m_{i,t-1} + Q_{t-1} \frac{S_t P_t^*}{P_t} \frac{P_{t-1}^*}{P_t^*} f_{i,t-1}^h - \tau_{i,t}^l + \pi_{i,t} \end{aligned} \tag{9}$$

where $x_{i,t}$ is i's domestic investment, $b_{i,t}$ is the real value of i's end-of-period domestic government bonds, $m_{i,t}$ is i's end-of period real domestic money holdings, $f_{i,t}^h$ is the real value of i's end-of-period internationally traded assets denominated in foreign currency (if $f_{i,t}^h < 0$, it denotes private foreign debt), r_t^k denotes the real return to the beginning-of-period domestic capital, $k_{i,t-1}$, $\widetilde{\omega}_{i,t}$ denotes i's real dividends received by domestic firms, w_t is the real wage rate, $R_{t-1} \geq 1$ denotes the gross nominal return to domestic government bonds between $t-1$ and t, $Q_{t-1} \geq 1$ denotes the gross nominal return to international assets between $t-1$ and t, $\tau_{i,t}^l$ are real lump-sum taxes/transfers to each household, $\pi_{i,t}$ is profits distributed in a lump-sum fashion to the domestic household by the financial intermediary (see below) in a lump-sum fashion and $0 \leq \tau_t^c$, τ_t^k, $\tau_t^n \leq 1$ are the tax rates on consumption, capital income and labour income respectively. Note that small letters denote real values, namely, $m_{i,t} \equiv \frac{M_{i,t}}{P_t}$, $b_{i,t} \equiv \frac{B_{i,t}}{P_t}$, $f_{i,t}^h \equiv \frac{F_{i,t}^h}{P_t^*}$, $w_t \equiv \frac{W_t}{P_t}$, $\widetilde{\omega}_{i,t} \equiv \frac{\widetilde{\Omega}_{i,t}}{P_t}$, $\tau_{i,t}^l \equiv \frac{T_{i,t}^l}{P_t}$, where capital letters denote nominal values.

The law of motion of physical capital for each household i is:

$$k_{i,t} = (1-\delta)k_{i,t-1} + x_{i,t} - \frac{\xi}{2}\left(\frac{k_{i,t}}{k_{i,t-1}} - 1\right)^2 k_{i,t-1} \tag{10}$$

where $0 < \delta < 1$ is the depreciation rate of capital and $\xi \geq 0$ is a parameter capturing adjustment costs related to physical capital.

Further details on the household's problem, its first-order conditions and implications for the price bundles are in Appendix 1.

3.3 Firms

This subsection presents firms in the domestic economy. There are N domestic firms indexed by $h = 1, 2, ..., N$. Each firm h produces a differentiated tradable good of variety h under monopolistic competition and Calvo-type nominal fixities.

3.3.1 Demand for firm's product

The demand for each domestic firm h's product, $y_t^H(h)$, is (see Appendix 2 for details):

$$y_t^H(h) = \left[\frac{P_t^H(h)}{P_t^H}\right]^{-\phi} y_t^H \tag{11}$$

where y_t^H is total product in the domestic country.

3.3.2 Firm's optimization problem

Nominal profits of each domestic firm h are defined as:

$$\widetilde{\Omega}_t(h) \equiv P_t^H(h)y_t^H(h) - r_t^k P_t^H(h)k_{t-1}(h) - W_t n_t(h) \tag{12}$$

where $k_{t-1}(h)$ and $n_t(h)$ denote respectively the current capital and labor inputs chosen by the firm.

Maximization is subject to the demand function, (11), and the production function:

$$y_t^H(h) = A_t[k_{t-1}(h)]^{\alpha}[n_t(h)]^{1-\alpha} \tag{13}$$

where A_t is an exogenous stochastic TFP process whose motion is defined below and $0 < \alpha < 1$ is a technology parameter.

In addition, following Calvo (1983), firms choose their prices facing a nominal fixity. In particular, in each period, each firm h faces an exogenous probability θ of not being able to

reset its price. A firm h, which is able to reset its price at time t, chooses its price $P_t^{\#}(h)$ to maximize the sum of discounted expected nominal profits for the next k periods in which it may have to keep its price fixed. This objective is given by:

$$E_t \sum_{k=0}^{\infty} \theta^k \Xi_{t,t+k} \widetilde{\Omega}_{t+k}(h) = E_t \sum_{k=0}^{\infty} \theta^k \Xi_{t,t+k} \left\{ P_t^{\#}(h)\, y_{t+k}^H(h) - \Psi_{t+k}\left(y_{t+k}^H(h)\right) \right\}$$

where $\Xi_{t,t+k}$ is a discount factor taken as given by the firm (but, in equilibrium, it equals the household's intertemporal marginal rate of substitution in consumption), $y_{t+k}^H(h) = \left[\frac{P_t^{\#}(h)}{P_{t+k}^H}\right]^{-\phi} y_{t+k}^H$ is the demand function in future periods and $\Psi_t(.)$ denotes the minimum nominal cost function for producing $y_t^H(h)$ at t so that $\Psi_t'(.)$ is the associated nominal marginal cost.

Further details on the firm's problem and its first-order conditions are in Appendix 2.

3.4 Government budget constraint

This subsection presents the government budget constraint in the domestic economy (details are in Appendix 3). The period budget constraint of the consolidated government sector expressed in real terms and aggregate quantities is:

$$\begin{aligned} b_t + \tfrac{S_t P_t^*}{P_t} f_t^g + m_t &= R_{t-1} \tfrac{P_{t-1}}{P_t} b_{t-1} + Q_{t-1} \tfrac{S_t P_t^*}{P_t} \tfrac{P_{t-1}^*}{P_t^*} f_{t-1}^g + \tfrac{P_{t-1}}{P_t} m_{t-1} + \\ &+ \tfrac{P_t^H}{P_t} g_t - \tau_t^c \left(\tfrac{P_t^H}{P_t} c_t^H + \tfrac{P_t^F}{P_t} c_t^F\right) - \tau_t^k \left(r_t^k \tfrac{P_t^H}{P_t} k_{t-1} + \widetilde{\omega}_{i,t}\right) - \tau_t^n w_t n_t - \tau_t^l \end{aligned} \tag{14}$$

where b_t is the end-of-period domestic real public debt, f_t^g is the end-of-period foreign real public debt expressed in foreign prices and m_t is the end-of-period stock of real money balances. Note that we use the definitions $c_t^H \equiv \sum_{i=1}^N c_{i,t}^H$, $c_t^F \equiv \sum_{i=1}^N c_{i,t}^F$, $k_{t-1} \equiv \sum_{i=1}^N k_{i,t-1}$, $\widetilde{\Omega}_t \equiv \sum_{i=1}^N \widetilde{\Omega}_{i,t}$, $n_t \equiv \sum_{i=1}^N n_{i,t}$, $F_{t-1}^h \equiv \sum_{i=1}^N F_{i,t-1}^h$, $B_{t-1} \equiv \sum_{i=1}^N B_{i,t-1}$ and $T_t^l \equiv \sum_{i=1}^N T_{i,t}^l$. As said above, small letters denote real variables, namely, $b_t \equiv \frac{B_t}{P_t}$, $m_t \equiv \frac{M_t}{P_t}$ and $f_t^g \equiv \frac{F_t^g}{P_t^*}$. Also, the government allocates its total expenditure among product varieties h by solving an identical problem with household i, so that $g_t(h) = \left[\frac{P_t^H(h)}{P_t^H}\right]^{-\phi} g_t$.

If we define total nominal public debt in the domestic country as $D_t \equiv B_t + S_t F_t^g$, so that in real terms $d_t \equiv b_t + \frac{S_t P_t^*}{P_t} f_t^g$, we have $b_t \equiv \lambda_t d_t$ and $\frac{S_t P_t^*}{P_t} f_t^g \equiv (1-\lambda_t) d_t$, where $0 \leq \lambda_t \leq 1$ is the fraction of domestic public debt held by domestic private agents and $0 \leq 1-\lambda_t \leq 1$ is the fraction of domestic public debt held by foreign private agents.

In each period, one of the fiscal instruments (τ_t^c, τ_t^k, τ_t^n, g_t, τ_t^l, λ_t, d_t) follows residually to satisfy the government budget constraint. We assume this role is played by the end-of-period total public debt, d_t.

3.5 World financial intermediary

We use a simple and popular model of financial frictions (see e.g. Uribe and Yue, 2006, Curdia and Woodford, 2009 and 2010, Benigno et al., 2014). International borrowing, or lending, takes place through a financial intermediary or a bank. This bank is located in the home country. The bank plays a traditional role only, which consists in collecting deposits from lenders and lending the funds to borrowers.

In particular, the bank raises funds from domestic private agents, $\left(f_t^h - f_t^g\right)$, at the rate Q_t and lends to foreign agents, $(f_t^{*g} - f_t^{*h})$, at the rate Q_t^*.[9] In addition, the bank faces operational costs, which are increasing and convex in the volume of the loan, $(f_t^{*g} - f_t^{*h})$. The profit of the bank is revenue minus cost where revenue is net of transcation or monitoring costs. Thus, the profit written in real terms in the domestic country is given by (details are in Appendix 4):

$$\pi_t = Q_{t-1}^* \left[\frac{P_{t-1}}{P_t}(f_{t-1}^{*g} - f_{t-1}^{*h}) - \frac{P_t^H}{P_t}\frac{P_{t-1}^H}{P_t^H}\frac{\psi}{2}(f_{t-1}^{*g} - f_{t-1}^{*h})^2 \right] - Q_{t-1}\frac{S_t}{S_{t-1}}\frac{P_{t-1}}{P_t}(f_{t-1}^{*g} - f_{t-1}^{*h}) \quad (15)$$

where $\frac{\psi}{2}(f_{t-1}^{*g} - f_{t-1}^{*h})^2$ is the real cost function and $\psi \geq 0$ is a parameter (see subsection 5.1 below for its value). The first term in the brackets on the RHS is the bank's return on the loan net of monitoring cost, while the last term is payments to the savers.[10]

The bank chooses the amount of its loan taking Q_t and Q_t^* as given. Then, the optimality condition of the bank with respect to the volume of the loan is (details are again in Appendix 4):

$$Q_{t-1}^* = \frac{Q_{t-1}\frac{S_t}{S_{t-1}}}{1 - \frac{P_{t-1}^H}{P_{t-1}}\frac{\psi}{2}(f_{t-1}^{*g} - f_{t-1}^{*h})} \quad (16)$$

where, in a currency union, $S_t \equiv 1$; thus, $Q_t^* > Q_t$ which means that borrowers pay a sovereign premium.

It needs to be stressed that the implied property in equation (16) - namely, that the interest rate, at which the country borrows from the rest of the world, is increasing in the nation's total

[9] Recall that f_t^h denotes private foreign assets and f_t^g denotes public foreign debt (i.e. public debt held by foreign agents) in the home country. Similarly in the foreign country. Thus, if it so happens that $\left(f_t^h - f_t^g\right)$ is positive, it denotes net foreign assets in the home country and if it so happens that $(f_t^{*g} - f_t^{*h})$ is positive, it denotes net foreign liabilities in the foreign country. In equilibrium, $(f^{*g} - f^{*h}) + \frac{S_t P_t^*}{P_t}(f^g - f^h) = 0$.

[10] Note that, as in Curdia and Woodford (2009 and 2010), any resources consumed by the bank for the monitoring of its financial operations are part of the aggregate demand for the Dixit-Stiglitz composite good (details are in Appendices 2 and 5). Also note that the bank is located in the home country so that its profits are distributed to private agents in the domestic country in a lump-sum fashion, where $\pi_t = \sum_{i=1}^N \pi_{i,t}$ in equilibrium.

foreign debt - is supported by a number of empirical studies (see e.g. European Commission, 2012). It should also be stressed that a similar type of endogeneity of the country premium can be produced by several other models, including models of default risk.[11]

3.6 Monetary and fiscal policy

We now specify monetary and fiscal policy. As said, we follow a rule-like approach to policy.

3.6.1 Single monetary policy under a currency union

If we had flexible exchange rates, the exchange rate would be an endogenous variable and the two countries' nominal interest rates, R_t and R_t^*, could be free to be set independently by the national monetary authoritis, say, to follow national Taylor-type rules (see section 6 for flexible exchange rates). Here, by contrast, to mimic the eurozone regime, we assume that only one of the interest rates, R_t, can follow a Taylor-type rule, while R_t^* is an endogenous variable replacing the exchange rate which becomes an exogenous policy variable (this modelling, where the union's central bank uses one of national governments' interest rates as its policy instrument, is similar to that in e.g. Galí and Monacelli (2008) and Benigno and Benigno (2008)).[12]

In particular, we assume a single monetary policy rule of the form:

$$\begin{aligned} \log\left(\frac{R_t}{R}\right) &= \phi_\pi\left(\eta\log\left(\frac{\Pi_t}{\Pi}\right)+(1-\eta)\log\left(\frac{\Pi_t^*}{\Pi^*}\right)\right)+ \\ &\quad +\phi_y\left(\eta\log\left(\frac{y_t^H}{y^H}\right)+(1-\eta)\log\left(\frac{y_t^{*H}}{y^{*H}}\right)\right) \end{aligned} \tag{17}$$

where ϕ_π and $\phi_y \geq 0$ are respectively feedback monetary policy coefficients on inflation and the output gap and $0 < \eta < 1$ is the weight given to the domestic country relative to the foreign country (see subsection 5.1 below for the values of these parameters) while variables without time subscripts denote steady state values.

[11] Default risk reflects the fear of repudiation of debt obligations but also the fear of new wealth taxes with retroactive effect on debt repayments (see Alesina et al., 1992, for an early study). As Corsetti et al. (2013) point out, there are two approaches to sovereign default. The first approach models it as a strategic choice of the government (see e.g. Eaton and Gersovitz, 1981, Arellano, 2008, and many others). The second approach assumes that default occurs when debt exceeds an endogenous fiscal limit (see Bi, 2012, and many others). Such issues are beyond the scope of our paper.

[12] For various ways of modelling monetary policy in a monetary union, see e.g. Dellas and Tavlas (2005) and Collard and Dellas (2006).

3.6.2 National fiscal policies

Countries can follow independent fiscal policies. As in the case of monetary policy above, we focus on simple rules meaning that national fiscal authorities react to a small number of easily observable macroeconomic indicators. In particular, in each country, we allow the main spending-tax policy instruments, namely, government spending as share of output, defined as s_t^g, and the tax rates on consumption, capital income and labor income, τ_t^c, τ_t^k and τ_t^n, to react to the public debt-to-output ratio as deviation from a target, as well as to the output gap, according to the linear rules:[13]

$$s_t^g - s^g = -\gamma_l^g \left(l_{t-1} - l\right) - \gamma_y^g \left(y_t^H - y^H\right) \tag{18}$$

$$\tau_t^c - \tau^c = \gamma_l^c \left(l_{t-1} - l\right) + \gamma_y^c \left(y_t^H - y^H\right) \tag{19}$$

$$\tau_t^k - \tau^k = \gamma_l^k \left(l_{t-1} - l\right) + \gamma_y^k \left(y_t^H - y^H\right) \tag{20}$$

$$\tau_t^n - \tau^n = \gamma_l^n \left(l_{t-1} - l\right) + \gamma_y^n \left(y_t^H - y^H\right) \tag{21}$$

1where we define:

$$l_t \equiv \frac{R_t \lambda_t D_t + Q_t \frac{S_{t+1}}{S_t} \left(1 - \lambda_t\right) D_t}{P_t^H y_t^H} \tag{22}$$

where $\gamma_l^q \geq 0$ and $\gamma_y^q \geq 0$, for $q \equiv (g, c, k, n)$, are respectively feedback fiscal policy coefficients on inherited public liabilities and the current output gap (see subsection 5.1 below for their values) while variables without time subscripts denote steady state values (which, in the case of fiscal policy instruments, will be the data averages). Notice that the rest of fiscal policy instruments (namely, lump-sum tranfers, τ^l, and the fraction of public debt held by domestic agents, λ) are set at their data average values all the time.

Fiscal policy in the foreign country is modelled similarly.

[13] For similar rules, see e.g Schmitt-Grohé and Uribe (2007) and Cantore et al. (2012). See also EMU Public Finances (2011) published by the European Commission for fiscal reaction functions used in practice.

3.7 Exogenous variables and shocks

We now specify the exogenous variables, $A_t, A_t^*, \tau_t^l, \lambda_t, \tau_t^{l*}, \lambda_t^*$ and $\frac{S_{t+1}}{S_t}$. We assume that stochasticity comes from shocks to TFP, A_t and A_t^*, only (we report however that our main results do not depend on this). The rest of the exogenous variables are kept constant over time.

Starting with A_t and A_t^*, we use stochastic $AR(1)$ processes of the form:

$$\log(A_t) = (1-\rho^a)\log(A) + \rho^a \log(A_{t-1}) + \varepsilon_t^\alpha \tag{23}$$

$$\log(A_t^*) = (1-\rho^{*a})\log(A^*) + \rho^{*a}\log\left(A_{t-1}^*\right) + \varepsilon_t^{*\alpha} \tag{24}$$

where $0 < \rho^a, \rho^{*a} < 1$ are persistence parameters, variables without time subscript denote long-run values and $\varepsilon_t^a \sim N\left(0,\ \sigma_a^2\right), \varepsilon_t^{*a} \sim N\left(0,\ \sigma_a^{*2}\right)$.

The exogenously set fiscal policy instruments, $\{\tau_t^l, \lambda_t, \tau_t^{l*}, \lambda_t^*\}_{t=0}^\infty$, or equivalently, if we express lump-sum transfers as share of output, $\{s_t^l, \lambda_t, s_t^{l*}, \lambda_t^*\}_{t=0}^\infty$,[14] are assumed to be constant and equal to their data average values. Finally, as said, in a currency union regime, $S_t \equiv 1$ at any t.

3.8 Equilibrium system in a monetary union

We now combine all the above to get the equilibrium system for any feasible policy. The system is defined to be a sequence of allocations, prices and policies such that: (i) households maximize utility; (ii) a fraction $(1-\theta)$ of firms maximize profits by choosing an identical price $P_t^\#$, while a fraction θ just set their previous period prices; (iii) the international bank maximizes its profit (iv) all constraints, including the government budget constraint and the balance of payments, are satisfied; (v) all markets clear, including the international asset market; (vi) policy instruments are set by rules.

The final equilibrium system is presented in detail in Appendix 5. It consists of 59 equations in 59 variables, $\{V_t, y_t^H, c_t, c_t^H, c_t^F, n_t, x_t, k_t, f_t^h, m_t, TT_t, \Pi_t, \Pi_t^H, \Theta_t, \Delta_t, w_t, mc_t, \widetilde{\omega}_t, r_t^k, d_t, \Pi_t^*, z_t^1, z_t^2, \pi_t, q_t, Q_t, l_t, V_t^*, y_t^{*H}, c_t^*, c_t^{H*}, c_t^{F*}, n_t^*, x_t^*, k_t^*, f_t^{h*}, m_t^*, \Pi_t^{H*}, \Theta_t^*, \Delta_t^*, w_t^*, mc_t^*, \widetilde{\omega}^*, r_t^{*k}, d_t^*, z_t^{1*}, z_t^{2*}, Q_t^*, l_t^*, R_t, s_t^g, \tau_t^c, \tau_t^k, \tau_t^n, R_t^*, s_t^{g*}, \tau_t^{c*}, \tau_t^{k*}, \tau_t^{n*}\}_{t=0}^\infty$. This is given the exogenous variables, $\{A_t, A_t^*, s_t^l, \lambda_t, s_t^{l*}, \lambda_t^*, \frac{S_{t+1}}{S_t}\}_{t=0}^\infty$, as defined in subsection 3.7, the values of feedback policy coefficients as defined in subsection (3.6) and initial conditions for the state variables.

[14] Thus, $s_t^l \equiv \frac{\tau_t^l}{y_t^H TT_t^{\nu-1}}$ and $s_t^{l*} \equiv \frac{\tau_t^{l*}}{y_t^{H*} TT_t^{1-\nu^*}}$.

4 Solution methodology

Our main goal in this paper is to compare a currency union to alternative policy regimes, like flexible exchange rates and a fiscal union. We therefore work as follows. First, using commonly employed parameter values and fiscal data from Germany and Italy, we numerically solve the above model of a currency union. This is in the next section (section 5). In turn, to the extent that the steady state solution of the currency union regime is empirically relevant (meaning that it can mimic the data averages over the euro area period of study), we will use this regime as a point of comparison to evaluate the hypothetical regimes of flexible exchange rates and a fiscal union. This is in sections 6 and 7.

More specifically, we solve the two-country model developed above under the three mentioned regimes (currency union, flexible exchange rates and fiscal union). In all cases, we depart from the steady state solution of the currency union model (in other words, the initial values of the predetermined variables will be those found by the steady state solution of the currency union model). Then, transition dynamics are driven by extrinsic shocks and changes in the policy regime (details are provided below as we solve for each regime).

Regarding transition results, we will compute second-order approximate solutions, around the associated steady state, by following the methodology of Schmitt-Grohé and Uribe (2004a). In doing so, we use the Dynare toolkit. Note that we focus on second-order approximate solutions because the model is stochastic and, as is known, first-order approximations can give spurious results when used to compare the welfare under alternative policies (see e.g. the review in Galí, 2008, pp. 110-111). We, nevetheless, report that, in our case, first-order approximations give similar results qualitatively (results are available upon request).

Finally, comparisons of alternative policy regimes will be in terms of expected lifetime discounted utility (or "welfare"). Welfare differences will also be expressed in terms of consumption equivalences, as is the tradition in the related literature (see e.g. Lucas, 1990). As said, the currency union will serve as the benchmark in these welfare comparisons.

5 Data, parameterization and solution of the monetary union model

This section solves numerically the model economy of section 3 by using data from Germany and Italy over 2001-2013. The data are from OECD Statistics and the Eurostat. As we shall see, the model's steady state solution will resemble the main empirical characteristics of the two countries over the euro years.

5.1 Parameter values and economic policy

The baseline parameter values and the data averages of fiscal policy variables are listed in Tables 1a and 1b respectively. The time unit is meant to be a year. The two countries differ in their discount factors (see β and β^* in Table 1a) and fiscal policy (see the fiscal instruments in Table 1b). In all other respects, the two countries are assumed to be symmetric. As we shall see, this is enough to make the solution of the model empirically relevant.

Regarding parameter values, the model's key parameters are the discount factors in the two countries, β and β^*, and the cost coefficient driving the wedge between the borrowing and the lending interest rate, ψ. The values of these parameters are calibrated to match the real interest rates and the net foreign asset position of the two countries in the euro period data. In particular, the values of β and β^* follow from the Euler equations in the two countries which, at the steady state, are reduced to:

$$\beta Q/\Pi = 1 \tag{25}$$

$$\beta^* Q^*/\Pi^* = 1 \tag{26}$$

where Q/Π and Q^*/Π^* are the real interest rates in the two countries.[15] Since $Q/\Pi < Q^*/\Pi^*$ in the data over the euro period, it follows $\beta = 0.9833 > \beta^* = 0.9780$. That is, the Germans are more patient than the Italians.

In turn, the optimality condition of the bank, (16), written at the steady state is (as said, $S \equiv 1$ in a currency union):

$$Q^* = \frac{Q}{1 - \frac{P^H}{P}\psi(f^{*g} - f^{*h})} \tag{27}$$

so that, given data from all other variables, we calibrate the value of the parameter ψ.

All other papameter values, as listed in Table 1a, are the same across countries and are set at values commonly used in related studies. We report that our main results are robust to changes in these values. Thus, although our numerical simulations below are not meant to provide a rigorous quantitative study, they illustrate the qualitative dynamic features of the model in a robust way.

[15] Here, $\Pi_t \equiv \frac{P_t}{P_{t-1}}$ and $\Pi_t^* \equiv \frac{P_t^*}{P_{t-1}^*}$ (see Appendix 5 for detailed definitions of variables).

Table 1a: Baseline parameter values

Parameter	Home	Foreign	Description
a, a^*	0.3	0.3	share of physical capital in production
ν, ν^*	0.5	0.5	home goods bias in consumption
μ, μ^*	3.42	3.42	money demand elasticity
δ, δ^*	0.1	0.1	capital depreciation rate
ϕ, ϕ^*	6	6	price elasticity of demand
φ, φ^*	1	1	inverse of Frisch labour elasticity
σ, σ^*	1	1	elasticity of intertemporal substitution in utility
θ, θ^*	0.2	0.2	price rigidity parameter
χ_m, χ_m^*	0.001	0.001	preference related to real money balances
χ_n, χ_n^*	5	5	preference parameter related to work effort
χ_g, χ_g^*	0.1	0.1	preference parameter related to public spending
ξ, ξ^*	0.01	0.01	adjustment cost parameter of physical capital
β, β^*	0.9833	0.9780	discount factor
ψ	0.072	-	cost parameter in international borrowing
σ_α, σ_{α^*}	0.01	0.01	standard deviation of TFP
ρ^α, ρ^{α^*}	0.92	0.92	persistence of TFP

Regarding fiscal (tax-spending) policy instruments in the two countries as defined in subsection 3.6.2 above, the steady state tax rates and government spending-to-output ratios are all set equal to their average values in the data in Germany and Italy (see Table 1b). Along the transition, fiscal instruments can also react to the current state of public debt and level of economic activity as deviations from their steady state values,[16] where this reaction is quantified by the feedback policy coefficients in the policy rules (18)-(21). Here, we simply set the feedback coefficient of government spending on public debt at 0.1 in both countries (i.e. $\gamma_l^g = \gamma_l^{*g} = 0.1$) which is necessary for dynamic stability in most experiments, while we switch off all other fiscal reactions to debt and output.[17] These baseline values of feedback fiscal policy coefficients are summarized in Table 1c. We report that our main results are robust to changes in these values (results are available upon request).

[16] Since policy instruments react to deviations of macroeconomic indicators from their steady state values, feedback policy coefficients do not play any role in steady state solutions. Also recall that "money is neutral" in the long run, so that the monetary and exchange rate policy regimes also do not matter to the real economy at steady state.

[17] These values are close to those found by optimized policy rules in related studies (see e.g. Schmitt-Grohé and Uribe (2007) and Philippopoulos et al. (2014). They are also consistent with calibrated or estimated values by previous research (see e.g. Leeper et al. (2009), Forni et al. (2010), Coenen et al. (2013), Cogan et al. (2013), Erceg and Linde (2013)).

Table 1b: Fiscal policy variables (data averages)

Variable	Home	Foreign	Description
τ^c, τ^{*c}	0.1934	0.1756	consumption tax rate
τ^k, τ^{*k}	0.2041	0.3118	capital income tax rate
τ^n, τ^{*n}	0.3833	0.421	labour income tax rate
s^g, s^{*g}	0.2131	0.2423	government spending on goods/services as share of GDP
$-s^l$, $-s^{*l}$	-0.2039	-0.2163	government transfers as share of GDP
λ, λ^*	0.52	0.61	share of public debt held by domestic agents

Regarding the single monetary policy as defined in subsection 3.6.1 above, we set the coefficient on inflation, ϕ_π, at 1.5 and the coefficient on output, ϕ_y, at zero.[18] We also set the weight given to the domestic country in the bank's rule, η, at the neutral value of 0.5.[19] These baseline values of feedback monetary policy coefficients are also summarized in Table 1c. We again report that our main results are robust to changes in these values (results are available upon request).

Table 1c: Baseline feedback policy coefficients

monetary and fiscal policy instruments	monetary reaction to inflation and output	home fiscal reaction to debt and output	foreign fiscal reaction to debt and output
R_t	$\phi_\pi = 1.5$ $\phi_y = 0$		
$s_t^g \quad s_t^{*g}$		$\gamma_l^g = 0.1$ $\gamma_y^g = 0$	$\gamma_l^{*g} = 0.1$ $\gamma_y^{*g} = 0$
$\tau_t^c \quad \tau_t^{*c}$		$\gamma_l^c = 0$ $\gamma_y^c = 0$	$\gamma_l^{*c} = 0$ $\gamma_y^{*c} = 0$
$\tau_t^k \quad \tau_t^{*k}$		$\gamma_l^k = 0$ $\gamma_y^k = 0$	$\gamma_l^{*k} = 0$ $\gamma_y^{*k} = 0$
$\tau_t^n \quad \tau_t^{*n}$		$\gamma_l^n = 0$ $\gamma_y^n = 0$	$\gamma_l^{*n} = 0$ $\gamma_y^{*n} = 0$

Notes: In the baseline parameterization, $\eta = 0.5$.

[18] See again previous footnote.

[19] Our results are not sensitive to this. For instance, we have also set η equal to domestic GDP relative to union-wide GDP.

5.2 Solution under a monetary union

The equilibrium system of a currency union was defined in subsection 3.8 and the associated steady state follows simply if we assume that variables do not change over time (details are in Appendix 5). Table 2 presents the steady state solution when we use the parameter values and the policy instruments in Tables 1a-b. At this steady state, the residually determined public financing variable is public debt in both countries. Table 2 also presents some key ratios in the German and Italian data and, as can be seen, the solved ratios are close to their respective values in the data. In particular, the solution can mimic rather well the data averages of public debt-to-GDP ratios and foreign debt-to-GDP ratios in the two countries. We also report that the equilibrium system of a currency union is dynamically stable around its steady state (see below for transition results under this policy regime).

Table 2: Steady state solution under a monetary union

Variables	Description	Home	Data	Foreign	Data
$u,\ u^*$	utility	0.0376	-	0.0315	-
y^H, y^{H*}	output	0.3912	-	0.3543	-
c, c^*	consumption	0.2314	-	0.2278	-
$n,\ n^*$	hours worked	0.3116	-	0.3063	-
k, k^*	capital	0.6655	-	0.4976	-
$w,\ w^*$	real wage rate	0.6976	-	0.7085	-
$r^k,\ r^{k*}$	real return to capital	0.1470	-	0.1780	-
$Q^* - Q$	interest rate premium	-	-	0.0055	0.0055
$\frac{c}{y^H TT^{1-\nu}},\ \frac{c^*}{y^{H*} TT_t^{\nu^*-1}}$	consumption as share of GDP	0.5633	-	0.6752	-
$\frac{k}{y^H},\ \frac{k^*}{y^{H*}}$	capital as share of GDP	1.7009	-	1.4045	-
$\frac{d}{TT^{\nu-1} y^H},\ \frac{d^*}{TT^{1-\nu^*} y^{*H}}$	total public debt as share of GDP	0.6907	0.6861	1.0871	1.08
$\frac{\left(\frac{(1-\lambda)d}{TT^{\nu-1}} - TT_t^{\nu^*} f^h\right)}{y^H},\ \frac{\frac{(1-\lambda^*)d^*}{TT^{1-\nu-\nu^*}} - f^{*h}}{TT_t^{\nu} y^{*H}}$	total foreign debt as share of GDP*	-0.2109	-0.2501	0.2114	0.2109

Notes: Parameters and policy variables as in Tables 1a-b.

6 Flexible exchange rates and monetary policy independence

This section resolves the model but now we assume flexible exchange rates and hence allow for independent or national monetary policies, other things equal.

6.1 Modelling flexible exchange rates

In terms of modelling, the only difference from the model in section 3 is that now the exchange rate between the two countries becomes an endogenous variable. Thus, R_t^* and S_t exchange places. The former was endogenous in section 3, while now it is the latter that becomes endogenous with the former being free to follow a national Taylor-type rule for the nominal interest rate. In other words, now we have an independent Taylor-type rule for the national nominal interest rate in each country:

$$\log\left(\frac{R_t}{R}\right) = \phi_\pi \log\left(\frac{\Pi_t}{\Pi}\right) + \phi_y \log\left(\frac{y_t^H}{y^H}\right) \tag{28}$$

$$\log\left(\frac{R_t^*}{R^*}\right) = \phi_\pi^* \log\left(\frac{\Pi_t^*}{\Pi^*}\right) + \phi_y^* \log\left(\frac{y_t^{*H}}{y^{*H}}\right) \tag{29}$$

where ϕ_π, ϕ_y, ϕ_π^*, $\phi_y^* \geq 0$ are feedback monetary policy coefficients on inflation and output in each country. As before with single policy, we set $\phi_\pi = \phi_\pi^* = 1.5$ and $\phi_y = \phi_y^* = 0$ (we report that our main results are not sensitive to changes in these feedback policy coefficients).

6.2 Solution under flexible exchange rates

The new equilibrium system is as in the case of the currency union except from the change in the list of endogenous and exogenous variables as described above (further details are in Appendix 6). Since money is neutral at the steady state, a switch to flexible exchange rates does not affect the steady state solution of real variables and hence the associated level of utility; they thus remain the same as in Table 2 above. Any differences between the currency union regime and the flexible exchange rate regime will arise in the transition only, during which monetary and exchange rate policies matter to the real variables thanks to Calvo-type nominal fixities.

Results for expected discounted lifetime utility (or "welfare") in a currency union and under flexible exchange rates are reported in Table 3. This is by using the baseline parameterization. Numbers in parentheses, below welfare levels, report the associated welfare difference between the two policy regimes expressed in terms of consumption equivalences (a positive number means that a switch to flexible exchange rates is welfare enhancing vis-a-vis the monetary union, and vice versa for a negative number).

Table 3: Expected discounted lifetile utility (welfare) under different regimes

Regimes	Welfare		
	Germany	Italy	Union-wide
Monetary union	2.2554	1.4317	1.8439
Independent mon. policies	2.2554	1.4318	1.8439
	(0)	($\simeq 0$)	(0)

Notes: Union-wide numbers are a weighted sum of country numbers where the weights are their relative outputs.

Our results in Table 3 imply that the welfare implications of switching to flexible exchange rates are negligible. Specifically, welfare differences between a currency union and flexible exchange rates show only at the fourth, or even higher, decimal point, so that the implied consumption equivalences are practically zero. We report that we have experimented with various changes - in parameter values and the model itself - and this quantitative result continues to hold. In other words, although one can find cases where the switch to flexible exchange rates is welfare superior to a monetary union at least for one country (e.g. our results show that Italy gains relative to the currency union when we assume that extrinsic volatility is much higher in Italy than in Germany[20]), or cases where the opposite happens meaning that this switch is counter-productive (e.g. our results show that Italy loses relative to the currency union when we assume that it cares strongly about the output gap in its national Taylor rule for the nominal interest rate), the associated welfare differences continue to be negligible quantitatively.[21] Actually, since our model abstains from potential credibility problems, typically arising in the case of independent monetary policy in inflation-prone countries like Italy in the pre-euro period, our findings cannot provide strong arguments for flexible exchange rates (credibility problems would strengthen even more the arguments for a currency union).[22]

[20] For instance, this happens when the standard deviation of the TFP shock is $\sigma_\alpha = 0$ in Germany, while it is $\sigma_{\alpha^*} = 0.1$ in Italy. In this case, in Italy, welfare is 1.6644 under a curreny union and it rises to 1.6648 only under flexible exchange rates. Germany, by contrast, becomes worse off under flexible exchange rates, athough changes are trivial again.

[21] Clerc et al. (2009) provide similar evidence when they model the benefits from flexible exchange rates vis-a-vis a currency union in the presence of non-synchronized shocks.

[22] On the other hand, Schmitt-Grohé and Uribe (2013) find larger benefits from an exchange rate devaluatiion although in a model with permanent nominal wage rigidities. Also, Kirsanova et al. (2009) discuss possible benefits from flexible exchange rates, when independent monetary policy is used to inflate away the real burden of public debt.

7 Adding a fiscal (transfer) union to the monetary union

This section returns to the currency union model of section 3 but now we also add an explicit transfer mechanism between countries. As argued in the Introduction, a transfer mechanism is an essential element of a fiscal union. As also argued in the Introduction, we distinguish two types of interregional transfers: transfers as insurance and transfers as redistribution. In the case of tranfers as insurance, temporary tranfers can go in either direction since all countries can be hit by random, positive or negative, temporary shocks. By contrast, in the case of transfers as redistribution, the relatively poor country systematically receives a fraction of the excess of union average output over its domestic output.

We will assume for simplicity that all interregional transfers take place directly between citizens rather than through their governments (inter-governmental transfers) or through a common budget (federal tranfers). In all cases, obviously, interregional transfers add up to zero across countries.

7.1 Modelling transfers

We now model the above two types of transfers. In turn, these transfers will be added to the model of a monetary union developed in section 3. This will be our fiscal union model.

7.1.1 Modelling transfers as insurance

We first consider a transfer mechanism that works as insurance against temporary shocks. In particular, imagine that the monetary union is at its status quo steady state (see Table 2) but it can deviate from it temporarily because of random (in our model, TFP) shocks hitting the two member-countries. These shocks cause deviations of current output (among all other endogenous variables) from its steady state value in each country. Then, there are transfers from one country to another conditioned on these temporary deviations in output.

Formally, since Italy's output can deviate from its steady state value because of shocks, we add $\gamma\left(y^{*H}-y_t^{*H}\right)\frac{P_t^{*H}}{P_t^*}$ on the revenue side of the household's budget constraint in Italy and, at the same time, we add $\frac{S_tP_t^*}{P_t}\gamma\left(y^{*H}-y_t^{*H}\right)\frac{P_t^{*H}}{P_t^*}$ on the expenditure side of the household's budget constraint in Germany, where $\gamma>0$ is a redistributive parameter. Similarly, since Germany's output can also deviate from its steady state value because of the very same shocks, we add $\gamma\left(y^{H}-y_t^{H}\right)\frac{P_t^{H}}{P_t}$ on the revenue side of the household's budget constraint in Germany and, at the same time, we add $\frac{P_t}{S_tP_t^*}\gamma\left(y^{H}-y_t^{H}\right)\frac{P_t^{H}}{P_t}$ on the expenditure side of the household's budget constraint in Italy. Further details and the new equations are in Appendix 7.

In the numerical solutions, the redistributive parameter, $\gamma > 0$, is calibrated so as the average redistributive transfer over time to be (almost) zero. We believe this makes sense: for transfers to correspond to insurance there should not be a presumption of transfers over time. With our baseline parameterization, this implies $\gamma = 0.05$. We report that our qualitative results are not sensitive to this parameter value.

7.1.2 Modelling transfers as redistribution

We now consider the case where there is a systematic one-way transfer from the relatively rich to the relatively poor country. The target in the transfer payment scheme is now the average of output in the two countries. This resembles practice in the EU (see the discussion in the Introduction).

Formally, the amount paid by Germany (which is the relatively rich country in the status quo steady state solution of a monetary union) and, at the same time, received by Italy (which is the relatively poor country in the same solution) is $\gamma\left(y_t^{union} - y_t^{*H}\frac{P_t^{*H}}{P_t^*}\frac{S_tP_t^*}{P_t}\right) > 0$, where $y_t^{union} = \frac{y_t^H\frac{P^H}{P}+y_t^{*H}\frac{P_t^{*H}}{P_t^*}\frac{S_tP_t^*}{P_t}}{2}$ denotes average output in the two countries.

Actually, since now transfers are systematically one-sided from the relatively rich to the relatively poor country, we will distinguish two sub-cases: one without moral hazard effets and one with moral hazard effects. By moral hazard, we mean that agents in the recipient country (Italy) internalize the interregional tranfers and this distorts their individual incentives to save and work.[23] Further details and the new equations are in Appendix 7.

Regarding the redistributive parameter, $\gamma > 0$, we will use the same value as in the case of transfers as insurance above (namely, $\gamma = 0.05$) but we will also report results with different, higher values.

7.2 Solution under a monetary plus fiscal (transfer) union

The new equilibrium system is as in the case of the currency union except from the addition of transfers and moral hazard effects as described above (details are in Appendix 7). In the case of transfers as insurance, the steady state solution remains as in Table 2 because shocks and hence transfers are temporary. In the case of transfers as redistribution, by contrast, the steady state solution changes because transfers are systematic (see Appendix 7 for the steady

[23] We could also incorporate moral hazard effects in the case of transfers of insurance. We have chosen not to do so simply because (we think) it is more natural to study such incentive effects in the context of systematic transfers.

state solution in this case).

Here, we report results for excepted discounted lifetime utility only. These results are shown in Table 4. In this table, for expositional convenience, we also repeat the results for the other two policy regimes studied above, namely, currency union and flexible exchange rates (see the first two rows respectively). In the case of transfers as redistribution, as said above, we distinguish two cases: one without moral hazard effets (see second row from the end) and one with moral hazard effects (see last row). All this is again with the baseline parameterization in Tables 1a-c. As said before, numbers in parentheses, below welfare levels, report the associated welfare difference between a policy regime and the benchmark case of the monetary union, where the welfare difference is expressed in consumption equivalences. For instance, a value of -0.045 in the last row means that Germany suffers a loss of 4.5% of consumption. Before we discuss our results, recall that these values are typically small (e.g. when Lucas, 1990, computes the lifetime welfare gain from a complete elimination of capital tax rates in the US, he finds a gain of 2.7% of consumption).

Table 4: Expected discounted lifetile utility (welfare) under different regimes

Regimes	Welfare		
	Germany	Italy	Union-wide
Monetary union	2.2554	1.4317	1.8439
Independent mon. policies	2.2554	1.4318	1.8439
	(0)	($\simeq 0$)	(0)
Mon. union plus transfers as insurance	2.2594	1.4324	1.8462
	($\simeq 0$)	($\simeq 0$)	($\simeq 0$)
Mon union plus transfers as redistribution	2.1050	1.7622	1.9337
	(-0.002)	(0.007)	(0.002)
Mon.union plus transfers as	-0.4764	1.2363	0.3793
redistribution (with moral hazard)	(-0.045)	(-0.004)	(-0.028)

Notes: See notes of Table 3.

Any welfare benefits from transfers as insurance appear at the third, or higher, decimal point only, so that the resulting welfare equivalences are practically zero. Thus, one can hardly claim that interregional risk-sharing is welfare improving. Recall that transfers as insurance are two-sided since both countries are hit by shocks. By contrast, the effects of tranfers as redistribution (see the last two rows) are non trivial. Such one–sided transfers hurt the donor country (Germany) in all cases, while what happens to the recipient country (Italy) depends on whether these transfers trigger moral hazard side-effects or not. If they do not trigger

moral hazard side-effects (see the second row from the bottom), the recipient country benefits; actually, the whole currency union benefits since the benefit of the recipient country more than outweighs the loss of the donor country. But if they do trigger moral hazard side-effects (see the bottom row), then even the recipient country loses. In other words, in the case of transfers as redistribution and with moral hazard problems, transfers are self-destructive for all member countries including those at the receiving end.

In Appendix 7, we also report sensitivity results with a higher value of the redistributive parameter, γ. All qualitative results remain as in Table 4. Quantitatively, a higher γ worsens the detrimental effects of transfers as redistribution in the presence of moral hazard.

8 Summary and possible extensions

This paper studied the implications of different policy regimes in a New Keynesian DSGE model consisting of two heterogeneous countries. We compared three debated policy regimes: a monetary union (used as a benchmark), flexible exchange rates and a fiscal (transfer) union within the monetary union.

Since the main results have already been listed in the Introduction, we close with some caveats and possible extensions. Here we studied three policy regimes only and we also modeled the fiscal union as a mechanism of interregional fiscal transfers. But, as also discussed in the Introduction, a fiscal union can have additional elements, like eurobonds and a union-wide bailout mechanism. Besides, there are other interesting policy regimes to study, like a comparison between a Single Market and a regime with barriers to trade in goods or assets. We leave such extensions for future research.

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Appendix to "Monetary union, even higher integration, or back to national currencies?"*

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Abstract

This paper quantifies the welfare differences among a monetary union, flexible exchange rates (economic disintegration) and a monetary plus fiscal transfer union (higher economic integration). The vehicle of analysis is a medium-scale New Keynesian DSGE model consisting of two heterogeneous countries. The model is solved using data from Germany and Italy. Our solutions imply that a switch to flexible exchange rates and independent monetary policies would have negligible welfare implications. A similar result applies when we add interregional fiscal tranfers as insurance. By contrast, the addition of fiscal tranfers as redistribution has non-trivial implications and these depend crucially on whether such one-sided transfers trigger moral hazard behavior or not.

Keywords: Fiscal union; monetary union; New Keynesian, DSGE.

JEL classification: E6, F3, H6.

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1 Appendix 1: Households

This Appendix presents the solution of the household's problem in the domestic country (the problem of the household in the foreign country is analogous except otherwise said). There are $i = 1, 2, ..., .N$ identical domestic households who act competitively.

1.1 Household's optimality conditions

Each domestic household i maximizes (7)-(8) subject to (1)-(6), (9) and (10) in the main text. We work in two steps. We first suppose that the household determines its desired consumption of composite goods, $c_{i,t}^H$ and $c_{i,t}^F$, and, in turn, chooses how to distribute its purchases of individual varieties, $c_{i,t}^H(h)$ and $c_{i,t}^F(f)$. The first-order conditions of each i include its constraints, as listed in the main text, plus:

$$\begin{aligned} &\tfrac{\partial u_{i,t}}{\partial c_{i,t}} \tfrac{\partial c_{i,t}}{\partial c_{i,t}^H} \tfrac{P_t}{P_t^H(1+\tau_t^c)} = \\ &= \beta E_t \tfrac{\partial u_{i,t+1}}{\partial c_{i,t+1}} \tfrac{\partial c_{i,t+1}}{\partial c_{i,t+1}^H} \tfrac{P_{t+1}}{P_{t+1}^H\left(1+\tau_{t+1}^c\right)} R_t \tfrac{P_t}{P_{t+1}} \end{aligned} \tag{1}$$

$$\begin{aligned} &\tfrac{\partial u_{i,t}}{\partial c_{i,t}} \tfrac{\partial c_{i,t}}{\partial c_{i,t}^H} \tfrac{P_t}{P_t^H(1+\tau_t^c)} \tfrac{S_t P_t^*}{P_t} = \\ &= \beta E_t \tfrac{\partial u_{i,t+1}}{\partial c_{i,t+1}} \tfrac{\partial c_{i,t+1}}{\partial c_{t+1}^H} \tfrac{P_{t+1}}{P_{t+1}^H\left(1+\tau_{t+1}^c\right)} Q_t \tfrac{S_{t+1} P_{t+1}^*}{P_{t+1}} \tfrac{P_t^*}{P_{t+1}^*} \end{aligned} \tag{2}$$

$$\begin{aligned} &\tfrac{\partial u_{i,t}}{\partial c_{i,t}} \tfrac{\partial c_{i,t}}{\partial c_{i,t}^H} \tfrac{1}{(1+\tau_t^c)} \left\{1 - \xi\left(\tfrac{k_{i,t}}{k_{i,t-1}} - 1\right)\right\} = \\ &= \beta E_t \tfrac{\partial u_{i,t+1}}{\partial c_{i,t+1}} \tfrac{\partial c_{i,t+1}}{\partial c_{i,t+1}^H} \tfrac{1}{\left(1+\tau_{t+1}^c\right)} \left\{ \begin{array}{c} (1-\delta) - \frac{\xi}{2}\left(\frac{k_{i,t+1}}{k_{i,t}} - 1\right)^2 + \\ \xi\left(\frac{k_{i,t+1}}{k_{i,t}} - 1\right)\frac{k_{i,t+1}}{k_{i,t}} + \left(1-\tau_{t+1}^k\right) r_{t+1}^k \end{array} \right\} \end{aligned} \tag{3}$$

$$\chi_m \frac{\partial u_{i,t}}{\partial m_{i.t}} = \frac{\partial u_{i,t}}{\partial c_{i,t}} \frac{\partial c_{i,t}}{\partial c_{i,t}^H} \frac{P_t}{P_t^H\left(1+\tau_t^c\right)} - \beta E_t \frac{\partial u_{i,t+1}}{\partial c_{i,t+1}} \frac{\partial c_{i,t+1}}{\partial c_{i,t+1}^H} \frac{P_{t+1}}{P_{t+1}^H\left(1+\tau_{t+1}^c\right)} \frac{P_t}{P_{t+1}} \tag{4}$$

$$-\chi_n \frac{\partial u_{i,t}}{\partial n_{i,t}} = \frac{(1-\tau_t^n)}{(1+\tau_t^c)} w_t \frac{\partial u_{i,t}}{\partial c_{i,t}} \frac{\partial c_{i,t}}{\partial c_{i,t}^H} \frac{P_t}{P_t^H} \tag{5}$$

$$\frac{c_{i,t}^H}{c_{i,t}^F} = \frac{\nu}{1-\nu} \frac{P_t^F}{P_t^H} \tag{6}$$

$$c_{i,t}^H(h) = \left[\frac{P_t^H(h)}{P_t^H}\right]^{-\phi} c_{i,t}^H \tag{7}$$

$$c_{i,t}^F(f) = \left[\frac{P_t^F(f)}{P_t^F}\right]^{-\phi} c_{i,t}^F \tag{8}$$

Equations (1)-(3) are respectively the Euler equations for domestic bonds, foreign assets and domestic capital, (4) is the optimality condition for money balances and (5) is the optimality condition for work hours. Finally, (6) shows the optimal allocation between domestic and foreign goods, while (7) and (8) show the optimal demand for each variety of domestic and foreign goods respectively.

1.2 Implications for price bundles

Equations (6), (7) and (8), combined with the household's budget constraints, imply that the three price indexes are:

$$P_t = (P_t^H)^{\nu}(P_t^F)^{1-\nu} \tag{9}$$

$$P_t^H = \left[\sum_{h=1}^{N} \kappa [P_t^H(h)]^{1-\phi}\right]^{\frac{1}{1-\phi}} \tag{10}$$

$$P_t^F = \left[\sum_{f=1}^{N} \kappa [P_t^F(f)]^{1-\phi}\right]^{\frac{1}{1-\phi}} \tag{11}$$

where $\kappa = 1/N$.

2 Appendix 2: Firms

This Appendix presents the solution of the firm's problem in the domestic country (the problem of the firm in the foreign country is analogous except otherwise said). There are $h = 1, 2, ..., .N$ differentiated domestic firms. Each firm h produces a differentiated good of variety h under monopolistic competition facing Calvo-type nominal fixities.

2.1 Demand for the firm's product

Each domestic firm h faces demand for its product, $y_t^H(h)$. The latter comes from domestic households' consumption and investment, $c_t^H(h)$ and $x_t(h)$, where $c_t^H(h) \equiv \sum_{i=1}^{N} c_{i,t}^H(h)$ and $x_t(h) \equiv \sum_{i=1}^{N} x_{i,t}(h)$, from the domestic government, $g_t(h)$, from the financial intermediary located in the domestic country, $q_t(h)$,[1] and from foreign households' consumption, $c_t^{F*}(h) \equiv$

[1] See also Curdia and Woodford (2009) for a similar modelling of resources consumed by banks.

$\sum_{i=1}^{N*} c_{i,t}^{F*}(h)$. Thus, the demand for the Dixit-Stiglitz good produced by each firm h is written as:

$$y_t^H(h) = c_t^H(h) + x_t(h) + g_t(h) + q_t(h) + c_t^{F*}(h) \tag{12}$$

where, for each component:

$$c_t^H(h) = \left[\frac{P_t^H(h)}{P_t^H}\right]^{-\phi} c_t^H \tag{13}$$

$$x_t(h) = \left[\frac{P_t^H(h)}{P_t^H}\right]^{-\phi} x_t \tag{14}$$

$$g_t(h) = \left[\frac{P_t^H(h)}{P_t^H}\right]^{-\phi} g_t \tag{15}$$

$$q_t(h) = \left[\frac{P_t^H(h)}{P_t^H}\right]^{-\phi} q_t \tag{16}$$

$$c_t^{F*}(h) = \left[\frac{P_t^{F*}(h)}{P_t^{F*}}\right]^{-\phi} c_t^{F*} \tag{17}$$

where, using the law of one price discussed above, we have in (17):

$$\frac{P_t^{F*}(h)}{P_t^{F*}} = \frac{\frac{P_t^H(h)}{S_t}}{\frac{P_t^H}{S_t}} = \frac{P_t^H(h)}{P_t^H} \tag{18}$$

Since, at the economy level, agreggate demand for the domestically produced good is:

$$y_t^H = c_t^H + x_t + g_t + q_t + c_t^{F*} \tag{19}$$

the above equations imply that the demand for each domestic firm's product is:

$$y_t^H(h) = c_t^H(h) + x_t(h) + g_t(h) + q_t(h) + c_t^{F*}(h) = \left[\frac{P_t^H(h)}{P_t^H}\right]^{-\phi} y_t^H \tag{20}$$

2.2 Firm's problem

Each domestic firm h maximizes nominal profits, $\widetilde{\Omega}_t(h)$:

$$\widetilde{\Omega}_t(h) = P_t^H(h)y_t^H(h) - r_t^k P_t^H(h)k_{t-1}(h) - W_t n_t(h) \tag{21}$$

This is subject to the production function:

$$y_t^H(h) = A_t[k_{t-1}(h)]^{\alpha}[n_t(h)]^{1-\alpha} \tag{22}$$

and, since the firm operates under imperfect competition, the product demand function:

$$y_t^H(h) = \left[\frac{P_t^H(h)}{P_t^H}\right]^{-\phi} y_t^H \tag{23}$$

In addition, following Calvo (1983), firms choose their prices facing a nominal fixity. In each period, firm h faces an exogenous probability θ of not being able to reset its price. A firm h, which is able to reset its price, chooses its price $P_t^{\#}(h)$ to maximize the sum of discounted expected nominal profits for the next k periods in which it may have to keep its price fixed.

2.3 Firm's optimality conditions

To solve the firm's problem, we work in two steps. We first solve a cost minimization problem, where each firm h minimizes its cost by choosing factor inputs given technology and prices. The solution will give a minimum nominal cost function, which is a function of factor prices and output produced by the firm. In turn, given this cost function, each firm, which is able to reset its price, solves a maximization problem by choosing its price.

The solution to the cost minimization problem gives the input demand functions**:**

$$w_t = mc_t(1-a)\frac{y_t(h)}{n_t(h)} \tag{24}$$

$$\frac{P_t^H}{P_t} r_t^k = mc_t a \frac{y_t(h)}{k_{t-1}(h)} \tag{25}$$

where mc_t is real marginal cost.

Then, the firm chooses its price to maximize nominal profits written as:

$$E_t \sum_{k=0}^{\infty} \theta^k \Xi_{t,t+k} \widetilde{\Omega}_{t+k}(h) = E_t \sum_{k=0}^{\infty} \theta^k \Xi_{t,t+k} \left\{ P_t^{\#}(h)\, y_{t+k}^H(h) - \Psi_{t+k}\left(y_{t+k}^H(h)\right) \right\}$$

where $\Xi_{t,t+k}$ is a discount factor taken as given by the firm, $y_{t+k}^H(h) = \left[\frac{P_t^{\#}(h)}{P_{t+k}^H}\right]^{-\phi} y_{t+k}^H$ and $\Psi_t(.)$ denotes the minimum nominal cost function for producing $y_t^H(h)$ at t so that $\Psi_t'(.)$ is the associated marginal cost (namely, $\Psi_t'(.) = mc_t P_t$).

The first-order condition gives:

$$E_t \sum_{k=0}^{\infty} \theta^k \Xi_{t,t+k} \left[\frac{P_t^{\#}(h)}{P_{t+k}^H}\right]^{-\phi} y_{t+k}^H \left\{ P_t^{\#}(h) - \frac{\phi}{\phi-1} \Psi_{t+k}' \right\} = 0 \tag{26}$$

Dividing by the aggregate price index, P_t^H, we have:

$$E_t \sum_{k=0}^{\infty} \theta^k [\Xi_{t,t+k} \left[\frac{P_t^{\#}(h)}{P_{t+k}^H}\right]^{-\phi} y_{t+k}^H \left\{ \frac{P_t^{\#}(h)}{P_t^H} - \frac{\phi}{\phi-1} mc_{t+k} \frac{P_{t+k}}{P_t^H} \right\}] = 0 \tag{27}$$

Therefore, the behaviour of each firm h is summarized by (24), (25) and (27). A recursive expression of this problem is presented below.

Note that each firm h, which can reset its price in period t, solves an identical problem, so $P_t^{\#}(h) = P_t^{\#}$ is independent of h, and each firm h, which cannot reset its price, just sets its previous period price $P_t^H(h) = P_{t-1}^H(h)$. Thus, the evolution of the aggregate price level is given by:

$$\left(P_t^H\right)^{1-\phi} = \theta \left(P_{t-1}^H\right)^{1-\phi} + (1-\theta) \left(P_t^{\#}\right)^{1-\phi} \tag{28}$$

3 Appendix 3: Government budget constraint

This Appendix presents the government budget constraint in some detail. In the domestic economy, the government budget constraint in nominal terms is:

$$\begin{gathered} B_t + M_t + S_t F_t^g = R_{t-1} B_{t-1} + M_{t-1} + \\ + Q_{t-1} S_t F_{t-1}^g + P_t^H g_t - \tau_t^c (P_t^H c_t^H + P_t^F c_t^F) - \tau_t^k (r_t^k P_t^H k_{t-1} + \widetilde{\Omega}_t) - \tau_t^n W_t n_t - T_t^l \end{gathered} \tag{29}$$

Now let $D_t \equiv B_t + S_t F_t^g$ denote total nominal public debt. This debt can be held both by domestic private agents, $\lambda_t D_t$, where in equilibrium $B_t = \lambda_t D_t$, and by foreign private agents, $S_t F_t^g = (1-\lambda_t) D_t$. Then, the government budget constraint can be written as:

$$\begin{gathered} D_t + M_t = R_{t-1}\lambda_{t-1}D_{t-1} + M_{t-1} + \\ +Q_{t-1}\tfrac{S_t}{S_{t-1}}(1-\lambda_{t-1})D_{t-1} + P_t^H g_t - \tau_t^c(P_t^H c_t^H + P_t^F c_t^F) \\ -\tau_t^k(r_t^k P_t^H k_{t-1} + \widetilde{\Omega}_t) - \tau_t^n W_t n_t - T_t^l \end{gathered} \tag{30}$$

and in real terms as:

$$\begin{gathered} d_t + m_t = R_{t-1}\lambda_{t-1}\tfrac{P_{t-1}}{P_t}d_{t-1} + \tfrac{P_{t-1}}{P_t}m_{t-1} + \\ +Q_{t-1}\tfrac{S_t P_t^*}{P_t}\tfrac{P_{t-1}^*}{P_t^*}\tfrac{P_{t-1}}{P_{t-1}^* S_{t-1}}(1-\lambda_{t-1})d_{t-1} + \tfrac{P_t^H}{P_t}g_t - \tau_t^c(\tfrac{P_t^H}{P_t}c_t^H + \tfrac{P_t^F}{P_t}c_t^F) - \tau_t^k(r_t^k\tfrac{P_t^H}{P_t}k_{t-1} + \widetilde{\omega}_t) - \tau_t^n w_t n_t - \tau_t^l \end{gathered} \tag{31}$$

Thus, the liabilities of the domestic government as a share of output are:

$$l_t \equiv \frac{R_t\lambda_t D_t + Q_t\frac{S_{t+1}}{S_t}(1-\lambda_t)D_t}{P_t^H y_t^H} \tag{32}$$

Similarly, the government budget constraint in nominal terms in the foreign country is:

$$\begin{gathered} B_t^* + M_t^* + \tfrac{F_t^{*g}}{S_t} = \tfrac{\phi^{*g}}{2}\left(\tfrac{F_t^{*g}}{P_t^* S_t} - \tfrac{F^{*g}}{SP^*}\right)^2 + R_{t-1}^* B_{t-1}^* + M_{t-1}^* + \\ +Q_{t-1}^*\tfrac{1}{S_t}F_{t-1}^{*g} + P_t^{*H}g_t^* - \tau_t^{*c}(P_t^{*H}c_t^{*H} + P_t^{*F}c_t^{*F}) - \tau_t^{*k}(r_t^{*k}P_t^{*H}k_{t-1}^* + \widetilde{\Omega}_t^*) - \tau_t^{*n}W_t^* n_t^* - T_t^{*l} \end{gathered} \tag{33}$$

Let denote D_t^* to be the total foreign public debt in foreign currency. This can be held by foreign private agents, $B_t^* = \lambda_t^* D_t^*$, and by domestic private agents, $\frac{F_t^{*g}}{S_t} = (1-\lambda_t^*)D_t^*$. Then, we have in nominal terms:

$$\begin{gathered} D_t^* + M_t^* = R_{t-1}^*\lambda_{t-1}^* D_{t-1}^* + M_{t-1}^* + \\ +Q_{t-1}^*\tfrac{S_{t-1}}{S_t}\left(1-\lambda_{t-1}^*\right)D_{t-1}^* + P_t^{*H}g_t^* - \tau_t^{*c}(P_t^{*H}c_t^{*H} + P_t^{*F}c_t^{*F}) \\ -\tau_t^{*k}(r_t^{*k}P_t^{*H}k_{t-1}^* + \widetilde{\Omega}_t^*) - \tau_t^{*n}W_t^* n_t^* - T_t^{*l} \end{gathered} \tag{34}$$

Thus, the liabilities of the foreign government as a share of output are:

$$l_t^* \equiv \frac{R_t^*\lambda_t^* D_t^* + Q_t^*\frac{S_t}{S_{t+1}}(1-\lambda_t^*)D_t^*}{P_t^{*H}y_t^{*H}} \tag{35}$$

4 Appendix 4: Financial intermediary or bank

The profit of the international bank from loans between $t-1$ and t is distributed at time t. In nominal terms, this profit is defined as:[2]

$$Q^*_{t-1}\left[(F^{*g}_{t-1}-F^{*h}_{t-1})-\frac{\psi}{2}P^H_{t-1}(f^{*g}_{t-1}-f^{*h}_{t-1})^2\right]-Q_{t-1}S_t\left(F^h_{t-1}-F^g_{t-1}\right) \tag{36}$$

where the real resources used by the bank are assumed to be consumed at the same time the interest payments/income are repaid/received, namely at time t, rather then when the loan contract was originated, namely at time $t-1$.

Dividing by P_t, the real profit, π_t, is:

$$\pi_t = Q^*_{t-1}\left[\frac{P_{t-1}}{P_t}(f^{*g}_{t-1}-f^{*h}_{t-1})-\frac{P^H_t}{P_t}\frac{\psi}{2}\frac{P^H_{t-1}}{P^H_t}(f^{*g}_{t-1}-f^{*h}_{t-1})^2\right]-Q_{t-1}\frac{S_tP^*_t}{P_t}\frac{P^*_{t-1}}{P^*_t}\left(f^h_{t-1}-f^g_{t-1}\right) \tag{37}$$

Since, in equilibrium, borrowing equals lending, namely, $F^{*g}_t - F^{*h}_t = S_t\left(F^h_t - F^g_t\right)$ or $f^{*g}_t - f^{*h}_t = \frac{S_tP^*_t}{P_t}\left(f^h_t - f^g_t\right)$ or in turn $f^{*g}_{t-1} - f^{*h}_{t-1} = \frac{S_{t-1}P^*_{t-1}}{P_{t-1}}\left(f^h_{t-1} - f^g_{t-1}\right)$, this is rewritten as:

$$\pi_t = Q^*_{t-1}\left[\frac{P_{t-1}}{P_t}(f^{*g}_{t-1}-f^{*h}_{t-1})-\frac{P^H_t}{P_t}\frac{\psi}{2}\frac{P^H_{t-1}}{P^H_t}(f^{*g}_{t-1}-f^{*h}_{t-1})^2\right]-Q_{t-1}\frac{S_t}{S_{t-1}}\frac{P_{t-1}}{P_t}(f^{*g}_{t-1}-f^{*h}_{t-1}) \tag{38}$$

If the volume of the loan, $(f^{*g}_{t-1} - f^{*h}_{t-1})$, is chosen optimally, the first-order condition is:

$$Q^*_{t-1} = \frac{Q_{t-1}\frac{S_t}{S_{t-1}}}{1-\frac{P^H_{t-1}}{P_{t-1}}\psi(f^{*g}_{t-1}-f^{*h}_{t-1})} \tag{39}$$

Finally, as said, the profit is distributed in lump-sum fashion at the start of period t to domestic households so that $\pi_t = \sum_{i=1}^N \pi_{i,t}$ in equilibrium.

[2] Thus, at the beginning of period t, agents carry over assets and liabilities from period $t-1$. Borrowers honor their preexisting obligations to lenders. In particular, in the international capital market, where transactions take place via the bank, the bank receives interest income from borrowers and pays off the lenders. The latter is the interest payments that the bank promised at $t-1$ to pay at t. The bank also pays the monitoring cost associated with these transactions.

5 Appendix 5: Equilibrium in a monetary union

This Appendix presents the equilibrium system in a monetary union, given feedback policy coefficients. We work in steps.

5.1 Equilibrium equations

The home country is summarized by the following equations:

$$\frac{\partial u_t}{\partial c_t}\frac{\partial c_t}{\partial c_t^H}\frac{P_t}{P_t^H\left(1+\tau_t^c\right)} = \beta\frac{\partial u_{t+1}}{\partial c_{t+1}}\frac{\partial c_{t+1}}{\partial c_{t+1}^H}\frac{P_{t+1}}{P_{t+1}^H\left(1+\tau_{t+1}^c\right)}R_t\frac{P_t}{P_{t+1}} \tag{40}$$

$$\begin{aligned}&\tfrac{\partial u_t}{\partial c_t}\tfrac{\partial c_t}{\partial c_t^H}\tfrac{1}{(1+\tau_t^c)}\tfrac{P_t}{P_t^H}\tfrac{S_tP_t^*}{P_t} = \\ &= \beta\tfrac{\partial u_{t+1}}{\partial c_{t+1}}\tfrac{\partial c_{t+1}}{\partial c_{t+1}^H}\tfrac{1}{\left(1+\tau_{t+1}^c\right)}\tfrac{P_{t+1}}{P_{t+1}^H}Q_t\tfrac{S_{t+1}P_{t+1}^*}{P_{t+1}}\tfrac{P_t^*}{P_{t+1}^*}\end{aligned} \tag{41}$$

$$\begin{aligned}&\tfrac{\partial u_t}{\partial c_t}\tfrac{\partial c_t}{\partial c_t^H}\tfrac{1}{(1+\tau_t^c)}\left\{1+\xi\left(\tfrac{k_t}{k_{t-1}}-1\right)\right\} = \\ &= \beta E_t\tfrac{\partial u_{t+1}}{\partial c_{t+1}}\tfrac{\partial c_{t+1}}{\partial c_{t+1}^H}\tfrac{1}{\left(1+\tau_{t+1}^c\right)}\left\{(1-\delta)-\tfrac{\xi}{2}\left(\tfrac{k_{t+1}}{k_t}-1\right)^2+\xi\left(\tfrac{k_{t+1}}{k_t}-1\right)\tfrac{k_{t+1}}{k_t}+\left(1-\tau_{t+1}^k\right)r_{t+1}^k\right\}\end{aligned} \tag{42}$$

$$\frac{\partial u_t}{\partial m_t} = \frac{\partial u_t}{\partial c_t}\frac{\partial c_t}{\partial c_t^H}\frac{P_t}{P_t^H\left(1+\tau_t^c\right)} - \beta\frac{\partial u_{t+1}}{\partial c_{t+1}}\frac{\partial c_{t+1}}{\partial c_{t+1}^H}\frac{P_{t+1}}{P_{t+1}^H\left(1+\tau_{t+1}^c\right)}\frac{P_t}{P_{t+1}} \tag{43}$$

$$\frac{\partial u_t}{\partial n_t} = \left(1-\tau_t^n\right)w_t\frac{\partial u_t}{\partial c_t}\frac{\partial c_t}{\partial c_t^H}\frac{P_t}{P_t^H\left(1+\tau_t^c\right)} \tag{44}$$

$$\frac{c_t^H}{c_t^F} = \frac{\nu}{1-\nu}\frac{P_t^F}{P_t^H} \tag{45}$$

$$k_t = (1-\delta)k_{t-1} + x_t - \frac{\xi}{2}\left(\frac{k_t}{k_{t-1}}-1\right)^2 k_{t-1} \tag{46}$$

$$c_t = \frac{\left(c_t^H\right)^\nu\left(c_t^F\right)^{1-\nu}}{\nu^\nu(1-\nu)^{1-\nu}} \tag{47}$$

$$w_t = mc_t(1-a)A_tk_{t-1}^an_t^{-a} \tag{48}$$

$$\frac{P_t^H}{P_t}r_t^k = mc_taA_tk_{t-1}^{a-1}n_t^{1-a} \tag{49}$$

$$\widetilde{\omega}_t = \frac{P_t^H}{P_t}y_t^H - \frac{P_t^H}{P_t}r_t^kk_{t-1} - w_tn_t \tag{50}$$

$$\sum_{k=0}^{\infty}\theta^k E_t\ \Xi_{t,t+k}\left[\frac{P_t^{\#}}{P_{t+k}^H}\right]^{-\phi} y_{t+k}^H\left\{\frac{P_t^{\#}}{P_t^H}\frac{P_t^H}{P_t}\frac{P_t}{P_{t-1}}-\frac{\phi}{(\phi-1)}mc_{t+k}\frac{P_t}{P_{t-1}}...\frac{P_{t+k}}{P_{t+k-1}}\right\}=0 \tag{51}$$

$$y_t^H=\frac{1}{\left(\frac{\widetilde{P}_t^H}{P_t^H}\right)^{-\phi}}A_t k_{t-1}^a n_t^{1-a} \tag{52}$$

$$\begin{aligned} b_t+m_t+\tfrac{S_tP_t^*}{P_t}f_t^g &= \tfrac{R_{t-1}b_{t-1}}{\Pi_t}+\tfrac{m_{t-1}}{\Pi_t}+Q_{t-1}\tfrac{S_tP_t^*}{P_t}\tfrac{P_{t-1}^*}{P_t^*}f_{t-1}^g+\\ +\tfrac{P_t^H}{P_t}g_t-\tau_t^c(\tfrac{P_t^H}{P_t}c_t^H&+\tfrac{P_t^F}{P_t}c_t^F)-\tau_t^k(r_t^k\tfrac{P_t^H}{P_t}k_{t-1}+\widetilde{\omega}_t)-\tau_t^n w_t n_t-\tau_t^l \end{aligned} \tag{53}$$

$$y_t^H=c_t^H+x_t+g_t+q_t+c_t^{F*} \tag{54}$$

$$\begin{aligned} &\tfrac{P_t^H}{P_t}(c_t^H+x_t+g_t-y_t^H)+\tfrac{P_t^F}{P_t}c_t^F+Q_{t-1}\tfrac{S_tP_t^*}{P_t}\tfrac{P_{t-1}^*}{P_t^*}\left(f_{t-1}^g-f_{t-1}^h\right)=\\ &=\tfrac{S_tP_t^*}{P_t}\left(f_t^g-f_t^h\right)+\pi_t \end{aligned} \tag{55}$$

$$\left(P_t^H\right)^{1-\phi}=\left[\theta\left(P_{t-1}^H\right)^{1-\phi}+(1-\theta)\left(P_t^{\#}\right)^{1-\phi}\right] \tag{56}$$

$$P_t=(P_t^H)^{\nu}(P_t^F)^{1-\nu} \tag{57}$$

$$P_t^F=S_tP_t^{H*} \tag{58}$$

$$P_t^*=(P_t^{*H})^{\nu^*}(P_t^H/S_t)^{1-\nu^*} \tag{59}$$

$$\left(\widetilde{P}_t^H\right)^{-\phi}=\left[\theta\left(\widetilde{P}_{t-1}^H\right)^{-\phi}+(1-\theta)\left(P_t^{\#}\right)^{-\phi}\right] \tag{60}$$

$$q_t=Q_{t-1}^*\frac{\psi}{2}\frac{P_{t-1}^H}{P_t^H}(f_{t-1}^{*g}-f_{t-1}^{*h})^2 \tag{61}$$

$$\pi_t=Q_{t-1}^*\left[\frac{P_{t-1}}{P_t}(f_{t-1}^{*g}-f_{t-1}^{*h})-\frac{P_t^H}{P_t}\frac{\psi}{2}\frac{P_{t-1}^H}{P_t^H}(f_{t-1}^{*g}-f_{t-1}^{*h})^2\right]-Q_{t-1}\frac{S_t}{S_{t-1}}\frac{P_{t-1}}{P_t}(f_{t-1}^{*g}-f_{t-1}^{*h}) \tag{62}$$

$$Q^*_{t-1} = \frac{Q_{t-1}\frac{S_t}{S_{t-1}}}{1 - \frac{P^H_{t-1}}{P_{t-1}}\psi(f^{*g}_{t-1} - f^{*h}_{t-1})} \tag{63}$$

where $\Xi_{t,t+k} \equiv \beta^k \frac{c^{-\sigma}_{t+k}}{c^{-\sigma}_t}\frac{P_t}{P_{t+k}}\frac{\tau^c_t}{\tau^c_{t+k}}$, $\frac{S_{t+1}}{S_t} = 1$ in a currency union model and recall that $\frac{P^H_t}{P_t} = \left(\frac{P^H_t}{P^F_t}\right)^{1-\nu}$.

The foreign country is summarized by the following equations:

$$\frac{\partial u^*_t}{\partial c^*_t}\frac{\partial c^*_t}{\partial c^{*H}_t}\frac{P^*_t}{P^{*H}_t\left(1+\tau^{*c}_t\right)} = \beta\frac{\partial u^*_{t+1}}{\partial c^*_{t+1}}\frac{\partial c^*_{t+1}}{\partial c^{*H}_{t+1}}\frac{P^*_{t+1}}{P^{*H}_{t+1}\left(1+\tau^{*c}_{t+1}\right)}R^*_t\frac{P^*_t}{P^*_{t+1}} \tag{64}$$

$$\begin{aligned}&\tfrac{\partial u^*_t}{\partial c^*_t}\tfrac{\partial c^*_t}{\partial c^{*H}_t}\tfrac{P^*_t}{P^{*H}_t(1+\tau^{*c}_t)}\tfrac{P_t}{S_tP^*_t} = \\ &= \beta\tfrac{\partial u^*_{t+1}}{\partial c^*_{t+1}}\tfrac{\partial c^*_{t+1}}{\partial c^{*H}_{t+1}}\tfrac{P^*_{t+1}}{P^{*H}_{t+1}\left(1+\tau^{*c}_{t+1}\right)}Q^*_t\tfrac{P_{t+1}}{S_{t+1}P^*_{t+1}}\tfrac{P_t}{P_{t+1}}\end{aligned} \tag{65}$$

$$\begin{aligned}&\tfrac{\partial u^*_t}{\partial c^*_t}\tfrac{\partial c^*_t}{\partial c^{*H}_t}\tfrac{1}{(1+\tau^{*c}_t)}\left\{1+\xi^*\left(\tfrac{k^*_t}{k^*_{t-1}}-1\right)\right\} = \\ &= \beta\tfrac{\partial u^*_{t+1}}{\partial c^*_{t+1}}\tfrac{\partial c^*_{t+1}}{\partial c^{*H}_{t+1}}\tfrac{1}{\left(1+\tau^{*c}_{t+1}\right)}\left\{(1-\delta^*) - \tfrac{\xi^*}{2}\left(\tfrac{k^*_{t+1}}{k^*_t}-1\right)^2 + \xi^*\left(\tfrac{k^*_{t+1}}{k^*_t}-1\right)\tfrac{k^*_{t+1}}{k^*_t} + \left(1-\tau^{*k}_{t+1}\right)r^{*k}_{t+1}\right\}\end{aligned} \tag{66}$$

$$\frac{\partial u^*_t}{\partial m^*_t} = \frac{\partial u^*_t}{\partial c^*_t}\frac{\partial c^*_t}{\partial c^{*H}_t}\frac{P^*_t}{P^{*H}_t\left(1+\tau^{*c}_t\right)} - \beta\frac{\partial u^*_{t+1}}{\partial c^*_{t+1}}\frac{\partial c^*_{t+1}}{\partial c^{*H}_{t+1}}\frac{P^*_{t+1}}{P^{*H}_{t+1}\left(1+\tau^{*c}_{t+1}\right)}\frac{P^*_t}{P^*_{t+1}} \tag{67}$$

$$-\frac{\partial u^*_t}{\partial n^*_t} = \left(1-\tau^{*n}_t\right)w_t\frac{\partial u^*_t}{\partial c^*_t}\frac{\partial c^*_t}{\partial c^{H*}_t}\frac{P^*_t}{P^{*H}_t\left(1+\tau^{*c}_t\right)} \tag{68}$$

$$\frac{c^{*H}_t}{c^{*F}_t} = \frac{\nu^*}{1-\nu^*}\frac{P^{*F}_t}{P^{*H}_t} \tag{69}$$

$$k^*_t = (1-\delta^*)k^*_{t-1} + x^*_t - \frac{\xi^*}{2}\left(\frac{k^*_t}{k^*_{t-1}}-1\right)^2 k^*_{t-1} \tag{70}$$

$$c^*_t = \frac{\left(c^{*H}_t\right)^{\nu^*}\left(c^{*F}_t\right)^{1-\nu^*}}{\nu^{*\nu^*}\left(1-\nu^*\right)^{1-\nu^*}} \tag{71}$$

$$w_t = mc_t^*(1-a^*)A_t^* k_{t-1}^{*a^*} n_t^{*-a^*} \tag{72}$$

$$\frac{P_t^{*H}}{P_t^*} r_t^{*k} = mc_t^* a^* A_t^* k_{t-1}^{*a-1} n_t^{*1-a} \tag{73}$$

$$\widetilde{\omega}_t^* = \frac{P_t^{*H}}{P_t^*} y_t^{*H} - \frac{P_t^{*H}}{P_t^*} r_t^{*k} k_{t-1}^* - \frac{W_t^*}{P_t^*} n_t^* \tag{74}$$

$$\sum_{k=0}^{\infty} (\theta^*)^k \, E_t \, \Xi_{t,t+k}^* \left[\frac{P_t^{*\#}}{P_{t+k}^{*H}}\right]^{-\phi} y_{t+k}^{*H} \left\{ \frac{P_t^{*\#}}{P_t^{*H}} \frac{P_t^{*H}}{P_t^*} \frac{P_t^*}{P_{t-1}^*} - \frac{\phi}{(\phi-1)} mc_{t+k}^* \frac{P_t^*}{P_{t-1}^*} ... \frac{P_{t+k}^*}{P_{t+k-1}^*} \right\} = 0 \tag{75}$$

$$y_t^{H*} = \frac{1}{\left(\frac{\widetilde{P}_t^{H*}}{P_t^{H*}}\right)^{-\phi}} A_t^* k_{t-1}^{*a^*} n_t^{*1-a^*} \tag{76}$$

$$\begin{aligned} b_t^* + m_t^* + \tfrac{P_t}{S_t P_t^*} f_t^{*g} = R_{t-1}^* b_{t-1}^* \tfrac{P_{t-1}^*}{P_t^*} + m_{t-1}^* \tfrac{P_{t-1}^*}{P_t^*} + Q_{t-1}^* \tfrac{P_t}{S_t P_t^*} \tfrac{P_{t-1}}{P_t} f_{t-1}^{*g} + \\ + \tfrac{P_t^{H*}}{P_t^*} g_t^* - \tau_t^{*c}(P_t^{*H} c_t^{*H} + P_t^{*F} c_t^{*F}) - \tau_t^{*k}(r_t^{*k} P_t^{*H} k_{t-1}^* + \widetilde{\omega}_t^*) - \tau_t^{*n} w_t^* n_t^* - \tau_t^{*l} \end{aligned} \tag{77}$$

$$y_t^{*H} = c_t^{*H} + x_t^* + g_t^* + c_t^F \tag{78}$$

$$\begin{aligned} \tfrac{P_t^{*H}}{P_t^*}(c_t^{*H} + x_t^* + g_t^* - y_t^{*H}) + \tfrac{P_t^{*F}}{P_t^*} c_t^{*F} + Q_{t-1}^* \tfrac{P_t}{S_t P_t^*} \tfrac{P_{t-1}}{P_t} \left(f_{t-1}^{*g} - f_{t-1}^{*h}\right) = \\ = \tfrac{P_t}{S_t P_t^*} \left(f_t^{*g} - f_t^{*h}\right) \end{aligned} \tag{79}$$

$$\left(P_t^{*H}\right)^{1-\phi^*} = \left[\theta^* \left(P_{t-1}^{*H}\right)^{1-\phi^*} + (1-\theta^*) \left(P_t^{*\#}\right)^{1-\phi^*}\right] \tag{80}$$

$$\left(\widetilde{P}_t^{*H}\right)^{-\phi^*} = \left[\theta^* \left(\widetilde{P}_{t-1}^{*H}\right)^{-\phi^*} + (1-\theta^*) \left(P_t^{*\#}\right)^{-\phi^*}\right] \tag{81}$$

where see below for number of equations and variables in this system.

5.2 Transformed variables

As in the related literature (see e.g. Schmitt-Grohé and Uribe (2005, 2007)), we transform some variables and introduce some new ones.

First, instead of price levels, we work with inflation rates and relative prices. Thus, we define $\Pi_t \equiv \frac{P_t}{P_{t-1}}$, $\Pi_t^* \equiv \frac{P_t^*}{P_{t-1}^*}$, $\Pi_t^H \equiv \frac{P_t^H}{P_{t-1}^H}$, $\Theta_t \equiv \frac{P_t^\#}{P_t^H}$, $\Delta_t \equiv \left(\frac{\widetilde{P}_t^H}{P_t^H}\right)^{-\phi}$, $\epsilon_t \equiv \frac{S_t}{S_{t-1}}$ and $TT_t \equiv \frac{P_t^F}{P_t^H}$. We also express some policy variables as shares of output. In particular, we define $b_t \equiv s_t^b y_t^H TT_t^{\nu-1}$, $s_t^l \equiv s_t^l y_t^H TT_t^{\nu-1}$ and $g_t \equiv s_t^g y_t^H$. So, in what follows, we use Π_t, Π_t^*, Π_t^H, Θ_t, Δ_t, ϵ_t, TT_t, s_t^g, s_t^l instead of P_t, P_t^*, P_t^H, $P_t^\#$, $\widetilde{P}_t$, S_t, P_t^F, g_t, τ_t^l respectively. Note that we also use (in the steady state only), $f_t^g \equiv s_t^f y_t^H \frac{1}{TT_t^{\nu*}}$.

Second, working as in Schmitt-Grohé and Uribe (2007), we rewrite the firm's optimality conditions in recursive form. In particular, instead of equation (80), we now use:

$$z_t^1 = \frac{\phi}{(\phi - 1)} z_t^2 \tag{82}$$

where

$$z_t^1 = \Theta_t^{1-\phi} y_t T T_t^{\nu-1} + \beta\theta E_t \frac{c_{t+1}^{-\sigma}}{c_t^{-\sigma}} \frac{1+\tau_t^c}{1+\tau_{t+1}^c} \left(\frac{\Theta_t}{\Theta_{t+1}}\right)^{1-\phi} \left(\frac{1}{\Pi_{t+1}^H}\right)^{1-\phi} z_{t+1}^1 \tag{83}$$

$$z_t^2 = \Theta_t^{-\phi} y_t mc_t + \beta\theta E_t \frac{c_{t+1}^{-\sigma}}{c_t^{-\sigma}} \frac{1+\tau_t^c}{1+\tau_{t+1}^c} \left(\frac{\Theta_t}{\Theta_{t+1}}\right)^{-\phi} \left(\frac{1}{\Pi_{t+1}^H}\right)^{-\phi} z_{t+1}^2 \tag{84}$$

thus, we add two more equations and two new endogenous variables, z_t^1 and z_t^2.

Third, again as in Schmitt-Grohé and Uribe (2007), in order to compute expected discounted lifetime utility, denoted as V_t, we add a new equation and a new endogenous variable, V_t:

$$V_t = \frac{c_t^{1-\sigma}}{1-\sigma} - \chi_n \frac{n_t^{1+\varphi}}{1+\varphi} + \chi_m \frac{m_t^{1-\mu}}{1-\mu} + \chi_g \frac{\left(s_t^g y_t^H\right)^{1-\zeta}}{1-\zeta} + \beta E_t V_{t+1} \tag{85}$$

We work similarly for the foreign country. That is, first, we use Π_t^{*H}, Θ_t^*, Δ_t^*, s_t^{*g}, s_t^{*l} instead of P_t^{*H}, $P_t^{*\#}$, $\widetilde{P}_t^*$, g_t, τ_t^{*l} respectively, second, we have for the foreign firm:

$$z_t^{*1} = \frac{\phi}{(\phi - 1)} z_t^{*2} \tag{86}$$

$$z_t^{*1} = \Theta_t^{*1-\phi^*} y_t^{*H} T T_t^{1-\nu^*} + \beta^*\theta^* E_t \frac{c_{t+1}^{*-\sigma^*}}{c_t^{*-\sigma^*}} \frac{1+\tau_t^{*c}}{1+\tau_{t+1}^{*c}} \left(\frac{\Theta_t^*}{\Theta_{t+1}^*}\right)^{1-\phi^*} \left(\frac{1}{\Pi_{t+1}^{*H}}\right)^{1-\phi^*} z_{t+1}^{*1} \tag{87}$$

$$z_t^{*2} = \Theta_t^{*-\phi^*} y_t^{*H} mc_t^* + \beta^*\theta^* E_t \frac{c_{t+1}^{*-\sigma^*}}{c_t^{*-\sigma^*}} \frac{1+\tau_t^{*c}}{1+\tau_{t+1}^{*c}} \left(\frac{\Theta_t^*}{\Theta_{t+1}^*}\right)^{-\phi^*} \left(\frac{1}{\Pi_{t+1}^{*H}}\right)^{-\phi^*} z_{t+1}^{*2} \tag{88}$$

and, thirdly, we have the new value function:

$$V_t^* = \frac{c_t^{*1-\sigma^*}}{1-\sigma^*} - \chi_n^* \frac{n_t^{*1+\varphi^*}}{1+\varphi^*} + \chi_m^* \frac{m_t^{*1-\mu^*}}{1-\mu^*} + \chi_g^* \frac{\left(s_t^{*g} y_t^{*H}\right)^{1-\zeta^*}}{1-\zeta^*} + \beta E_t V_{t+1}^* \tag{89}$$

Finally, given the above, notice that we make use of the following equations:

$$\frac{P_t}{S_t P_t^*} = TT_t^{1-\nu-\nu^*}$$

$$TT_t = \frac{P_t^F}{P_t^H} = \frac{\frac{P_t^F}{S_t}}{\frac{P_t^H}{S_t}} = \frac{P_t^{*H}}{P_t^{*F}}$$

$$\frac{P_t^{*H}}{P_t^*} = \frac{P_t^{*H}}{\left(P_t^{*H}\right)^{\nu^*}\left(P_t^{*F}\right)^{1-\nu^*}} = \left(\frac{P_t^{*H}}{P_t^{*F}}\right)^{1-\nu^*} = TT_t^{1-\nu^*}$$

$$\frac{P_t^{*F}}{P_t^*} = \frac{P_t^{*F}}{\left(P_t^{*H}\right)^{\nu^*}\left(P_t^{*F}\right)^{1-\nu^*}} = \left(\frac{P_t^{*F}}{P_t^{*H}}\right)^{\nu^*} = \left(\frac{1}{TT_t}\right)^{\nu^*}$$

5.3 Final equilibrium system in a monetary union

Using the above, we now present the final equilibrium system (given feedback policy coefficients).

The domestic country is summarized by the following equations:

$$V_t = \frac{c_t^{1-\sigma}}{1-\sigma} - \chi_n \frac{n_t^{1+\varphi}}{1+\varphi} + \chi_m \frac{m_t^{1-\mu}}{1-\mu} + \chi_g \frac{\left(s_t^g y_t^H\right)^{1-\zeta}}{1-\zeta} + \beta E_t V_{t+1} \tag{90}$$

$$\beta E_t \frac{c_{t+1}^{-\sigma}}{\left(1+\tau_{t+1}^c\right)} \frac{R_t}{\Pi_{t+1}} = \frac{c_t^{-\sigma}}{\left(1+\tau_t^c\right)} \tag{91}$$

$$\beta E_t \frac{c_{t+1}^{-\sigma}}{\left(1+\tau_{t+1}^c\right)} \frac{Q_t TT_{t+1}^{v^*+\nu-1}}{\Pi_{t+1}^*} = \frac{c_t^{-\sigma}}{\left(1+\tau_t^c\right)} TT_t^{v^*+\nu-1} \tag{92}$$

$$\beta E_t \frac{c_{t+1}^{-\sigma}}{\left(1+\tau_{t+1}^c\right)} TT_{t+1}^{\nu-1} \left\{1-\delta-\frac{\xi}{2}\left(\frac{k_{t+1}}{k_t}-1\right)^2+\xi\left(\frac{k_{t+1}}{k_t}-1\right)\frac{k_{t+1}}{k_t}+\left(1-\tau_{t+1}^k\right) r_{t+1}^k\right\} = \\ = \frac{c_t^{-\sigma}}{(1+\tau_t^c)} TT_t^{\nu-1}\left[1+\xi\left(\frac{k_t}{k_{t-1}}-1\right)\right] \tag{93}$$

$$\chi_m m_t^{-\mu} = \frac{c_t^{-\sigma}}{(1+\tau_t^c)} - \beta E_t \frac{c_{t+1}^{-\sigma}}{\left(1+\tau_{t+1}^c\right)} \frac{1}{\Pi_{t+1}} \tag{94}$$

$$\chi_n n_t^{\varphi} = \left(1-\tau_t^n\right) w_t \frac{c_t^{-\sigma}}{(1+\tau_t^c)} \tag{95}$$

$$\frac{c_t^H}{c_t^F} = \frac{\nu}{1-\nu} TT_t \tag{96}$$

$$k_t = (1-\delta)k_{t-1} + x_t - \frac{\xi}{2}\left(\frac{k_t}{k_{t-1}}-1\right)^2 k_{t-1} \tag{97}$$

$$c_t = \frac{\left(c_t^H\right)^{\nu}\left(c_t^F\right)^{1-\nu}}{(\nu)^{\nu}(1-\nu)^{1-\nu}} \tag{98}$$

$$w_t = mc_t(1-a)A_t k_{t-1}^a n_t^{-a} \tag{99}$$

$$\frac{1}{TT_t^{1-v}} r_t^k = mc_t a A_t k_{t-1}^{a-1} n_t^{1-a} \tag{100}$$

$$\widetilde{\omega}_t = \frac{1}{TT_t^{1-v}} y_t^H - \frac{1}{TT_t^{1-v}} r_t^k k_{t-1} - w_t n_t \tag{101}$$

$$z_t^1 = \frac{\phi}{(\phi-1)} z_t^2 \tag{102}$$

$$y_t^H = \frac{1}{\Delta_t} A_t k_{t-1}^a n_t^{1-a} \tag{103}$$

$$d_t + m_t = \frac{R_{t-1}}{\Pi_t}\lambda_{t-1} d_{t-1} + \frac{Q_{t-1} TT_t^{v+v^*-1}}{\Pi_t^*} \frac{1}{TT_{t-1}^{v+v^*-1}}(1-\lambda_{t-1}) d_{t-1} + \\ + \frac{1}{\Pi_t} m_{t-1} + TT_t^{\nu-1} s_t^g y_t^H - \tau_t^c\left(\frac{1}{TT_t^{1-v}} c_t^H + TT_t^v c_t^F\right) - \\ - \tau_t^k\left(r_{t-1}^k \frac{1}{TT_t^{1-v}} k_{t-1} + \widetilde{\omega}_t\right) - \tau_t^n w_t n_t - TT_t^{\nu-1} s_t^l y_t^H \tag{104}$$

$$(1-\lambda_t)d_t - TT_t^{\nu^*+\nu-1} f_t^h + \pi_t + TT_t^{\nu-1} q_t = -TT_t^{\nu-1} c_t^{F*} + TT_t^{\nu} c_t^F + \\ + \frac{Q_{t-1} TT_t^{\nu^*+\nu-1}}{\Pi_t^*} \left(\frac{1}{TT_{t-1}^{v+v^*-1}} (1-\lambda_{t-1}) d_{t-1} - f_{t-1}^h \right) \tag{105}$$

$$y_t^H = c_t^H + x_t + s_t^g y_t^H + q_t + c_t^{F*} \tag{106}$$

$$\left(\Pi_t^H\right)^{1-\phi} = \theta + (1-\theta)\left(\Theta_t \Pi_t^H\right)^{1-\phi} \tag{107}$$

$$\frac{\Pi_t}{\Pi_t^H} = \left(\frac{TT_t}{TT_{t-1}}\right)^{1-\nu} \tag{108}$$

$$\frac{TT_t}{TT_{t-1}} = \frac{\epsilon_t \Pi_t^{*H}}{\Pi_t^H} \tag{109}$$

$$\frac{\Pi_t^*}{\Pi_t^{*H}} = \left(\frac{TT_{t-1}}{TT_t}\right)^{1-\nu^*} \tag{110}$$

$$\Delta_t = \theta \Delta_{t-1} \left(\Pi_t^H\right)^{\phi} + (1-\theta)\left(\Theta_t\right)^{-\phi} \tag{111}$$

$$z_t^1 = \Theta_t^{1-\phi} y_t TT_t^{\nu-1} + \beta\theta E_t \frac{c_{t+1}^{-\sigma}}{c_t^{-\sigma}} \frac{1+\tau_t^c}{1+\tau_{t+1}^c} \left(\frac{\Theta_t}{\Theta_{t+1}}\right)^{1-\phi} \left(\frac{1}{\Pi_{t+1}^H}\right)^{1-\phi} z_{t+1}^1 \tag{112}$$

$$z_t^2 = \Theta_t^{-\phi} y_t mc_t + \beta\theta E_t \frac{c_{t+1}^{-\sigma}}{c_t^{-\sigma}} \frac{1+\tau_t^c}{1+\tau_{t+1}^c} \left(\frac{\Theta_t}{\Theta_{t+1}}\right)^{-\phi} \left(\frac{1}{\Pi_{t+1}^H}\right)^{-\phi} z_{t+1}^2 \tag{113}$$

$$q_t = Q_{t-1}^* \frac{\psi}{2} \frac{P_{t-1}^H}{P_t^H} (f_{t-1}^{*g} - f_{t-1}^{*h})^2 \tag{114}$$

$$\pi_t = Q_{t-1}^* \left[\frac{P_{t-1}}{P_t} (f_{t-1}^{*g} - f_{t-1}^{*h}) - \frac{P_t^H}{P_t} \frac{\psi}{2} \frac{P_{t-1}^H}{P_t^H} (f_{t-1}^{*g} - f_{t-1}^{*h})^2 \right] - Q_{t-1} \frac{S_t}{S_{t-1}} \frac{P_{t-1}}{P_t} (f_{t-1}^{*g} - f_{t-1}^{*h}) \tag{115}$$

$$Q^*_{t-1} = \frac{Q_{t-1}\frac{S_t}{S_{t-1}}}{1 - \frac{P^H_{t-1}}{P_{t-1}}\psi(f^{*g}_{t-1} - f^{*h}_{t-1})} \tag{116}$$

Next, the foreign country is summarized by the following equations:

$$V^*_t = \frac{c^{*1-\sigma^*}_t}{1-\sigma^*} - \chi^*_n \frac{n^{*1+\varphi^*}_t}{1+\varphi^*} + \chi^*_m \frac{m^{*1-\mu^*}_t}{1-\mu^*} + \chi^*_g \frac{\left(s^{*g}_t y^{*H}_t\right)^{1-\zeta^*}}{1-\zeta^*} + \beta^* E_t V^*_{t+1} \tag{117}$$

$$\beta^* E_t \frac{c^{*-\sigma}_{t+1}}{\left(1+\tau^{*c}_{t+1}\right)} \frac{R^*_t}{\Pi^*_{t+1}} = \frac{c^{*-\sigma}_t}{(1+\tau^{*c}_t)} \tag{118}$$

$$\beta^* E_t \frac{c^{*-\sigma}_{t+1}}{\left(1+\tau^{*c}_{t+1}\right)} \frac{Q^*_t TT^{1-\nu-v^*}_{t+1}}{\Pi_{t+1}} = \frac{c^{*-\sigma}_t}{(1+\tau^{*c}_t)} TT^{1-\nu-v^*}_t \tag{119}$$

$$\begin{aligned}\beta E_t TT^{1-\nu^*}_{t+1} \frac{c^{*-\sigma}_{t+1}}{\left(1+\tau^{*c}_{t+1}\right)} \left\{1 - \delta^* - \frac{\xi^*}{2}\left(\frac{k^*_{t+1}}{k^*_t} - 1\right)^2 + \xi^*\left(\frac{k^*_{t+1}}{k^*_t} - 1\right)\frac{k^*_{t+1}}{k^*_t} + \left(1-\tau^{*k}_{t+1}\right) r^{*k}_{t+1}\right\} = \\ = TT^{1-\nu^*}_t \frac{c^{*-\sigma}_t}{(1+\tau^{*c}_t)}\left[1 + \xi^*\left(\frac{k^*_t}{k^*_{t-1}} - 1\right)\right]\end{aligned} \tag{120}$$

$$\chi^*_m m^{*-\mu^*}_t = \frac{c^{*-\sigma}_t}{(1+\tau^{*c}_t)} - \beta^* E_t \frac{c^{*-\sigma}_{t+1}}{\left(1+\tau^{*c}_{t+1}\right)} \frac{1}{\Pi^*_{t+1}} \tag{121}$$

$$\chi^*_n n^{*\varphi^*}_t = \left(1 - \tau^{*n^*}_t\right) w^*_t \frac{c^{*-\sigma}_t}{(1+\tau^{*c}_t)} \tag{122}$$

$$\frac{c^{*H}_t}{c^{*F}_t} = \frac{\nu^*}{1-\nu^*}\frac{1}{TT_t} \tag{123}$$

$$k^*_t = (1-\delta^*)k^*_{t-1} + x^*_t - \frac{\xi^*}{2}\left(\frac{k^*_t}{k^*_{t-1}} - 1\right)^2 k^*_{t-1} \tag{124}$$

$$c^*_t = \frac{\left(c^{*H}_t\right)^{\nu^*}\left(c^{*F}_t\right)^{1-\nu^*}}{(\nu^*)^{\nu^*}(1-\nu^*)^{1-\nu^*}} \tag{125}$$

$$w^*_t = mc^*_t(1-a^*)A^*_t k^{*a^*}_{t-1} n^{*-a^*}_t \tag{126}$$

$$TT^{1-v^*}_t r^{*k}_t = mc^*_t a^* A^*_t k^{*a^*-1}_{t-1} n^{*1-a^*}_t \tag{127}$$

$$\widetilde{\omega}^*_t = TT^{1-v^*}_t y^{*H}_t - TT^{1-v^*}_t r^{*k}_t k^*_{t-1} - w^*_t n^*_t \tag{128}$$

$$z_t^{*1} = \frac{\phi^*}{(\phi^* - 1)} z_t^{*2} \tag{129}$$

$$y_t^{*H} = \frac{1}{\Delta_t^*} A_t^* k_{t-1}^{*a^*} n_t^{*1-a^*} \tag{130}$$

$$\begin{aligned} d_t^* + m_t^* = & \tfrac{R_{t-1}^*}{\Pi_t^*} \lambda_{t-1}^* d_{t-1}^* + \tfrac{Q_{t-1}^* TT_t^{1-v-v^*}}{\Pi_t} \tfrac{1}{TT_{t-1}^{1-v-v^*}} (1 - \lambda_{t-1}^*) d_{t-1}^* + \\ & + \tfrac{1}{\Pi_t^*} m_{t-1}^* + TT_t^{1-\nu^*} s_t^{*g} y_t^{*H} - \tau_t^{*c} (TT_t^{1-v^*} c_t^{*H} + \tfrac{1}{TT_t^{v^*}} c_t^{*F}) - \\ & - \tau_t^{*k} (r_{t-1}^{*k} TT_t^{1-v^*} k_{t-1}^* + \widetilde{\omega}_t^*) - \tau_t^{*n} w_t^* n_t^* - s_t^{*l} y_t^{*H} TT_t^{1-\nu^*} \end{aligned} \tag{131}$$

$$y_t^{*H} = c_t^{*H} + x_t^* + s_t^{*g} y_t^{*H} + c_t^F \tag{132}$$

$$\begin{aligned} (1 - \lambda_t^*) d_t^* - TT_t^{1-\nu^*-\nu} f_t^{*h} = & -TT_t^{1-\nu^*} c_t^F + TT_t^{-\nu^*} c_t^{F^*} \\ & + \tfrac{Q_{t-1}^* TT_t^{1-\nu^*-\nu}}{\Pi_t} \left(\tfrac{1}{TT_{t-1}^{1-v-v^*}} (1 - \lambda_{t-1}^*) d_{t-1}^* - f_{t-1}^{h^*} \right) \end{aligned}$$

$$\left(\Pi_t^{*H}\right)^{1-\phi^*} = \theta^* + (1 - \theta^*) \left(\Theta_t^* \Pi_t^{*H}\right)^{1-\phi^*} \tag{133}$$

$$\Delta_t^* = \theta^* \Delta_{t-1}^* \left(\Pi_t^{*H}\right)^{\phi^*} + (1 - \theta^*) \left(\Theta_t^*\right)^{-\phi^*} \tag{134}$$

$$z_t^{*1} = \Theta_t^{*1-\phi^*} y_t^{*H} TT_t^{1-\nu^*} + \beta^* \theta^* E_t \frac{c_{t+1}^{*-\sigma^*}}{c_t^{*-\sigma^*}} \frac{1 + \tau_t^{*c}}{1 + \tau_{t+1}^{*c}} \left(\frac{\Theta_t^*}{\Theta_{t+1}^*} \right)^{1-\phi^*} \left(\frac{1}{\Pi_{t+1}^{*H}} \right)^{1-\phi^*} z_{t+1}^{*1} \tag{135}$$

$$z_t^{*2} = \Theta_t^{*-\phi^*} y_t^{*H} mc_t^* + \beta^* \theta^* E_t \frac{c_{t+1}^{*-\sigma^*}}{c_t^{*-\sigma^*}} \frac{1 + \tau_t^{*c}}{1 + \tau_{t+1}^{*c}} \left(\frac{\Theta_t^*}{\Theta_{t+1}^*} \right)^{-\phi^*} \left(\frac{1}{\Pi_{t+1}^{*H}} \right)^{-\phi^*} z_{t+1}^{*2} \tag{136}$$

where $S_t F^g = (1 - \lambda_t) D_t$, $F^g = \frac{(1-\lambda_t) D_t}{S_t}$, $\frac{F^g}{P_t^*} = \frac{(1-\lambda_t) D_t}{P_t^* S_t}$, $f_t^g = (1 - \lambda_t) d_t \frac{P_t}{P_t^* S_t} = (1 - \lambda_t) d_t \frac{1}{TT_t^{\nu^*+\nu-1}}$,

$\frac{F^{*g}}{S_t} = (1 - \lambda_t^*) D_t^*$, $F^{*g} = (1 - \lambda_t) D_t^* S_t$, $\frac{F^{*g}}{P_t} = \frac{(1-\lambda_t^*) D_t^* S_t}{P_t}$, $f_t^{*g} = (1 - \lambda_t^*) d_t^* \frac{S_t P_t^*}{P_t} = (1 - \lambda_t^*) d_t^* TT_t^{\nu^*+\nu-1}$,

$\frac{P_{t-1}^H}{P_{t-1}} = TT_{t-1}^{\nu-1}$, $\frac{S_t P_t^*}{P_t} = TT_t^{\nu^*+\nu-1}$, $\frac{P_t}{S_t P_t^*} = TT_{t+1}^{1-\nu-v^*}$, $\frac{P_t^{*H}}{P_t^*} = \left(\frac{P_t^{*H}}{P_t^{*F}}\right)^{1-\nu^*}$, $TT_t = \frac{P_t^F}{P_t^H} =$

$\frac{\frac{P_t^F}{S_t}}{\frac{P_t^H}{S_t}} = \frac{P_t^{*H}}{P_t^{*F}}$, $\frac{P_t^{*F}}{P_t^{*H}} = \frac{1}{TT_t}$, $\epsilon_t = \frac{S_t}{S_{t-1}}$.

We finally have the feedback monetary and fiscal policy rules:

$$\begin{aligned}\log\left(\frac{R_t}{R}\right) &= \phi_\pi\left(\eta\log\left(\frac{\Pi_t}{\Pi}\right)+(1-\eta)\log\left(\frac{\Pi_t^*}{\Pi^*}\right)\right)+\\ &\quad +\phi_y\left(\eta\log\left(\frac{y_t^H}{y^H}\right)+(1-\eta)\log\left(\frac{y_t^{*H}}{y^{*H}}\right)\right)\end{aligned} \tag{137}$$

$$s_t^g - s^g = -\gamma_l^g\left(l_{t-1}-l\right) - \gamma_y^g\left(y_t^H - y^H\right) \tag{138}$$

$$\tau_t^c - \tau^c = \gamma_l^c\left(l_{t-1}-l\right) + \gamma_y^c\left(y_t^H - y^H\right) \tag{139}$$

$$\tau_t^k - \tau^k = \gamma_l^k\left(l_{t-1}-l\right) + \gamma_y^k\left(y_t^H - y^H\right) \tag{140}$$

$$\tau_t^n - \tau^n = \gamma_l^n\left(l_{t-1}-l\right) + \gamma_y^n\left(y_t^H - y^H\right) \tag{141}$$

$$s_t^{*g} - s^{*g} = -\gamma_l^{*g}\left(l_{t-1}^*-l^*\right) - \gamma_y^{*g}\left(y_t^{*H} - y^{*H}\right) \tag{142}$$

$$\tau_t^{*c} - \tau^{*c} = \gamma_l^{*c}\left(l_{t-1}^*-l^*\right) + \gamma_y^{*c}\left(y_t^{*H} - y^{*H}\right) \tag{143}$$

$$\tau_t^{*k} - \tau^{*k} = \gamma_l^{*k}\left(l_{t-1}^*-l^*\right) + \gamma_y^{*k}\left(y_t^{*H} - y^{*H}\right) \tag{144}$$

$$\tau_t^{*n} - \tau^{*n} = \gamma_l^{*n}\left(l_{t-1}^*-l^*\right) + \gamma_y^{*n}\left(y_t^{*H} - y^{*H}\right) \tag{145}$$

$$l_t = \frac{R_t \lambda_t d_t + Q_t \epsilon_{t+1} (1 - \lambda_t) d_t}{TT_t^{\nu-1} y_t^H} \tag{146}$$

$$l_t^* = \frac{R_t^* \lambda_t^* d_t^* + Q_t^* \frac{1}{\epsilon_{t+1}} (1 - \lambda_t^*) d_t^*}{TT_t^{1-\nu^*} y_t^{*H}} \tag{147}$$

Therefore, we have 27 equations for the home country, 21 equations for the foreign country and 11 equations for the policy reules. This is 59 equations in total. We also have 59 endogenous variables, which are $\{V, y^H, c, c^H, c^F, n, x, k, f^h, m, TT, \Pi, \Pi^H, \Theta, \Delta, w, mc, \widetilde{\omega}, r^k, d, \Pi^*, z^1, z^2, Q, \pi, q\}$ and $\{R, s^g, \tau^c, \tau^k, \tau^n, l\}$ for the home country, and $\{V^*, y^{*H}, c^*, c^{*H}, c^{*F}, n^*, x^*, k^*, f^{*h}, m^*, \Pi^{*H}, \Theta^*, \Delta^*, w^*, mc^*, \widetilde{\omega}^*, r^{*k}, d^*, z^{*1}, z^{*2}, Q^*, R^*\}$ and $\{s^{*g}, \tau^{*c}, \tau^{*k}, \tau^{*n}, l^*\}$ for the foreign country. This is given given the exogenous variables, $\{\epsilon, \lambda, s^l, \lambda^*, s^{*l}, A, A^*\}$, initial conditions for the state variables and the values of the feedback (monetary and fiscal) policy coefficients in the policy rules.

Notice that, since all market-clearing conditions have been already included, the above system also satisfies the international asset market-clearing condition, $(f^{*g} - f^{*h}) + \frac{S_t P_t^*}{P_t}(f^g - f^h) = 0$. This can be seen if we add up the two balance of payments above; this will give $(f^{*g} - f^{*h}) + \frac{S_t P_t^*}{P_t}(f^g - f^h) = 0$ residually.

5.4 Steady state and transition in a monetary union

The steady state system follows directly from the above defined system when variables do not change over time. At steady state, regarding monetary policy, we set $\Pi = \Pi^* = 1$ and let the nominal interest rates to follow residually from the Euler for bonds in each country. Regarding fiscal policy, the residual policy instrument is total public debt in each country. To get the transition path, we approximate the dynamic system around its steady state solution, as explained in the main text. In this regime, transition dynamics are driven by shocks.

6 Appendix 6: Flexible exchange rates

The equilibrium system with flexible exchange rates is the same as that with a currency union except that ϵ and R^* exchange places, which means that now ϵ becomes endogenous while R^* follows a Taylor-type rule as R does. To get the steady state system and the transition path, we work as said above. Since money is neutral in the steady state, the steady state solution is the same as in the currency union regime (at least for real variables and welfare).

7 Appendix 7: Monetary plus fiscal (transfer) union

This Appendix presents some details on the fiscal (transfer) union regime. As said, we distinguish two types of interregional transfers.

7.1 Transfers as insurance

The benchmark is the currency union system. In the case of transfers as insurance, the balance of payments in Germany changes to:

$$\begin{gathered}(1-\lambda_t)d_t - TT_t^{\nu^*+\nu-1}f_t^h + \pi_t + TT_t^{\nu-1}q_t = -TT_t^{\nu-1}c_t^{F*} + TT_t^{\nu}c_t^F + \\ +\frac{Q_{t-1}TT_t^{\nu^*+\nu-1}}{\Pi_t^*}\left(\frac{1}{TT_{t-1}^{v+v^*-1}}(1-\lambda_{t-1})d_{t-1} - f_{t-1}^h\right) + \gamma\left(y^{*H} - y_t^{*H}\right)\frac{P_t^{*H}}{P_t^*} - \gamma\left(y^H - y_t^H\right)\frac{P_t^H}{P_t}\end{gathered} \tag{148}$$

and the balance of payments in Italy changes to:

$$\begin{gathered}(1-\lambda_t^*)d_t^* - TT_t^{1-\nu^*-\nu}f_t^{*h} = -TT_t^{1-\nu^*}c_t^F + TT_t^{-\nu^*}c_t^{F^*} \\ +\frac{Q_{t-1}^*TT_t^{1-\nu^*-\nu}}{\Pi_t}\left(\frac{1}{TT_{t-1}^{1-v-v^*}}(1-\lambda_{t-1}^*)d_{t-1}^* - f_{t-1}^{h^*}\right) - \gamma\left(y^{*H} - y_t^{*H}\right)\frac{P_t^{*H}}{P_t^*} + \gamma\left(y^H - y_t^H\right)\frac{P_t^H}{P_t}\end{gathered}$$

To get the steady state system and the transition path, we work as said above. Since shocks and hence insurance transfers are temporary, the steady state solution remains the same as in the currency union regime.

7.2 Transfers as redistribution

Again, the benchmark is the currency union system. In the case of transfers as redistribution, the balance of payments in Germany changes to:

$$\begin{gathered}(1-\lambda_t)d_t - TT_t^{\nu^*+\nu-1}f_t^h + \pi_t + TT_t^{\nu-1}q_t = -TT_t^{\nu-1}c_t^{F*} + TT_t^{\nu}c_t^F + \\ +\frac{Q_{t-1}TT_t^{\nu^*+\nu-1}}{\Pi_t^*}\left(\frac{1}{TT_{t-1}^{v+v^*-1}}(1-\lambda_{t-1})d_{t-1} - f_{t-1}^h\right) + \gamma\left(y_t^{union} - y_t^{*H}\frac{P_t^{*H}}{P_t^*}\right)\end{gathered} \tag{149}$$

and the balance of payments in Italy changes to:

$$\begin{gathered}(1-\lambda_t^*)d_t^* - TT_t^{1-\nu^*-\nu}f_t^{*h} = -TT_t^{1-\nu^*}c_t^F + TT_t^{-\nu^*}c_t^{F^*} \\ +\frac{Q_{t-1}^*TT_t^{1-\nu^*-\nu}}{\Pi_t}\left(\frac{1}{TT_{t-1}^{1-v-v^*}}(1-\lambda_{t-1}^*)d_{t-1}^* - f_{t-1}^{h^*}\right) - \gamma\left(y_t^{union} - y_t^{*H}\frac{P_t^{*H}}{P_t^*}\right)\end{gathered}$$

If, in addition, we have moral hazard effects, the Euler equation for capital and the labor supply condition in the receiving country change to:

$$\beta E_t T T_{t+1}^{1-\nu^*} \frac{c_{t+1}^{*-\sigma}}{\left(1+\tau_{t+1}^{*c}\right)} \left\{ \begin{array}{c} 1-\delta^* - \frac{\xi^*}{2}\left(\frac{k_{t+1}^*}{k_t^*}-1\right)^2 + \xi^*\left(\frac{k_{t+1}^*}{k_t^*}-1\right)\frac{k_{t+1}^*}{k_t^*} + \\ \left(1-\tau_{t+1}^{*k}\right) r_{t+1}^{*k} - \frac{P_{t+1}^{*H}}{P_{t+1}^*}\gamma^{IT}\frac{1}{\Delta_{t+1}^*}a^* A_{t+1}^* k_t^{*a^*-1} n_{t+1}^{*1-a^*} \end{array} \right\} = \\ = T T_t^{1-\nu^*} \frac{c_t^{*-\sigma}}{\left(1+\tau_t^{*c}\right)} \left[1+\xi^*\left(\frac{k_t^*}{k_{t-1}^*}-1\right)\right] \tag{150}$$

$$\chi_n^* n_t^{*\varphi^*} = \frac{c_t^{*-\sigma}}{\left(1+\tau_t^{*c}\right)}[\left(1-\tau_t^{*n^*}\right) w_t^* - \frac{P_t^{*H}}{P_t^*}\gamma^{IT}\frac{1}{\Delta_t^*}(1-a^*)A_t^* k_{t-1}^{*a^*} n_t^{*-a^*}] \tag{151}$$

To get the steady state system and the transition path, we work as said above. Since redistributive transfers are systematic, the steady state solution differs from that of a currency union. Numerical steady state solutions with redistributive transfers (without, and with, moral hazard) are reported right below in the next subsection.

7.3 Steady state solutions with transfers as redistribution

Table A1: Steady state solution with transfers as redistribution without moral hazard effects

Variables	Description	Home	Foreign
$u,\ u^*$	steady-state utility	0.0326	0.0366
y^H, y^{H*}	output	0.392064	0.353538
c, c^*	consumption	0.230406	0.22882
$n,\ n^*$	hours worked	0.312246	0.305637
k, k^*	capital	0.666854	0.496561
$w,\ w^*$	real wage rate	0.696103	0.709986
$r^k,\ r^{k*}$	real return to physical capital	0.146983	0.177993
$Q^* - Q$	interest rate premium	-	0.0055
$\frac{c}{y^H TT^{1-\nu}},\ \frac{c^*}{y^{H*} TT_t^{\nu^*-1}}$	consumption as share of GDP	0.5585	0.6810
$\frac{k}{y^H},\ \frac{k^*}{y^{H*}}$	physical capital as share of GDP	1.7009	1.4045
$TT_t^{\nu^*} \frac{f_t^h}{y_t^H},\ \frac{1}{TT_t^{\nu}} \frac{f_t^{*h}}{y_t^{*H}}$	private foreign assets as share of GDP	0.2442	0.2190
$\frac{d}{TT^{\nu-1} y^H},\ \frac{d^*}{TT^{1-\nu^*} y^{*H}}$	total public debt as share of GDP	0.6609	1.1075
$\frac{TT_t^{\nu^*}\left(\frac{(1-\lambda)d}{TT^{\nu+\nu^*-1}} - f^h\right)}{y^H},\ \frac{\frac{(1-\lambda^*)d^*}{TT^{1-\nu-\nu^*}} - f^{*h}}{TT_t^{\nu} y^{*H}}$	total foreign debt as share of GDP	0.211749	-0.0592009

Notes: see Tables 1a-c for parameter values and exogenous policy instruments.

Table A2: Steady state solution with transfers as redistribution with moral hazard effects

Variables	Description	Home	Foreign
$u,\ u^*$	steady-state utility	-0.0169	0.0159
y^H, y^{H*}	output	0.392997	0.316356
c, c^*	consumption	0.219549	0.220082
$n,\ n^*$	hours worked	0.312989	0.285588
k, k^*	capital	0.66844	0.401661
$w,\ w^*$	real wage rate	0.664878	0.711848
$r^k,\ r^{k*}$	real return to physical capital	0.146983	0.196905
$Q^* - Q$	interest rate premium	-	0.0055
$\frac{c}{y^H TT^{1-\nu}},\ \frac{c^*}{y^{H*} TT_t^{\nu^*-1}}$	consumption as share of GDP	0.5071	0.7664
$\frac{k}{y^H},\ \frac{k^*}{y^{H*}}$	physical capital as share of GDP	1.7009	1.2696
$TT_t^{\nu^*} \frac{f_t^h}{y_t^H},\ \frac{1}{TT_t^{\nu}} \frac{f_t^{*h}}{y_t^{*H}}$	private foreign assets as share of GDP	0.0906	0.2438
$\frac{d}{TT^{\nu-1} y^H},\ \frac{d^*}{TT^{1-\nu^*} y^{*H}}$	total public debt as share of GDP	0.6275	1.2355
$\frac{TT_t^{\nu^*}\left(\frac{(1-\lambda)d}{TT^{\nu+\nu^*-1}} - f^h\right)}{y^H},\ \frac{\frac{(1-\lambda^*)d^*}{TT^{1-\nu-\nu^*}} - f^{*h}}{TT_t^{\nu} y^{*H}}$	total foreign debt as share of GDP	0.0850653	0.236636

Notes: see Tables 1a-c for parameter values and exogenous policy instruments.

7.4 Transfers as redistribution (sensitivity analysis)

Here we report results when γ rises to $\gamma = 0.1$. This implies that transfers from Germany to Italy, as share of Italian GDP, are equal to 0.2%.

Table A3: Expected discounted lifetime utility (welfare) under different regimes (γ=0.1)

Regimes	Welfare		
	Germany	Italy	Union-wide
Monetary union	2.2554	1.4317	1.8439
Independent mon. policies	2.2554	1.4318	1.8439
Mon. union plus transfers as insurance	2.2676	1.4328	1.8505
Mon. union plus transfers as redistribution	1.9704	2.1012	2.0357
Mon. union plus transfers as redistribution (with moral hazard)	-4.4050	1.2640	-1.5727