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Demonstration of an optical spring in the 100 g mirror regime

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Abstract

Optical rigidity will play an important role in future generations of gravitational wave (GW) interferometers which employ high laser power in order to reach and exceed the standard quantum limit. Several experiments have demonstrated the optical spring effect for very low weight mirror masses. In this paper we extend this to a mass and frequency regime more directly applicable to GW detectors. Using a 100 g end mirror mass we demonstrate an optical spring resonant at 500 Hz and a stiffness of $9.4 \times 10^5 \text{ N m}^{-1}$. The 100 g to 1 kg mass regime may also be useful for the application as a readout mirror for optical bar or optical lever configurations.

Keywords: optical spring, interferometry, gravitational wave

(Some figures may appear in colour only in the online journal)

1. Introduction

The first generation of laser-interferometric gravitational wave (GW) detectors (LIGO [1], Virgo [2], GEO 600 [3] and Tama [4]) have successfully demonstrated the long-term operation km-scale Michelson interferometers, delivering observational data of unprecedented strain sensitivity. The next generation of GW detectors is currently under construction and commissioning (advanced LIGO [5], GEO-HF [6], KAGRA [7] and advanced Virgo [8]) and is expected to achieve about a ten-fold increased sensitivity. Over most of their frequency



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range these advanced instruments will be limited by so-called quantum noise, which consists of photon shot noise at the high-frequency end and photon radiation pressure noise at the low frequency of the observing band.

Optical spring techniques [9] have various applications in further reducing quantum noise, and in the long term future even in surpassing the standard quantum limit (SQL) [10]. Detuned signal recycling [11] produces an optical spring, and the resulting change to the opto-mechanical dynamics of the interferometer provides one example of how the SQL may be beaten [12]. Such an optical spring resonance with detuned signal recycling was observed at the Caltech 40 m prototype [13]. Other interferometer topologies based on the principle of optical rigidity include optical bar [14], optical lever [15] and local readout configurations [16]. All of these techniques are currently under consideration [17, 18] for upgrades of Advanced LIGO or third generation observatories such as ET [19, 20].

In this article we describe the demonstration of an optical spring in a three-mirror coupled cavity. Our experiments extend the range in which optical springs have been demonstrated for Fabry-Pérot cavities, from 1 g [21] up to a mass range of 100 g and to a 10 m prototype-system scale, bringing it therefore one step closer to application in future GW observatories.

2. Optical spring generation

To observe opto-mechanical coupling between the mirrors of an optical cavity, the radiation pressure force exerted by the internal cavity field must be comparable to, or greater than, the mechanical restoring force of the pendulums [9]. Since the radiation pressure force is inherently weak, this requires high-finesse cavities to maximise the amount of stored light. In our system two cavities were combined in a three-mirror coupled cavity configuration, analogous to that of the power recycling and arm cavities used in GW detectors, to form a coupled system of high-finesse and allowing high power build up in the ‘arm cavity (AC)’ (see figure 1).

As the stored light in a three mirror coupled cavity interacts with the mirrors, it imparts a radiation pressure force and modifies the dynamics of the suspended mirrors. We relate the cavity length, or more specifically its detuning, to the optical power with coupling considered to be optimal when there is maximum power in the AC. The steady power in the AC P_0 is given by

$$P_0 = \left| \frac{\tau_c}{1 + \rho_c \rho_{ab} e^{-2i\varphi}} \right|^2 \left(\frac{\tau_a}{1 - \rho_a \rho_b} \right)^2 P_{\text{laser}}, \quad (1)$$

where $\tau_{(c,a)}$ are the transmission efficiencies of the recycling mirror and input cavity mirror respectively, $\rho_{(c,a,b)}$ are the reflection efficiencies of the recycling mirror, input and end cavity mirrors respectively, ρ_{ab} is the static reflectivity of the cavity as seen by the incoming light, defined by

$$\rho_{ab} = \rho_a - \frac{\tau_a^2 \rho_b e^{-2i\varphi}}{1 - \rho_a \rho_b e^{-2i\varphi}} \quad (2)$$

and P_{laser} is the laser power impinging on the recycling cavity mirror. If we arrange that the recycling cavity is exactly on resonance and held such that the power recycling cavity resonance condition is decoupled from the AC resonance [25], equation (1) can then be expressed in terms of the AC detuning θ , as

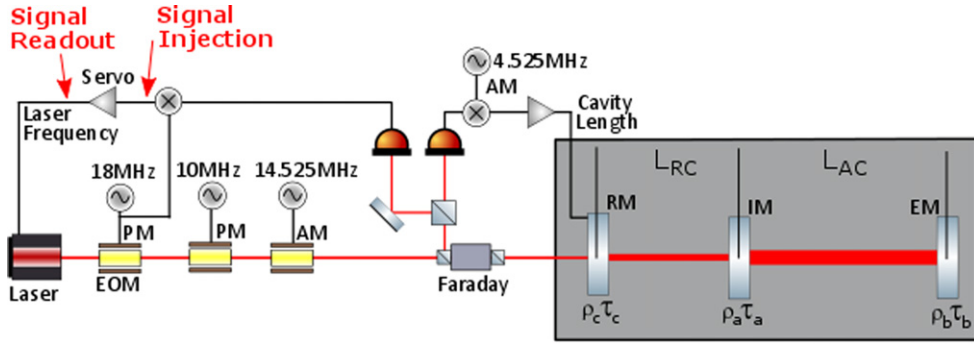


Figure 1. Schematic diagram of two co-aligned cavities.

Table 1. System properties for the Glasgow 10 m prototype in the optical spring experiment. Mirror reflectivity and transmission values are based on specification from manufacture along with confirmation from modelling during experimental setup with measurements of cavity finesse and transmitted light power.

Parameter	Value	Description
τ_c^2	0.04950	Recycling mirror transmission
τ_a^2	0.01228	Input mirror transmission
τ_b^2	0.00002	End mirror transmission
ρ_c^2	0.95045	Recycling mirror reflectivity
ρ_a^2	0.98772	Input mirror reflectivity
ρ_b^2	0.99994	End mirror reflectivity
M_c	1.5 kg	Recycling mirror mass
M_a	2.7 kg	Input mirror mass
M_b	0.1 kg	End mirror mass
L_{RC}	5.16 m	Recycling cavity length
L_{AC}	9.87 m	Arm length
γ_{RC}	29.05 MHz	Recycling cavity linewidth
γ_{AC}	31.88 kHz	Arm cavity linewidth
P_{laser}	0.15 W	Input power
λ	1064 nm	Laser light wavelength

$$P_0 = g_{ca}^2 g_{ab}^2 P_{laser} \frac{1}{1 + F_c \sin^2 \theta}, \quad (3)$$

where F_c is the coefficient of finesse for the coupled cavity system and g_{ca} and g_{ab} are the amplitude gains of the recycling and arm cavities respectively. The recycling cavity can thus be considered simply as an additional source of power gain and the system is reduced in complexity such that the optomechanical coupling behaves like that of a simple cavity.

Such systems have been considered in detail [9, 26] and the dynamics of a detuned cavity of linewidth, γ , are well known. The quasi-static optical spring constant for such systems (where the optical spring frequency is $f_{opt} \ll \gamma$), can be defined as

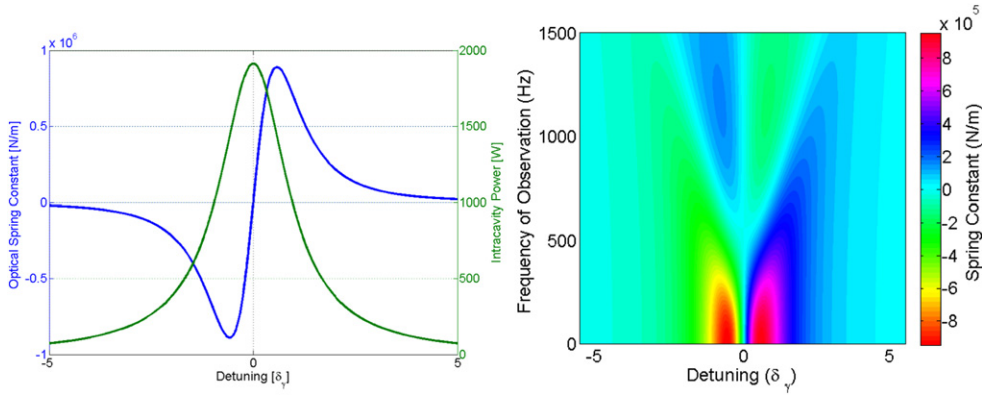


Figure 2. (Left) Calculated intracavity power (green line) and the optical spring constant (blue line) as a function of detuning for a coupled Fabry–Pérot cavity defined by the parameters in table 1. (Right) Optical spring strength as a function of cavity detuning and frequency of observation, based on the complete frequency dependent spring strength equation (5).

$$K_{\text{opt}} = -\frac{2}{c} \frac{dP}{dL} = \frac{8\pi P_0 F_c \theta}{c\lambda(1 + F_c \theta^2)^2}. \quad (4)$$

However, if we consider our experimental parameters (table 1), the expected AC power build up is around 2 kW, providing a resonant radiation pressure force on the cavity end mirror $F_{\text{RP}} \approx 13 \mu\text{N}$. It follows that the maximum optical spring strength obtained for a cavity detuning of $\delta_\gamma \approx 0.29$ (detuning parameter expressed as fractional linewidth) is $K = 9.4 \times 10^5 \text{ N m}^{-1}$ with a corresponding optical spring frequency of $f_{\text{opt}} = 496 \text{ Hz}$. Using the same parameters the linewidth of the coupled cavity is 3305 Hz) and is therefore sufficiently narrow that any measurement process to obtain the optomechanical response will induce motion at comparable frequencies. Hence, the assumption of quasi-static detuning is not sufficient for the experimental system under consideration and the cavity response time must be included in the analysis.

The frequency dependent optical rigidity has been studied [9, 27], and upon simplification can be shown to be

$$K_{\text{opt}}(\omega) = K_{\text{opt}} \frac{1 + \delta_\gamma^2}{(1 + i\omega\gamma)^2 + \delta_\gamma^2}. \quad (5)$$

The complete description of the frequency dependent optical rigidity is complex with the real components attributed to rigidity and imaginary parts describing a velocity dependent damping force. The full form of the optical spring constant is valid for all mirror frequencies, and we see that for sufficiently slow mirror motion ($\omega \rightarrow 0$) the imaginary term disappears resulting in the expression obtained earlier for static detuning.

Figure 2 (right) shows the expected change to the strength of the optical spring constant with respect to detuning and frequency of observation, based on the numerical model. At frequencies greater than the cavity linewidth the spring constant is predicted to change sign. In other words there is a fundamental limit to the observation of an optical spring that is directly related to the response time of the cavity.

3. Description of the experimental setup

3.1. Optical layout of three-mirror coupled cavity

The three-mirror coupled cavity configuration was installed in the Glasgow 10 m prototype interferometer laboratory. Each mirror was suspended as a multi-stage pendulum to provide isolation from seismic vibrations, and housed in a vacuum system to eliminate the effects of acoustic noise and air-currents.

The Glasgow system typically houses 2.7 kg mirrors (test masses) and utilises lower circulating power such that the effects from radiation pressure are negligible, but in this experiment we wanted to accentuate the spring effect. To this end, the AC end mirror was reduced to 100 g to bring the optical spring resonance into an appropriate frequency band to allow unambiguous observation (around 500 Hz) and extend the mass range from those explored in previous experiments [9, 22, 23].

The cavity mirrors were suspended as triple pendulums, providing considerable isolation against ground motion above the fundamental longitudinal pendulum resonance (at around 0.6 Hz). Through careful design choices regarding masses, wire thickness, length and positioning of break-off points it was possible to keep the rigid body modes below 30 Hz, leaving an unobstructed region near the target optical spring frequency range at around a few hundred Hz.

3.2. Coupled cavity control scheme

It is necessary to control the relative positions of the mirrors to maintain the resonant condition for maximum power build up in the AC. In this experiment it was desirable to decouple the length sensing signals from the two cavities and so a scheme was implemented whereby the AC and power recycling cavity length sensing and control signals were separately detected using a combination of the conventional Pound Drever Hall approach and a doubly resonant amplitude-modulated sub-carrier. This orthogonalised control scheme allows for large deviations of each cavity from the centre of resonance, while maintaining decoupling between the coupled cavity control signals. A detailed explanation of the approach used is described in [25].

To introduce detuning, and hence create an optical spring, a DC offset was introduced at the error point of the AC loop, producing a static frequency offset between the laser and the AC. As the AC feedback control is provided by the laser frequency stabilisation loop and has a high bandwidth (100 kHz), this loop maintains the lock and thus ensures stability of the system during detuning.

To measure the response of the optical spring to a perturbation of the system, a test signal was added to the DC offset in the AC control loop. The effect of the optomechanical spring can subsequently be observed by measuring the cavity transfer function.

It should be noted that both the optical spring strength and cavity control signals are dependant on the optical power stored in the cavity. Thus any effect which can alter the stored power during the course of the measurement, such as long-term drifts in cavity alignment or Sidle–Sigg effects [28], will cause fluctuations away from the intended detuning as determined by the DC offset. From observation of power changes on the stored light power during measurements the detuning uncertainty is approximately $0.03 \delta_\gamma$.

Figure 3 presents the largest observed optical spring frequency, f_{opt} , located at around 496 Hz. As the reduced mass of the coupled two mirror system is known, $m_r = 96.4$ g, we can determine the rigidity for an optical spring at this frequency by

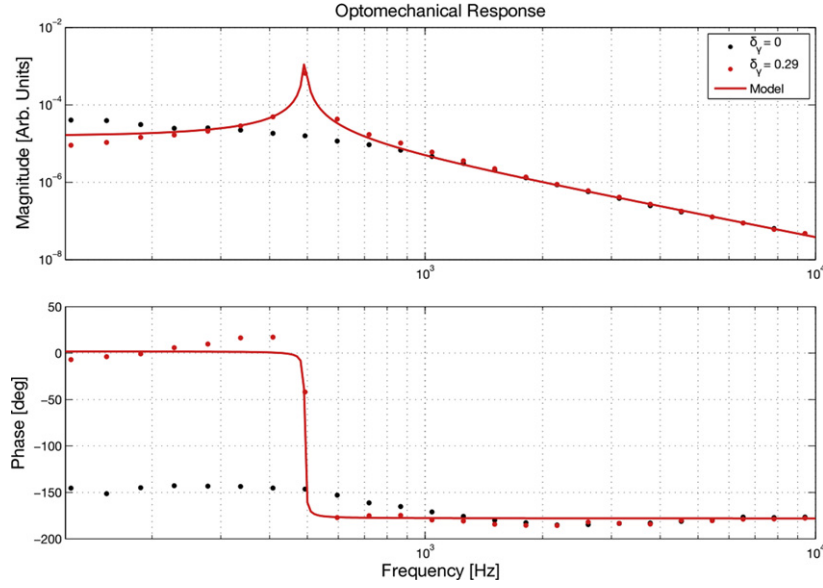


Figure 3. Full-span frequency response of the coupled arm cavity for no detuning (black) and at maximum detuning (red) before the cavity would fall out of lock. A clear resonance feature at approximately 500 Hz is observed with the phase plot below indicating a distinct 180° phase flip at the optical spring frequency. The measured optical spring feature and frequency agree well with the modelled response for equivalent detuning (red dashed).

$$K = \left(2\pi f_{\text{opt}}\right)^2 m_r = 9.4 \times 10^5 \text{ N m}^{-1}. \quad (6)$$

Furthermore, using a beam analyser directly after the input mirror, the approximate beam area was found to be $A = 8.24 \mu\text{m}^2$. By neglecting the expansion of the beam inside the cavity, we can attribute an effective Young's modulus to the stiffness of the light coupling the two cavity mirrors as

$$E = \frac{KL_{\text{AC}}}{A} \approx 1 \times 10^{12} \text{ Pa}. \quad (7)$$

To put the result from equation (7) into perspective, we can compare it to the Young's modulus of natural diamond $E_{\text{diamond}} = 1.05 \times 10^{12} \text{ Pa}$ [29], indicating that our investigations on optical rigidity with a fully suspended coupled cavity enabled a effective coupling medium of about the same stiffness as diamond.

To investigate variation of the stiffness of the optical spring with cavity detuning we operated the system with several different DC offsets and estimated the spring frequency in each case. It was found that the clearest marker of the resonant frequency, in the presence of measurement noise, was the point at which the phase trace crossed 90°. Comparison of five measured resonant frequencies plotted as a function of detuning is shown with the numerical model in figure 4.

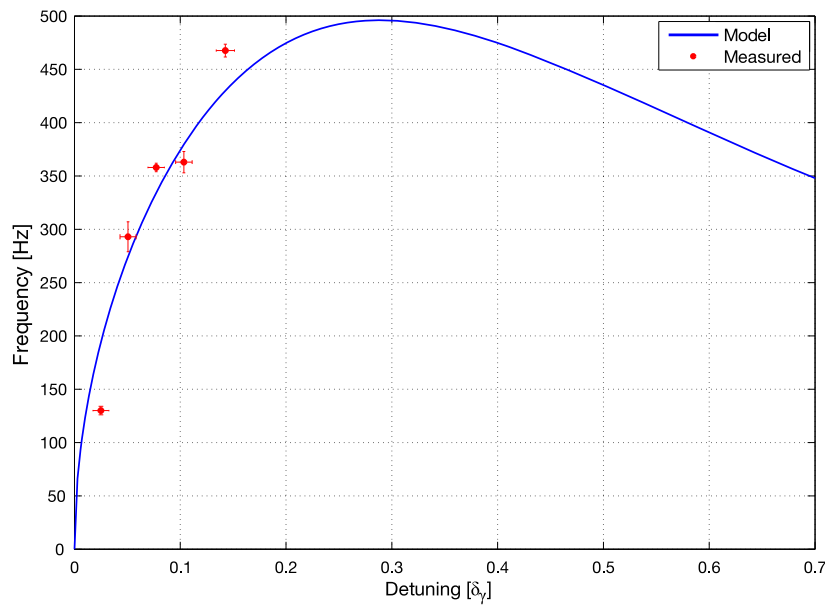


Figure 4. Measured optical spring locations (red) for known injected offsets to laser frequency, and modelled predictions (blue).

4. Summary and future work

The largest optical spring observed at $f_{\text{opt}} = 496$ Hz corresponds to an optical spring constant of $K_{\text{opt}} = 9.4 \times 10^5 \text{ N m}^{-1}$, for which an effective coupling medium between the cavity mirrors would have a Young's modulus essentially that of diamond.

The main limitation to our measurements was due to residual laser frequency noise. In future experiments, lowering this noise, for example by using a pre-stabilised laser, would allow much shorter measurement times and reduced sensitivity to drift. Lowering mirror losses and increasing the injected optical power would allow even stiffer springs, more appropriate to the 10 kg-scale masses employed in GW detectors.

Acknowledgments

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References

- [1] Abbott B P *et al* 2009 *Rep. Prog. Phys.* **72** 076901
- [2] Accadia T *et al* 2011 *Class. Quantum Grav.* **28** 114002
- [3] Luck H *et al* 2006 *Class. Quantum Grav.* **23** S71–8
- [4] Takahashi R and (the TAMA Collaboration) 2004 *Class. Quantum Grav.* **21** S403–408
- [5] Abadie J *et al* 2015 *Class. Quantum Grav.* **32** 074001
- [6] Willke B *et al* 2006 *Class. Quantum Grav.* **23** S207–14
- [7] Aso Y *et al* 2013 *Phys. Rev. D* **88** 043007

- [8] Acernese F *et al* 2015 *Class. Quantum Grav.* **32** 024001
- [9] Corbitt T, Ottaway D, Innerhofer E, Pelc J and Mavalvala N 2006 *Phys. Rev. A* **74** 021802
- [10] Braginsky V B and Khalili F Y 1992 *Quantum Measurements* (Cambridge: Cambridge University Press)
- [11] Meers B J 1988 *Phys. Rev. D* **38** 2317–26
- [12] Buonanno A and Chen Y 2002 *Phys. Rev. D* **65** 042001
- [13] Miyakawa O *et al* 2006 *Phys. Rev. D* **74** 022001
- [14] Braginsky V B and Khalili F Y 1996 *Phys. Lett. A* **218** 167–74
- [15] Khalili F Y 2002 *Phys. Lett. A* **298** 308–14
- [16] Rehbein H *et al* 2008 *Phys. Rev. D* **78** 062003
- [17] LIGO Scientific Collaboration 2011 *Instrument Science White Paper* <https://dcc.ligo.org/cgi-bin/private/DocDB/ShowDocument?docid=62186>
- [18] Mueller-Ebhard H *et al* 2009 Review of quantum non-demolition schemes for the Einstein telescope available at <https://tds.ego-gw.it/itf/tds/file.php?callFile=ET-010-09.pdf>
- [19] Punturo M *et al* 2010 *Class. Quantum Grav.* **27** 084007
- [20] Hild S *et al* 2011 Sensitivity studies for third-generation gravitational wave observatories **28** 094013
- [21] Corbitt T *et al* 2007 *Phys. Rev. Lett.* **98** 150802
- [22] Sheard B S *et al* 2004 *Phys. Rev. A* **69** 051801
- [23] Kippenberg T J *et al* 2005 *Phys. Rev. Lett.* **95** 033901
- [24] Torrie C 2001 *PhD Thesis* University of Glasgow, Glasgow, Scotland, UK
- [25] Huttner S H *et al* 2008 *Class. Quantum Grav.* **25** 235003
- [26] Sato S *et al* 2000 *Appl. Opt.* **39** 4616
- [27] Braginsky V B and Vyatchanin S P 2008 *Phys. Lett. A* **293** 228
- [28] Sidles J A and Sigg D 2006 *Phys. Lett. A* **354** 167
- [29] Savvides N and Bell T J 1993 *Thin Solid Films* **228** 289–92