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Nonlinearities in the CAPM: Evidence from developed and emerging markets

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Abstract: This paper examines the forecasting ability of the non-linear specifications of the market model. We propose a conditional Two-moment market model with a time-varying systematic covariance (*beta*) risk in the form of a mean reverting process of the state space model via Kalman Filter algorithm. In addition, we account for the systematic component of *co-skewness* and *co-kurtosis* by considering

higher moments. The analysis is implemented using data from the stock indices of several developed and emerging stock markets. The empirical findings favour the time-varying market model approaches which outperform linear model specifications, both in terms of model fit and predictability. Precisely, higher moments are necessary for datasets which involve structural changes and/or market inefficiencies which are common in most of the emerging stock markets.

Key words: CAPM, co-skewness, co-kurtosis, cubic market model, quadratic market model, time-varying market model.

JEL Codes: C18, C22, C61, G12

1 Introduction

A long criticism on the usefulness of the traditional Capital Asset Pricing Model (CAPM) model has been addressed in the literature of arbitrage pricing models that propose risk factors on firm fundamentals (Fama and French, 2004) or non-linearities on the model specification. Specifically, researchers focus on the examination of the dynamics of asset pricing models, either by addressing the importance and the theoretical intuition of documented stylized factors or by quantifying the time series properties of the data generating process and/or the estimated parameter set. Consequently, some doubts on the mechanism with which informational efficiency of stock exchanges is examined have been raised. The review papers of Schwert (2003) and Malkiel (2003) highlight this criticism and provide evidence that several of the stylized facts tend to be weaker after the papers which highlighted them were published,

or viewed as short-term aberrations of a long-term efficient market.

Any decision about the usefulness of the Two-Moment CAPM on the examination of the Efficient Market Hypothesis should be made conditionally on the validity of the assumptions of this model and on the securities' pricing mechanism. Indeed, normally distributed returns, independence and homoscedasticity of security prices and linearity of asset pricing models reflect a stock exchange of perfection. Additionally, the foundations behind the pricing mechanism that CAPM suggests rely on the existence of rational investors with unconditional and unlimited leverage opportunities who form a one-period investment and financing decision and on the existence of a market portfolio which consists of all possible financial products of the underlying stock market.

Consequently, it is questionable whether investors care only about the 1-period portfolio returns and whether they do not care about the covariance of their portfolio returns with other factors relevant with labor income, investment opportunities, business risk and political risk, diminishing thus the usefulness of CAPM. Moreover, the difficulty to reflect the minimum variance frontier on a market proxy of the market portfolio has "boomerang" effects. Several extensions have been proposed in the literature, including Blume (1970) and Black et al. (1972) who implement the CAPM on portfolios rather than on individual securities, Fama and MacBeth (1973) who test the significance of the risk premia with a two stage regressions analysis, Black (1972) who proposes a zero-beta model, Merton's (1973) intertemporal CAPM, Breeden's (1979) consumption CAPM and Roll (1977) with the arbitrage asset pricing model. However, even if the authors of the above-mentioned papers by the empirical examination of the CAMP model have verified a clear relationship between beta

and asset returns, several asymmetries were detected against the rationality of the CAPM. Specifically, aggressive firms underperform those expected by CAPM while several other firm fundamentals contribute substantially on the explanation of the cross sectional asset returns, such as the firm size (Banz (1981)), liquidity (Amihud and Mendelson (1986)) and momentum (Jegadeesh and Titman (1993)). It is worth mentioned that as Subrahmanyam (2010) explains more than 50 variables have been used in the literature to explain asset returns. The intuition behind the utilization of these variables and/or factors is not crystal clear and this concerns many economists about the usefulness of the extensions of the CAPM.

This paper addresses the limitations and the usefulness of the Two-Moment CAPM and proposes non-linear extensions with higher moments that account for the skewness and the kurtosis components of asset returns. This extension is of crucial importance for trading, asset allocation and risk management and the examination of the informational efficiency of securities' prices. Our objective is the comparative analysis of linear and non-linear market models in terms of predictive ability. We allow for exogenous structural changes, reflecting the October 2008 financial crisis period and most importantly we utilize data from developed and emerging stock markets to identify potential patterns of the predictability of asset pricing models in relation to the inefficiencies that are penetrated in less developed or emerging stock markets. Specifically we compare the Linear Market Model (consistent with the Two-Moment CAPM including systematic covariance (*beta*)) with six non-linear extensions. The first two are new reformulated forms of Higher order Data Generating Processes (DGP's) as simple polynomial extensions of the Linear Market Model that involve fractional moments, i.e the Quadratic Market Model (consistent with the Three-Moment CAPM including systematic covariance (*beta*) and skewness (*co-skewness*)) and the Cubic

Market Model (consistent with the Four-Moment CAPM including systematic covariance, systematic skewness and systematic kurtosis (*co-kurtosis*)). The third model is the Generalized Additive Model (GAM) (relaxing some of the assumptions underpinning polynomial models). The last three approaches consist of the time-varying versions of the Linear Market Model and polynomial extensions in the mean reverting specification of the state space model via Kalman Filter Algorithm (Kalman, 1960), i.e. the time-varying Linear Market Model (allowing for only time-varying systematic covariance), the time-varying Quadratic Market Model (allowing for time-varying systematic covariance and time-varying systematic skewness) and the time-varying Cubic Market Model (allowing for time-varying systematic covariance, time-varying systematic skewness and time-varying systematic kurtosis).

The comparative analysis which is conducted in this paper sheds much light on the necessity of non-linear models in the explanation of asset prices. Time-varying model specifications outperform the unconditional models while structural changes of financial time series are better absorbed within higher moments of the CAPM. Finally, we provide evidence in favour of higher moment model specification when dealing with data of emerging stock markets, underlying the importance of non-linear models when analysing market inefficiencies.

Our paper contributes to the literature of CAPM usefulness in a number of ways. Firstly, it explains the usefulness of CAPM using higher order moments and nonlinearities to support the expected utility foundations of asset pricing models. Secondly, it proposes an innovative Generalized Additive Model (GAM) application in the CAPM framework. Thirdly, it proposes a non-linear model with fractional moments $([R_m - R_f]^\Upsilon)$ where Υ takes any positive value) instead of integers that represent

the second and third moments. Finally, it applies the mean reverting specification of the state space model via Kalman Filter methodology in the proposed Quadratic Market Model (QMM) and Cubic Market Model (CMM) accounting for the time varying characteristics of the systematic covariance, the systematic skewness and the systematic kurtosis, namely time-varying Quadratic Market Model (TvQMM) and time-varying Cubic Market Model (TvCMM).

The rest of the paper is organised as follows: section 2 provides a revision of the literature review, section 3 considers that data set for our analysis, section 4 explains the research methodology, section 5 discusses our empirical findings while section 6 concludes the paper.

2 Literature Review

While Sharpe's (1964) capital asset pricing model (CAPM) under specific, and often heroic, assumptions, lay on the security market line that comprises exclusively the beta (systematic covariance) risk with respect to the market portfolio, a hypothetical portfolio (Roll, 1977), it fails to provide consistency through time and/or across firm fundamentals. A significant contribution on the former aspect of this literature is Merton's (1973) intertemporal capital asset pricing model (ICAPM), according to which investors optimize their portfolios considering the intertemporal relationship of expected returns with future state variables. The latter inconsistency has motivated many researchers among them Fama and French (1993) to propose extensions that account for several stylized financial facts that associate investors' expectations with firm fundamentals. While the statistical significance of these characteristics, that do

not, necessarily, represent state variables of concern to investors, on multifactor models, enhance the criticism against CAPM, Ang and Chen (2007) argue that they could be fully accounted for by a one-factor with time varying factor loadings, providing evidence in favour to the conditional CAPM.

Another significant contribution to the CAPM literature was developed by Kraus and Litzenberger (1976) in order to relax the two restrictive assumptions of the CAPM; i.e. normally distributed asset returns and the quadratic utility function (in terms of the mean and variance terms only; that is why it is called a Two-Moment CAPM in this paper). In doing so, Kraus and Litzenberger (1976) identify a Three-Moment CAPM, which extends the Two-Moment CAPM, by incorporating a skewness term which, itself, characterizes the degree of symmetry of a return distribution around its mean (seeing as it plays an important role in asset pricing). Kraus and Litzenberger's (1976) Three-Moment CAPM, in equilibrium, can be represented as

$$E(R_i) - R_f = c_1\beta_{im} + c_2\gamma_{im}, \quad (2.1)$$

where $E(R_i)$ and R_f are the expected asset return and the risk-free rate, respectively. Here, β_{im} and γ_{im} are the systematic covariance (*beta*) and systematic skewness (*co-skewness*) risk measures with c_1 and c_2 being the market prices or risk premiums for these systematic risk measures, respectively, whose theoretical details are provided in Appendix A. For the purpose of computing these systematic risk measures, Kraus and Litzenberger (1976) specifies the Higher order Data Generating Process (DGP), namely the Quadratic Market Model (consistent with the Three-Moment CAPM (2.1)). This is represented by

$$R_i - R_f = \kappa_i + \alpha_{1i}(R_m - R_f) + \alpha_{2i}(R_m - E(R_m))^2 + \varepsilon_i. \quad (2.2)$$

The systematic risk measures (β_{im} and γ_{im}) can be expressed in terms of (α_{1i} and

α_{2i}) as follows

$$\beta_{im} = \alpha_{1i} + \frac{\alpha_{2i}S(R_m)^3}{\sigma(R_m)^2}, \quad (2.3)$$

$$\gamma_{im} = \alpha_{1i} + \frac{\alpha_{2i}\{K(R_m)^4 - \sigma(R_m)^4\}}{S(R_m)^3}, \quad (2.4)$$

where $\sigma(R_m) = E[(R_m - E(R_m))^2]^{1/2}$, $S(R_m) = E[(R_m - E(R_m))^3]^{1/3}$ and $K(R_m) = E[(R_m - E(R_m))^4]^{1/4}$ are the central moments of the rate of returns on the market portfolio.

Another significant research framework incorporates kurtosis (which characterizes the relative peakness and flatness of a distribution compared with the normal distribution) in the Three-Moment CAPM. This is called the Four-Moment CAPM and was investigated by Fang and Lai (1997), Hwang and Satchell (1999), Christie-David and Chaudhry (2001), Galagedera and Silvapulle (2002), Ranaldo and Favre (2005), Liow and Chan (2005), Jurczenko and Maillet (2006) and Javid (2009). The Four-Moment CAPM, in equilibrium, can be presented as

$$E(R_i) - R_f = c_1\beta_{im} + c_2\gamma_{im} + c_3\delta_{im}. \quad (2.5)$$

Letting β_{im} , γ_{im} , and δ_{im} , be, respectively, the systematic covariance (*beta*), systematic skewness (*co-skewness*) and systematic kurtosis (*co-kurtosis*), and letting c_1 , c_2 and c_3 be the market prices, or risk premiums, for systematic covariance (β_{im}), systematic skewness (γ_{im}); systematic kurtosis (δ_{im}), respectively, Their theories details are provided in Appendix A. In the literature, two prominent papers were published by Fang and Lai (1997) and Hwang and Satchell (1999) which identify their own Cubic Market Models (consistent with the Four-Moment CAPM (2.5)) as a Higher order DGP for the purpose of calculating these systematic risk measures. The first

Cubic Market Model was developed by Fang and Lai (1997) and can be presented by

$$R_{it} = \kappa_i + \beta_{im}(R_{mt}) - \gamma_{im}(R_{mt})^2 + \delta_{im}(R_{mt})^3 + \varepsilon_{it}. \quad (2.6)$$

Here, R_{it} and R_{mt} are the excess rates of return on asset i ($i = 1, \dots, n$) and market portfolio at time t ($t = 1, \dots, T$). The multiple regression coefficients are identical to the parameters β_{im} , γ_{im} , and δ_{im} in equation 2.5.

The latter Cubic Market Model (which is consistent with the Four-Moment CAPM (2.5) developed by Hwang and Satchell (1999) can be represented by

$$R_{it} - R_{ft} = \kappa_i + \alpha_{1i}(R_{mt} - R_{ft}) + \alpha_{2i}(R_{mt} - E(R_m))^2 + \alpha_{3i}(R_{mt} - E(R_m))^3 + \varepsilon_{it}. \quad (2.7)$$

Furthermore, the systematic risk measures (β_{im} , γ_{im} , and δ_{im}) can be expressed in terms of (α_{1i} , α_{2i} and α_{3i}) as follows

$$\beta_{im} = \alpha_{1i} + \frac{\alpha_{2i}S(R_m)^3 + \alpha_{3i}K(R_m)^4}{\sigma(R_m)^2}, \quad (2.8)$$

$$\gamma_{im} = \alpha_{1i} + \frac{\alpha_{2i}\{K(R_m)^4 - \sigma(R_m)^4\} + \alpha_{3i}\{\theta(R_m)^5 - S(R_m)^3\sigma(R_m)^2\}}{S(R_m)^3}, \quad (2.9)$$

$$\delta_{im} = \alpha_{1i} + \frac{\alpha_{2i}\{\theta(R_m)^5 - \sigma(R_m)^2S(R_m)^3\} + \alpha_{3i}\{\omega(R_m)^6 - S(R_m)^6\}}{K(R_m)^4}, \quad (2.10)$$

where $\sigma(R_m) = E[(R_m - E(R_m))^2]^{1/2}$, $S(R_m) = E[(R_m - E(R_m))^3]^{1/3}$, $K(R_m) = E[(R_m - E(R_m))^4]^{1/4}$, $\theta(R_m) = E[\{R_m - E(R_m)\}^5]^{1/5}$ and $\omega(R_m) = E[\{R_m - E(R_m)\}^6]^{1/6}$ are the central moments of the rate of returns on the market portfolio.

To sum up, the several formulations of Higher order DGPs aim to illustrate the link between the Higher order DGPs and their equivalent Higher-Moment CAPMs for reducing the multicollinearity of the systematic risk measures in the Higher-Moment

CAPMs in the financial literature. Neslihanoglu (2014) proposed newly formulated forms of the Higher order DGPs as simple polynomial extensions of the Linear Market Model; namely, the Quadratic Market Model (allowing for systematic skewness and systematic skewness) and the Cubic Market Model (allowing for systematic covariance, systematic skewness and systematic kurtosis). In doing so, Higher order DGPs prefer when R_f is used instead of $E(R_m)$ in the second and third order of equation 2.7 whose theoretical details are provided in Appendix A. These DGPs are examined in this paper.

3 Data

For the purposes of our analysis we utilize data from 18 global market indices from the Morgan Stanley Capital International Incorporation and obtained from the Thomson Reuters Financial Datastream database. The data are of weekly frequency and refer to 9 stock indices from developed economies and 9 from emerging economies covering the period from 17/07/2002 to 18/07/2012. Specifically, the set of developed economies consists of France, Germany, Italy, Japan, Norway, Sweden, Switzerland, UK and US while the set of emerging economies includes Brazil, Chile, India, Korea, Malaysia, Mexico, Poland, Russia and South Africa. Table 1 (Table 5.1 for the PhD (Neslihanoglu, 2014)) presents the regional characteristics of the utilized markets.

Insert Table [1]

In all cases the Morgan Stanley Capital International (MSCI) World Index and the three-month US dollar London Interbank Offered Rate (LIBOR) interest rate are used as a proxies for the market portfolio and the risk-free rate, respectively. The MSCI

index consists of 1,606 constituents including the large and mid-cap segmentations across 24 developed markets. All indices are expressed in US dollars. The one-week returns between time t to $t-1$ for i refers to the MSCI World market and 18 global markets from the first difference of in the logarithm of Wednesday closing price index (i.e. P_{it} in week t) is calculated as

$$R_{it} = \log(P_{it}) - \log(P_{it-1}), \quad (3.1)$$

for $t = 2, \dots, T$ (Neslihanoglu, 2014). The weekly risk-free rate calculated from three-month US dollar London Interbank Offered Rate (LIBOR) interest rate (Mergner, 2009) as follows

$$R_{ft} = \left(1 + \frac{LIBOR_t}{100}\right)^{1/52} - 1, \quad (3.2)$$

where $LIBOR_t$ are in percentage per annum and R_{ft} are the weekly risk-free rate in week t .

Insert Table [2]

Insert Table [3]

Tables 2 (Table 5.2 for the PhD (Neslihanoglu, 2014)) and 3 provide several descriptive statistics about the distributional form of time series data during the three different period analyses in this paper: the entire period from July 2002 to July 2012, from July 2002 to before the October 2008 financial crisis, and from after the October 2008 financial crisis to July 2012. Specifically, these tables represent information about the first four moments while these tables report the Jargue Bera (JB) and the Ljung-Box (LB) test statistics for the normality and the autocorrelation of asset returns, respectively.

From the descriptive analysis in Tables 2 and 3 it is shown that emerging markets'

indices compensate investors with higher returns due to the higher risk that they undertake, while it is apparent that data embed left skewness and leptokurtosis. The magnitude of left skewness as well as of the fat tails of returns' distributions is greater for emerging markets. This is consistent with the inefficiencies that emerging markets exhibit compared to more developed ones.

4 Research Methodology

4.1 Higher DGPs

For the purpose of assessing the necessity for the Higher-Moment CAPMs, a newly formulated form of the Cubic Market Model (consistent with the Four-Moment CAPM (eq. (2.5)) including systematic covariance (*beta*), systematic skewness (*co-skewness*) and systematic kurtosis (*co-kurtosis*)) in the form of a polynomial extension of the Linear Market Model (consistent with the Two-Moment CAPM) which was proposed by Neslihanoglu (2014) is applied to the stock market data in this paper. The more theoretical details about the link between the Four-Moment CAPM (eq. (2.5)) and the newly formulated form of the Cubic Market Model (CMM) (eq. (4.1)) are provided in Appendix A. This model, being a third order polynomial in excess market returns, can be represented by

$$R_{it} - R_{ft} = \kappa_i + \alpha_{1i}(R_{mt} - R_{ft}) + \alpha_{2i}(R_{mt} - R_{ft})^2 + \alpha_{3i}(R_{mt} - R_{ft})^3 + \varepsilon_{it}. \quad (4.1)$$

Here, R_{it} and R_{mt} are the country i 's stock market returns and MSCI World market returns at time t ($t = 1, \dots, T$), respectively. R_{ft} is the risk-free rate at time t . And ε_{it} are the residuals with $\varepsilon_{it} \sim N(0, \sigma_i^2)$, $E(\varepsilon_{it}\varepsilon_{kt})=0$, for $i \neq k$ and $E(\varepsilon_{it}\varepsilon_{i,t+j})=0$,

for $j > 0$. Here, the regression intercept, κ_i , accounts for the unexpected risk. Of the regression slopes, α_{1i} accounts for the systematic covariance (*proxy for* β_{im}), α_{2i} accounts for the systematic skewness (*proxy for* γ_{im}), and α_{3i} accounts for the systematic kurtosis (*proxy for* δ_{im}). These details are provided in Appendix A.

To show the link between the Four-Moment CAPM (2.5) and the Cubic Market Model (4.1), the systematic risk measures (β_{im} , γ_{im} , and δ_{im}) can be expressed in terms of (α_{1i} , α_{2i} and α_{3i}) as

$$\begin{aligned}\beta_{im} &= \frac{E[(R_i - E(R_i))(R_m - E(R_m))]}{E[(R_m - E(R_m))^2]} = \alpha_{1i} \\ &+ \alpha_{2i} \frac{E[((R_m - R_f)^2 - E(R_m - R_f)^2)(R_m - E(R_m))]}{E[(R_m - E(R_m))^2]} \\ &+ \alpha_{3i} \frac{E[((R_m - R_f)^3 - E(R_m - R_f)^3)(R_m - E(R_m))]}{E[(R_m - E(R_m))^2]},\end{aligned}\tag{4.2}$$

$$\begin{aligned}\gamma_{im} &= \frac{E[(R_i - E(R_i))(R_m - E(R_m))^2]}{E[(R_m - E(R_m))^3]} = \alpha_{1i} \\ &+ \alpha_{2i} \frac{E[((R_m - R_f)^2 - E(R_m - R_f)^2)(R_m - E(R_m))^2]}{E[(R_m - E(R_m))^3]} \\ &+ \alpha_{3i} \frac{E[((R_m - R_f)^3 - E(R_m - R_f)^3)(R_m - E(R_m))^2]}{E[(R_m - E(R_m))^3]},\end{aligned}\tag{4.3}$$

$$\begin{aligned}\delta_{im} &= \frac{E[(R_i - E(R_i))(R_m - E(R_m))^3]}{E[(R_m - E(R_m))^4]} = \alpha_{1i} \\ &+ \alpha_{2i} \frac{E[((R_m - R_f)^2 - E(R_m - R_f)^2)(R_m - E(R_m))^3]}{E[(R_m - E(R_m))^4]} \\ &+ \alpha_{3i} \frac{E[((R_m - R_f)^3 - E(R_m - R_f)^3)(R_m - E(R_m))^3]}{E[(R_m - E(R_m))^4]}.\end{aligned}\tag{4.4}$$

Its proofs are provided in Appendix A. Note that the Four-Moment CAPM could only be employed if the DGP was at least cubic that is, α_{3i} should be statistically significantly different from zero. If not, there will be collinearity in the systematic

risk measures (β_{im} , γ_{im} and δ_{im}) of the Four-Moment CAPM. It is worth noting that the Quadratic Market Model ($\alpha_{3i} = 0$ in (4.1), consistent with the Three-Moment CAPM) and the Linear Market Model ($\alpha_{2i} = 0$ and $\alpha_{3i} = 0$ in (4.1), consistent with the Two-Moment CAPM) are reduced forms of the Cubic Market Model. The details of their systematic risk measures have been omitted for the sake of brevity, with their systematic risk measures being provided in Appendix A.

4.2 Generalized Additive Model

The generalized additive model (GAM) generated by Hastie and Tibshirani (1990) is first time that a comparator to the polynomial models given by (4.1) have been employed in CAPM studies seeing if the latter's rigid parametric shapes are too restrictive. This comparison allows us to assess the necessity for even higher moments in the CAPMs, such as five, six, etc. In CAPM studies, the GAM function can be represented as

$$R_{it} - R_{ft} = \kappa_i + f_i(R_{mt} - R_{ft}) + \varepsilon_{it}. \quad (4.5)$$

Here, $\varepsilon_{it} \sim N(0, \sigma_i^2)$ with $E(\varepsilon_{it}\varepsilon_{kt})=0$, for $i \neq k$, and $E(\varepsilon_{it}\varepsilon_{it+j})=0$, for $j > 0$, and $f_i(R_{mt} - R_{ft})$ is a smooth function of $R_{mt} - R_{ft}$. The parameter estimation procedure used in generalized additive models (GAMs) is briefly outlined in my PhD thesis (Neslihanoglu, 2014).

4.3 Time-varying Higher DGPs

There now exists widespread evidence for the instability of the systematic risk measures (β_{im} , γ_{im} and δ_{im}) in the Four Moment CAPM in the literature. For the purpose of assessing this instability and estimating its time-varying systematic risk measures,

the Cubic Market Model (4.1), which can be extended by allowing α_{1i} , α_{2i} , and α_{3i} not to be constant over time. This model is called a time-varying Cubic Market Model (TvCMM) which is consistent with the conditional Four-Moment CAPM (allowing for time-varying systematic risk measures, time-varying systematic covariance (β_{imt}), time-varying systematic skewness (γ_{imt}), and the time-varying systematic kurtosis (δ_{imt})). These time-varying systematic risk measures (β_{imt} , γ_{imt} , and δ_{imt}) are the time-varying extensions of equations 4.2, 4.3, and 4.4, respectively. The time-varying Cubic Market Model (TvCMM) can be achieved by using the model defined below, where estimation is achieved via the mean reverting specification of the state space model via Kalman Filter (KFMR) algorithm which was outlined in my PhD thesis (Neslihanoglu, 2014). The model has a state space form and has been modified to become an observation equation which can be expressed as

$$R_{it} - R_{ft} = \kappa_i + \alpha_{1it}(R_{mt} - R_{ft}) + \alpha_{2it}(R_{mt} - R_{ft})^2 + \alpha_{3it}(R_{mt} - R_{ft})^3 + \varepsilon_{it}. \quad (4.6)$$

Here, $\varepsilon_{it} \sim N(0, H_i)$. The state equations can be expressed as

$$\alpha_{1it} = \bar{\alpha}_{1i} + \phi_{1i}(\alpha_{1i,t-1} - \bar{\alpha}_{1i}) + w_{1it}, \quad w_{1it} \sim N(0, Q_{1i}), \quad (4.7)$$

$$\alpha_{2it} = \bar{\alpha}_{2i} + \phi_{2i}(\alpha_{2i,t-1} - \bar{\alpha}_{2i}) + w_{2it}, \quad w_{2it} \sim N(0, Q_{2i}), \quad (4.8)$$

$$\alpha_{3it} = \bar{\alpha}_{3i} + \phi_{3i}(\alpha_{3i,t-1} - \bar{\alpha}_{3i}) + w_{3it}, \quad w_{3it} \sim N(0, Q_{3i}), \quad (4.9)$$

with priors

$$\alpha_{1i0} \sim N(\mu_{\alpha_{1i}}, \Sigma_{\alpha_{1i}}), \quad \alpha_{2i0} \sim N(\mu_{\alpha_{2i}}, \Sigma_{\alpha_{2i}}), \quad \alpha_{3i0} \sim N(\mu_{\alpha_{3i}}, \Sigma_{\alpha_{3i}}). \quad (4.10)$$

where the parameters of these distributions are estimated from the data as part of an estimation algorithm.

Here, the regression intercept, κ_i (the *proxy for unexpected risk*) and the regression slopes, α_{1it} , α_{2it} , and α_{3it} , with α_{1it} accounting for the time-varying systematic co-

variance (*proxy for* β_{imt}), α_{2it} accounting for the time-varying systematic skewness (*proxy for* γ_{imt}), and α_{3it} accounting for the time-varying systematic kurtosis (*proxy for* δ_{imt}). Note that the conditional Four-Moment CAPM is only appropriate if the time-varying DGP is at least cubic; that is if α_{3it} is statistically significantly different from zero. If not, then collinearity will exist in the time-varying systematic risk measures (β_{imt} , γ_{imt} , and δ_{imt}).

It is worth noting that the time-varying Quadratic Market Model ($\alpha_{3it} = 0$ in (4.6)) and the time-varying Linear Market Model ($\alpha_{2it} = 0$ and $\alpha_{3it} = 0$ in (4.6)) are reduced forms of the time-varying Cubic Market Model. The time-varying Quadratic Market Model (consistent with the conditional Three-Moment CAPM) and the time-varying Linear Market Model (consistent with the conditional Two-Moment CAPM) are extensions of the results proved in Appendix A. Note that the software used for implementing the time-varying Higher order DGPs using the KFMR algorithm was the **R** software as described Neslihanoglu (2014).

The fit of these models is compared using two different summaries of the errors; i.e. the Mean Absolute Error (MAE) and the Mean Squared Error (MSE). These are defined as follows

$$MAE = \frac{1}{T} \sum_{t=1}^T \left| (\widehat{R_{it} - R_{ft}}) - (R_{it} - R_{ft}) \right|, \quad (4.11)$$

$$MSE = \frac{1}{T} \sum_{t=1}^T \left((\widehat{R_{it} - R_{ft}}) - (R_{it} - R_{ft}) \right)^2. \quad (4.12)$$

Moreover, the MAE and MSE in the in-sample (out-of-sample) procedure provides a measure of the modelling (forecasting) ability of these models. According to these modelling (forecasting) errors, the models with the lowest MAE and MSE values

indicate better modelling (forecasting) performance (Neslihanoglu, 2014).

We also applied the Diebold-Mariano Test (Diebold and Mariano (1995)) based on MAE and MSE as the measures in the in-sample (out-of-sample) procedure for the robustness checking of the modelling (forecasting) accuracy between two different models. The test statistic is defined as

$$DB = \frac{d}{\text{Var}(\bar{d})}, \quad (4.13)$$

$$d = \left| \left(\widehat{(R_{it} - R_{ft})} - (R_{it} - R_{ft}) \right)_a \right|^w - \left| \left(\widehat{(R_{it} - R_{ft})} - (R_{it} - R_{ft}) \right)_b \right|^w. \quad (4.14)$$

Here, $\left(\widehat{(R_{it} - R_{ft})} - (R_{it} - R_{ft}) \right)_a$ and $\left(\widehat{(R_{it} - R_{ft})} - (R_{it} - R_{ft}) \right)_b$ are the residuals for the two different models i.e. a and b for the t^{th} ($t = 1, \dots, n$). Here, w is equal to 1 when using the MAE as the measure and equal to 2 when using the MSE as the measure as described (Choudhry and Wu, 2009). Here, given the null hypothesis of there being no different levels of modelling (forecasting) accuracy between the two different models, DB follows a t distribution with $n - 1$ degrees of freedom.

We also discuss the diagnostic procedures of checking how well the assumptions of the regression model are satisfied. The assumptions underlying the regression model are that the residuals are normally distributed, independent, and have constant variance. These assumptions need to be checked using various diagnostic tests based on the residuals. These tests are primarily taken from Harvey (1989), Durbin and Koopman (2001), Faraway (2004) and Neslihanoglu (2014).

The standardised residuals can be represented as

$$s_t = \frac{\left(\widehat{(R_{it} - R_{ft})} - (R_{it} - R_{ft}) \right)}{\sqrt{\text{Var} \left(\widehat{(R_{it} - R_{ft})} - (R_{it} - R_{ft}) \right)}}. \quad (4.15)$$

where $(R_{it} - R_{ft})$ is the observed response and $(\widehat{R_{it} - R_{ft}})$ is the fitted value for the t^{th} ($t = 1, \dots, n$) unit. Note that the standardised residuals $\{s_t\}_{t=1}^n$ will have approximately $N(0, 1)$ distributions if the linear Gaussian model holds.

For the purpose of testing the normality of the residuals, the Jarque-Bera test (*JB*; see (Jarque and Bera, 1980)) is a goodness-of-fit test of whether the skewness and kurtosis of the data are appropriate for a Gaussian distribution. The sample skewness and kurtosis of the standardised residuals are represented by

$$S = \frac{\frac{1}{n} \sum_{t=1}^n (s_t - \bar{s})^3}{\left(\frac{1}{n} \sum_{t=1}^n (s_t - \bar{s})^2\right)^{3/2}}, \quad K = \frac{\frac{1}{n} \sum_{t=1}^n (s_t - \bar{s})^4}{\left(\frac{1}{n} \sum_{t=1}^n (s_t - \bar{s})^2\right)^2}, \quad (4.16)$$

where $\bar{s}$ is the mean of the standardised residuals, $\{s_t\}_{t=1}^n$ with the Jarque-Bera (*JB*) test statistic being defined as

$$JB = n \left\{ \frac{S^2}{6} + \frac{(K - 3)^2}{24} \right\}. \quad (4.17)$$

Here, under the null hypothesis of normality, *JB* follows a chi-squared distribution with 2 degrees of freedom.

For the purpose of testing for heteroskedasticity (non-constant variance), the simplest diagnostic test statistic is defined (Durbin and Koopman, 2001) as follows

$$Het(h) = \frac{\sum_{t=n-h+1}^n s_t^2}{\sum_{t=1}^h s_t^2}. \quad (4.18)$$

Here, given the null hypothesis of homoscedasticity (constant variance), *Het(h)* follows a $F_{h,h}$ distribution for some preset positive integer h which is the nearest integer to $n/3$.

For the purpose of testing for temporal autocorrelation, the Ljung-Box (*LB*) test (often referred to as the portmanteau test), developed by Ljung and Box (1978), is

used. The test statistic is as follows

$$LB(L) = n(n+2) \sum_{k=1}^L \frac{\rho_k^2}{n-k}, \quad (4.19)$$

where L is the number of lags being tested and ρ_k , the sample autocorrelation of the standardised residuals at lag k . This is defined as

$$\rho_k = \frac{\sum_{t=k+1}^n (s_t - \bar{s})(s_{t-k} - \bar{s})}{\sum_{t=1}^n (s_t - \bar{s})^2}, \quad k = 1, 2, \dots \quad (4.20)$$

Here, given the null hypothesis of no autocorrelation, $LB(L)$ follows a chi-squared distribution with L degrees of freedom.

5 Empirical Findings

5.1 Model Fit

This section presents a comparison of the in-sample model fit performance for the Linear Market Model (LMM), the Quadratic Market Model (QMM), the Cubic Market Model (CMM), the generalized additive model (GAM), the time-varying Linear Market Model (TvLMM), the time-varying Quadratic Market Model (TvQMM) and the time-varying Cubic Market Model (TvCMM) for 18 global markets during three different time periods: the entire period from July 2002 to July 2012, from July 2002 to before the October 2008 financial crisis, and from after the October 2008 financial crisis to July 2012. The model fit comparison is conducted in terms of two different measures of errors, the MAE (eq. (4.11)) and MSE (eq. (4.12)). The model fit results are given in Tables 4, 5 and 6 which display the MAE and MSE results for the three different time periods, respectively.

A comparison of the seven model results in terms of MAE (and MSE) for developed markets display that the time-varying Linear Market Model (TvLMM) (consistent with the conditional Two-Moment CAPM including only time-varying beta risk) seems to be preferable in 8 (8) out of 9 for the entire period (see in Table 4), in 7 (8) out of 9 for the period before the October 2008 financial crisis (see in Table 5), and in 8 (8) out of 9 for the period after the October 2008 financial crisis (see in Table 6), respectively. Overall, the time-varying Linear Market Model (having either the lowest, or one of the lowest, median MAE and MSE results) improves on the Linear Market Model (having the highest, or one of the highest, median MAE and MSE) in terms of MAE (MSE) for developed markets on average (median) by 21.3% (44.1%) for the entire period, by 18.4% (34.2%) for the period before the October 2008 financial crisis, and by 22.4% (39.1%) for the period after the October 2008 financial crisis.

A comparison of the results in terms of the MAE (and MSE) for emerging markets display that the time-varying Linear Market Model (TvLMM) seems to be preferable in 7 (7) out of 9 for the entire period (see in Table 4), in 6 (6) out of 9 for the period before the October 2008 financial crisis (see in Table 5), and in 5 (4) out of 9 for the period after the October 2008 financial crisis (see in Table 6), respectively. Overall, the time-varying Linear Market Model (having either the lowest, or one of the lowest, median MAE and MSE) improves on the Linear Market Model (having the highest, or one of the highest, median MAE and MSE) in terms of MAE (MSE) for the emerging markets on average (median) by 32.4% (57.9%) for the entire period, by 17.4% (33.4%) for the period before the October 2008 financial crisis, and by 12.7% (21.6%) for the period after the October 2008 financial crisis.

Insert Table [4]

Insert Table [5]

Insert Table [6]

Note that all models performs better for the developed markets than for the emerging markets in all periods. This may be due the fact that the emerging models are more volatile than the developed markets, as evidenced in Tables 2 and 3. In addition, the relative improvements in performance of the time-varying Linear Market Model (TvLMM) compared to the Linear Market Model (LMM) in emerging markets is higher than that in the developed markets for the entire period. For the period after the October 2008 financial crisis, however, the relative improvements in the emerging markets are lower than that in the developed markets. Also, the improvements in both the developed and emerging are relatively similar for the period before the October 2008 financial crisis.

It is worth noting that the GAM (when assessing the performance of even higher moments) outperforms the Higher order DGPs, namely the Quadratic Market Model (QMM) and Cubic Market Model (QMM) in terms of median MAE and MSE for both the developed and emerging markets for both the entire period and for the period before the October 2008 financial crisis, whereas the performance of the GAM was generally worse than the Higher order DGPs and close, or nearly equal to, the results obtained by the Linear Market Model for the period after the October 2008 financial crisis. This suggests that even Higher-Moment CAPMs (allowing for even higher moments, i.e. sixth, seventh etc.) can be considerable for both the entire period and for the period before the October 2008 financial crisis, but that they, nevertheless, are not required for the period after the October 2008 crisis (where the beta risk measure can suffice).

To sum up, the time-varying Linear Market Model (TvLMM) is the preferable model for both the developed and emerging markets for all three different time periods.

5.2 Out-of-Sample Forecasting

In this section, the out-of-sample forecasting performance of the same models are compared. Here, a rolling window technique is used for evaluating the predictive performance of the models. The more theoretical details of this are discussed in my PhD thesis (Neslihanoglu, 2014). In this case, the length of the rolling window is 5 years for the entire data and 2 years for the period before the October 2008 financial crisis, with the period after the October 2008 financial crisis being used to predict parameters one-week ahead. The length of the prediction period is 1 year over the period from July 27, 2011 to July 18, 2012 for the entire data, October 3, 2007 to September 24, 2008 for the period before the October 2008 financial crisis, and from July 27, 2011 to July 18, 2012 for the period after the October 2008 financial crisis. The MAE and MSE values between the predicted and actual returns on the stock markets are calculated over 52 values for all markets for each time period and model. The out-of-sample forecasting results (in terms of the MAE and MSE) are given in Tables 7, 8 and 9 for each time period, respectively.

Insert Table [7]

Insert Table [8]

Insert Table [9]

A comparison of the same models' results in terms of MAE (MSE) for the developed markets display that the time-varying Linear Market Model (TvLMM) (including only time-varying beta risk) can be the preferable model in 7 (5) out of 9 for the

entire period (see in Table 7), in 7 (7) out of 9 for the period before the October 2008 financial crisis (see in Table 8), and 2 (1) out of 9 for the period after the October 2008 financial crisis (see in Table 9) where the time-varying Cubic Market Model (TvCMM) (consistent with the conditional Four-Moment CAPM) seems to be preferable model in 7 (8) out of the 9 developed markets. Overall, the time-varying Linear Market Model (TvLMM) (having either the lowest, or one of the lowest, median MAE and MSE) improves on the Linear Market Model (LMM) (having either the highest, or one of the highest, median MAE and MSE) in terms of MAE (MSE) for the developed markets on average (median) by 38% (56.1%) for the entire period, and by 39.5% (65.8%) for the period before the October 2008 financial crisis. For the period after the October 2008 financial crisis, on the other hand, the time-varying Cubic Market Model (TvCMM) (having the lowest median MAE and MSE) improves on the Linear Market Model (having the highest median MAE and MSE) in terms of MAE (MSE) for the developed markets on average (median) by 22.9% (40.6%).

A comparison of the results for the same models in terms of MAE (MSE) for the emerging markets displays that the time-varying Linear Market Model (TvLMM) seems to be preferable in 7 (7) out of 9 for the entire period (see in Table 7), in 4 (5) out of 9 for the period before the October 2008 financial crisis (see in Table 8), but not for the period after the October 2008 financial crisis (see in Table 9), where the time-varying Cubic Market Model (TvCMM) seems to be the preferable model for all 9 emerging markets. Overall, the time-varying Linear Market Model (TvLMM) (having either the lowest, or one of the lowest, median MAE and MSE) improves on the Linear Market Model (LMM) (having the highest, or one of the highest, median MAE and MSE) in terms of MAE (MSE) for the emerging markets on average (median) by 23.3% (49.2%) for the entire period, and by 23.8% (32.2%) for

the period before the October 2008 financial crisis. The time-varying Cubic Market Model (TvCMM) (having the lowest median MAE and MSE) improves on Linear Market Model (having the highest median MAE and MSE) in terms of MAE (MSE) for the emerging markets on average (median) by 4.6% (31.7%) for the period after the October 2008 financial crisis.

It is worth noting that all models for the emerging markets for all periods perform worse than that for the developed markets. This is possibly due to there being outliers which are more common in the emerging markets than in the developed markets (Neslihanoglu, 2014). Moreover, the relative improvements made to the performances of the time-varying Linear Market Model or the time-varying Cubic Market Model compared to the Linear Market Model in emerging markets is lower than that in developed markets for all time periods, which, in turn, may also be due to the aforementioned outliers.

It is worth noting that the predictive performance of the GAM was worse than that of the Higher order DGPs, namely the Quadratic and Cubic Market Models in terms of median MAE and MSE for both the developed and emerging markets for all three time periods, with the exception of the emerging market for the entire period and both the developed and emerging markets for the period before the October 2008 financial crisis. This suggests that even higher moments can be significant for the developed (for the period before the October 2008 financial crisis) and emerging markets (for the entire period and for the period before the October 2008 financial crisis).

To sum up, the time-varying Linear Market Model (TvLMM) is the preferable model for both the developed and emerging markets for all the different time periods, with the exception of the period after the October 2008 financial crisis where the time-

varying Cubic Market Model (TvCMM) seems to be the preferable model for both the developed and emerging markets.

5.3 Robustness Checking

This section discusses the robustness checking of the modelling and forecasting accuracy between the two aforementioned models by using the Diebold-Mariano Test (Diebold and Mariano (1995)) in terms of both MAE and MSE (eq.4.13) as the measures for the in-sample (provided in Tables 10, 11 and 12) and out-of-sample procedure (provided in Tables 13, 14 and 15) for 9 developed and 9 emerging markets for the three aforementioned time periods. Here, the null hypothesis is that there are no different levels of modelling (forecasting) accuracy for the two aforementioned models. An alternative hypothesis is that there are different levels of modelling (forecasting) accuracy for the two aforementioned models in the in-sample (out-of-sample) procedure for each of the stock markets for each of the time periods. The tables display the number of developed markets out of the total developed markets and the number of emerging markets out of the total emerging markets that reject the null hypothesis by using the Diebold-Mariano Test at the 5% significance level.

Insert Table [10]

Insert Table [11]

Insert Table [12]

Insert Table [13]

Insert Table [14]

Insert Table [15]

Based on the MAE and MSE values for both the in-sample and out-of-sample proce-

cedure for each time period, it can clearly be determined that the time-varying Linear Market Model (TvLMM) (which generally exhibits best modelling and forecasting performance) is statistically significant at the different levels of modelling and forecasting accuracy for the Linear Market Model, the Higher order DGPs and the GAM for both developed and emerging markets, except for when one takes into consideration the forecasting accuracy for the developed markets in the out-of-sample procedure for the period before the October 2008 financial crisis period. Also, the time-varying market models are generally statistically significant at the different levels of modelling accuracy for each other in the in-sample procedure, but not generally statistically significant at the different levels of forecasting accuracy for each other in the out-of-sample procedure when compared to the in-sample procedure. To sum up, the time-varying Linear Market Model (TvLMM) seems to be the preferable model, especially for emerging markets.

5.4 Graphical Summary

This section discussed the the scatter plots which depicted the relationship between each stock market excess return and the MSCI World market excess return for both developed and emerging markets for the entire period from July 2002 to July 2012 as seen in Figures 1 and 2, respectively. To save the space remaining, the sub-periods before and after the October 2008 financial crisis and out-of-sample procedure are not displayed here. These plots include the fitted models for the Linear Market Model (LMM) (generally exhibiting the worst modelling performance), the GAM function (GAM with degrees of freedom) (generally exhibiting a better modelling performance than other Higher order DGPs) and the time-varying Linear Market

Model (TvLMM)(generally exhibiting the best modelling performance). The remaining fitted model curves are not shown for ease of presentation.

Insert Figure [1]

Insert Figure [2]

It can be concluded that the time-varying Linear Market Model (TvLMM) provides a much closer fit to the stock markets data (especially for emerging markets) than to the other models due to the time-varying relationship estimated between $R_{it} - R_{ft}$ and $R_{mt} - R_{ft}$ which allows one to capture the short-term volatility in this relationship. Note that, for the vast majority of the data, with the exception of extreme values (which are common in the emerging markets than in the developed markets) in the data sample, the time-varying Linear Market Model (TvLMM) and the GAM exhibit estimated relationships which are close to that of the Linear Market Model (LMM).

To sum up, the time-varying Linear Market Model (TvLMM) with time-varying systematic covariance (*beta*) risk seems to be the most appropriate model, especially for emerging markets.

5.5 Best Forecasting Model

The best forecasting model for the developed and emerging markets for the different aforementioned time periods are discussed here. Thus, the time-varying Linear Market Model (TvLMM), which generally exhibits the best predictive performance for both the developed and emerging markets for each time period, is examined in greater detail here. Tables 16, 17 and 18 present the time-varying Linear Market Model (TvLMM) via KFMR parameter estimates (with *standard errors*) defined in equations (4.6) and (4.7) for each time period, respectively.

Insert Table [16]

Insert Table [17]

Insert Table [18]

The average estimated $\hat{H}_i$ and $\hat{Q}_i$ values for the developed markets are lower than those of the developed markets for all three time periods. This may be due to the fact that the developed markets are more stable than the emerging markets overall, as evidenced in Tables 2 and 3. Also, the average estimated values of $\hat{Q}_i$ are higher than those of $\hat{H}_i$, meaning that the state variance ($\hat{Q}_i$) captures the volatility of the stock market excess returns more than the observation variance ($\hat{H}_i$).

The average temporal autocorrelation ($\hat{\phi}_i$) in the time-varying systematic covariance (*beta*) risk for the emerging markets is higher than for the developed markets for each time period. This value are much closer to 0 than to 1. This suggests that the time-varying systematic covariance risks change rapidly due to there being low autocorrelation.

It is worth noting that the average estimated regression intercept, $\hat{\kappa}_i$, of all 18 global markets is positive and is close to zero for all three time periods. This case can be an anticipated result for $\hat{\kappa}_i$ in the Linear Market Model (consistent with Two-Moment CAPM). This is likely to be a consequence of the risk-free rate (R_{ft}) being subtracted before estimation (see (Campbell et al., 1997)). Note that the average $\hat{\kappa}_i$ is negative (-0.001) for the developed markets for the period after the October 2008 financial crisis. This indicates that the actual return on developed country i's stock market is lower than the expected return from the time-varying Linear Market Model during that same period. The estimated average mean of the time-varying systematic covariance $\hat{\alpha}_{1it}$ for all of the 18 global markets for each time period is

positive, and is higher than 1. This means that the stock market is more volatile than the MSCI World market portfolio. Moreover, the standard errors of the time-varying systematic covariance ($\hat{\alpha}_{1it}$) risk in the emerging markets are generally higher than that of the developed markets. This case provides a wider range of time-varying systematic covariance ($\hat{\alpha}_{1it} = \hat{\beta}_{imt}$) risks in the emerging markets than in the developed markets. This suggests that the relationship between excess returns in the emerging markets (and the MSCI World market portfolio as a whole) is less consistent than that of the developed markets (and the MSCI World market portfolio as a whole) (Neslihanoglu, 2014).

Tables 16, 17 and 18 provide the diagnostic test statistics (with $p - value$) of the time-varying Linear Market Model (TvLMM) for both the developed and emerging markets for each time period, respectively. Furthermore, according to the Jarque-Bera (JB) test (equations (4.16) and (4.17)), these residuals are not normally distributed at the 5% significance level for most markets for each time period, implying that the time-varying Linear Market Model is poor in terms of non-normal residuals. According to the H test (equation (4.18)), the null hypothesis of no heteroskedasticity cannot be rejected at the 5% significance level for most markets for each time period, implying that the time-varying Linear Market Model can be adequate in terms of no heteroskedasticity for most markets for each time period. According to the Ljung-Box (LB) test (equations (4.19) and (4.20)), the null hypothesis of no autocorrelation can be rejected at the 5% significance level for most markets for each time period, meaning that the time-varying Linear Market Model is not adequate in terms of no autocorrelation. Note that the assumptions of normally distributed and independent (no autocorrelation) residuals are generally violated here; therefore, the performance of the KFMR can be affected. The possible extensions of KFMR

are discussed in the PhD thesis of Neslihanoglu (2014). For example, the Gaussian distribution can be replaced by a heavy-distribution, such as a t distribution, or by an asymmetric distribution, such as a skewed- t distribution, in order that the KFMR may deal with the non-normally distributed residuals (Durbin and Koopman, 2001). In addition, the time-varying intercept term (κ_i) using a random walk specification of the state space model via Kalman Filter (KFRW) is another possible approach given that the slope parameter ($\alpha_{1it} = \beta_{imt}$) is also used in order to handle the dependent (autocorrelation) residuals.

6 Conclusions

This paper examines the forecasting ability of non-linear specifications of the market model. The analysis is implemented using data from stock indices of several developed and emerging stock exchanges. The empirical findings are in favour of time-varying market model approaches which outperform linear model specifications both in terms of model fit and predictability, especially for emerging stock exchanges.

This comparative analysis sheds much light on the necessity of non-linear models in the explanation of stock market returns. Time-varying model specifications outperform the unconditional models, with the structural changes of the financial time series being better absorbed within the higher moments of the CAPM. This is apparent in the in-sample model fit and in the out-of-sample forecasting ability of the examined models.

Finally, we provide evidence in favour of the higher moment model specification when dealing with data regarding emerging stock markets, thereby underlying the impor-

tance of non-linear models when analysing market inefficiencies.

A Appendix

The proof of the link between Higher order DGPs and Higher-Moment CAPMs

This section displays the proof that a link exists between the newly formulated forms of the Higher order Data Generating Processes (DGPs) as simple polynomial extensions of the Linear Market Model; namely, the Quadratic Market Model (which allows for systematic covariance and systematic skewness), the Cubic Market Model (which allows for systematic covariance, systematic skewness and systematic kurtosis), and their equivalent Higher-Moment CAPMs (the Three- and Four-Moment CAPMs). This link is referred to often throughout this paper. This proof is primarily taken Chapter 2 of my PhD Thesis (Neslihanoglu, 2014).

The Cubic Market Model is consistent with the Four-Moment CAPM

The Four-Moment CAPM, in equilibrium, can be represented as

$$E(R_i) - R_f = c_1\beta_{im} + c_2\gamma_{im} + c_3\delta_{im}, \quad (\text{A.1})$$

where $E(R_i)$ and R_f are the expected asset return on asset i and the risk-free rate, respectively. The systematic risk measures, systematic covariance (β_{im}), systematic

skewness (γ_{im}), and systematic kurtosis (δ_{im}) are defined as

$$\beta_{im} = \frac{E[(R_i - E(R_i))(R_m - E(R_m))]}{E[(R_m - E(R_m))^2]} = \frac{Cov(R_i, R_m)}{E[(R_m - E(R_m))^2]}, \quad (\text{A.2})$$

$$\gamma_{im} = \frac{E[(R_i - E(R_i))(R_m - E(R_m))^2]}{E[(R_m - E(R_m))^3]} = \frac{Cos(R_i, R_m)}{E[(R_m - E(R_m))^3]}, \quad (\text{A.3})$$

$$\delta_{im} = \frac{E[(R_i - E(R_i))(R_m - E(R_m))^3]}{E[(R_m - E(R_m))^4]} = \frac{Cok(R_i, R_m)}{E[(R_m - E(R_m))^4]}. \quad (\text{A.4})$$

Let c_1 , c_2 , and c_3 be the market prices or risk premiums for systematic covariance (β_{im}), systematic skewness (γ_{im}) and systematic kurtosis (δ_{im}), respectively; these are given by

$$c_1 = \frac{dE(R_i)}{d\sigma(R_i)}\sigma(R_m) \quad (\text{A.5})$$

$$c_2 = \frac{dE(R_i)}{dS(R_i)}S(R_m), \quad (\text{A.6})$$

$$c_3 = \frac{dE(R_i)}{dK(R_i)}K(R_m). \quad (\text{A.7})$$

Here, $\sigma(\cdot)$, $S(\cdot)$ and $K(\cdot)$ refer to the standard deviation (*volatility*), skewness and kurtosis of the market portfolio return, R_m , and the asset returns, R_m , respectively.

For example, those for the market portfolio return, R_m , are defined as

$$\sigma(R_m) = E[(R_m - E(R_m))^2]^{1/2}, \quad (\text{A.8})$$

$$S(R_m) = E[(R_m - E(R_m))^3]^{1/3}, \quad (\text{A.9})$$

$$K(R_m) = E[(R_m - E(R_m))^4]^{1/4}. \quad (\text{A.10})$$

Here, $\sigma^2(\cdot)$, $S^3(\cdot)$ and $K^4(\cdot)$ are the second, third, and fourth respective central moments of the market portfolio return, R_m . In the financial literature and throughout this paper, however, $S(\cdot)$ and $K(\cdot)$ are called skewness and kurtosis, respectively. Note that the derivation of the Four-Moment CAPM has been omitted for the sake of brevity. For greater detail about this derivation, see my PhD thesis (Neslihanoglu, 2014).

The Cubic Market Model is given by

$$R_i - R_f = \kappa_i + \alpha_{1i}(R_m - R_f) + \alpha_{2i}(R_m - R_f)^2 + \alpha_{3i}(R_m - R_f)^3 + \varepsilon_i, \quad (\text{A.11})$$

which is a simple polynomial regression of order three of the response, $R_i - R_f$ on the covariate $(R_m - R_f)$. The errors are assumed to be independent and identically distributed with $E(\varepsilon_i) = 0$ and $Var(\varepsilon_i) = \sigma^2$.

To show the link between the Four-Moment CAPM and the Cubic Market Model, the systematic risk measures (β_{im} , γ_{im} and δ_{im}) can be expressed as

$$\begin{aligned} \beta_{im} &= \frac{E[(R_i - E(R_i))(R_m - E(R_m))]}{E[(R_m - E(R_m))^2]} = \alpha_{1i} \\ &+ \alpha_{2i} \frac{E[((R_m - R_f)^2 - E(R_m - R_f)^2)(R_m - E(R_m))]}{E[(R_m - E(R_m))^2]} \\ &+ \alpha_{3i} \frac{E[((R_m - R_f)^3 - E(R_m - R_f)^3)(R_m - E(R_m))]}{E[(R_m - E(R_m))^2]}, \end{aligned} \quad (\text{A.12})$$

$$\begin{aligned} \gamma_{im} &= \frac{E[(R_i - E(R_i))(R_m - E(R_m))^2]}{E[(R_m - E(R_m))^3]} = \alpha_{1i} \\ &+ \alpha_{2i} \frac{E[((R_m - R_f)^2 - E(R_m - R_f)^2)(R_m - E(R_m))^2]}{E[(R_m - E(R_m))^3]} \\ &+ \alpha_{3i} \frac{E[((R_m - R_f)^3 - E(R_m - R_f)^3)(R_m - E(R_m))^2]}{E[(R_m - E(R_m))^3]}, \end{aligned} \quad (\text{A.13})$$

$$\begin{aligned} \delta_{im} &= \frac{E[(R_i - E(R_i))(R_m - E(R_m))^3]}{E[(R_m - E(R_m))^4]} = \alpha_{1i} \\ &+ \alpha_{2i} \frac{E[((R_m - R_f)^2 - E(R_m - R_f)^2)(R_m - E(R_m))^3]}{E[(R_m - E(R_m))^4]} \\ &+ \alpha_{3i} \frac{E[((R_m - R_f)^3 - E(R_m - R_f)^3)(R_m - E(R_m))^3]}{E[(R_m - E(R_m))^4]}. \end{aligned} \quad (\text{A.14})$$

Note that the Four-Moment CAPM can only be employed if the DGP is at least cubic that is, α_{3i} should be statistically significantly different from zero. If not, there will be collinearity in the systematic risk measures (β_{im} , γ_{im} and δ_{im}) of the Four-Moment CAPM.

Proof: For systematic covariance, β_{im} : Start by taking the expectations of the Cubic Market Model which gives

$$\begin{aligned} E(R_i - R_f) &= \kappa_i + \alpha_{1i}E(R_m - R_f) \\ &+ \alpha_{2i}E(R_m - R_f)^2 + \alpha_{3i}E(R_m - R_f)^3 + E(\varepsilon_i). \end{aligned} \quad (\text{A.15})$$

Subtract from equation (A.11) to (A.15),

$$\begin{aligned} (R_i - R_f) - E(R_i - R_f) &= \alpha_{1i}((R_m - R_f) - E(R_m - R_f)) \\ &+ \alpha_{2i}((R_m - R_f)^2 - E(R_m - R_f)^2) \\ &+ \alpha_{3i}((R_m - R_f)^3 - E(R_m - R_f)^3) + \varepsilon_i, \\ R_i - E(R_i) &= \alpha_{1i}(R_m - E(R_m)) \\ &+ \alpha_{2i}((R_m - R_f)^2 - E(R_m - R_f)^2) \\ &+ \alpha_{3i}((R_m - R_f)^3 - E(R_m - R_f)^3) + \varepsilon_i. \end{aligned} \quad (\text{A.16})$$

Multiply both sides of equation (A.16) by $R_m - E(R_m)$,

$$\begin{aligned} (R_i - E(R_i))(R_m - E(R_m)) &= \alpha_{1i}(R_m - E(R_m))(R_m - E(R_m)) \\ &+ \alpha_{2i}((R_m - R_f)^2 - E(R_m - R_f)^2)(R_m - E(R_m)) \\ &+ \alpha_{3i}((R_m - R_f)^3 - E(R_m - R_f)^3)(R_m - E(R_m)) \\ &+ \varepsilon_i(R_m - E(R_m)). \end{aligned} \quad (\text{A.17})$$

Take the expected values both sides of equation (A.17)

$$\begin{aligned} E[(R_i - E(R_i))(R_m - E(R_m))] & \\ &= \alpha_{1i}E[(R_m - E(R_m))(R_m - E(R_m))] \\ &+ \alpha_{2i}E[((R_m - R_f)^2 - E(R_m - R_f)^2)(R_m - E(R_m))] \\ &+ \alpha_{3i}E[((R_m - R_f)^3 - E(R_m - R_f)^3)(R_m - E(R_m))]. \end{aligned} \quad (\text{A.18})$$

Divide both sides in equation (A.18) by the variance of the market return,

$E[(R_m - E(R_m))^2]$. This gives

$$\begin{aligned} \beta_{im} &= \frac{E[(R_i - E(R_i))(R_m - E(R_m))]}{E[(R_m - E(R_m))^2]} = \alpha_{1i} \\ &+ \alpha_{2i} \frac{E[((R_m - R_f)^2 - E(R_m - R_f)^2)(R_m - E(R_m))]}{E[(R_m - E(R_m))^2]} \\ &+ \alpha_{3i} \frac{E[((R_m - R_f)^3 - E(R_m - R_f)^3)(R_m - E(R_m))]}{E[(R_m - E(R_m))^2]}. \end{aligned} \quad (\text{A.19})$$

For systematic skewness, γ_{im} : Similarly, multiply both side of equation (A.16) by

$(R_m - E(R_m))^2$

$$\begin{aligned} &(R_i - E(R_i))(R_m - E(R_m))^2 \\ &= \alpha_{1i}(R_m - E(R_m))(R_m - E(R_m))^2 \\ &+ \alpha_{2i}((R_m - R_f)^2 - E(R_m - R_f)^2)(R_m - E(R_m))^2 \\ &+ \alpha_{3i}((R_m - R_f)^3 - E(R_m - R_f)^3)(R_m - E(R_m))^2 \\ &+ \varepsilon_i(R_m - E(R_m))^2. \end{aligned} \quad (\text{A.20})$$

Take the expected values both sides of equation (A.20)

$$\begin{aligned} &E[(R_i - E(R_i))(R_m - E(R_m))^2] \\ &= \alpha_{1i}E[(R_m - E(R_m))(R_m - E(R_m))^2] \\ &+ \alpha_{2i}E[((R_m - R_f)^2 - E(R_m - R_f)^2)(R_m - E(R_m))^2] \\ &+ \alpha_{3i}E[((R_m - R_f)^3 - E(R_m - R_f)^3)(R_m - E(R_m))^2]. \end{aligned} \quad (\text{A.21})$$

Divide both sides of equation (A.21) by the third central moment of the market

return, $E[(R_m - E(R_m))^3]$. This results in

$$\begin{aligned} \gamma_{im} &= \frac{E[(R_i - E(R_i))(R_m - E(R_m))^2]}{E[(R_m - E(R_m))^3]} = \alpha_{1i} \\ &+ \alpha_{2i} \frac{E[((R_m - R_f)^2 - E(R_m - R_f)^2)(R_m - E(R_m))^2]}{E[(R_m - E(R_m))^3]} \\ &+ \alpha_{3i} \frac{E[((R_m - R_f)^3 - E(R_m - R_f)^3)(R_m - E(R_m))^2]}{E[(R_m - E(R_m))^3]}. \end{aligned} \quad (\text{A.22})$$

For systematic kurtosis, δ_{im} : Finally, multiply both sides of equation (A.16) by

$$(R_m - E(R_m))^3$$

$$\begin{aligned} &(R_i - E(R_i))(R_m - E(R_m))^3 \\ &= \alpha_{1i}(R_m - E(R_m))(R_m - E(R_m))^3 \\ &+ \alpha_{2i}((R_m - R_f)^2 - E(R_m - R_f)^2)(R_m - E(R_m))^3 \\ &+ \alpha_{3i}((R_m - R_f)^3 - E(R_m - R_f)^3)(R_m - E(R_m))^3 \\ &+ \varepsilon_i(R_m - E(R_m))^3. \end{aligned} \quad (\text{A.23})$$

Take the expected values both sides of equation (A.23)

$$\begin{aligned} &E[(R_i - E(R_i))(R_m - E(R_m))^3] \\ &= \alpha_{1i}E[(R_m - E(R_m))(R_m - E(R_m))^3] \\ &+ \alpha_{2i}E[((R_m - R_f)^2 - E(R_m - R_f)^2)(R_m - E(R_m))^3] \\ &+ \alpha_{3i}E[((R_m - R_f)^3 - E(R_m - R_f)^3)(R_m - E(R_m))^3]. \end{aligned} \quad (\text{A.24})$$

Divide both sides in equation (A.24) by the fourth central moment of the market

return, $E[(R_m - E(R_m))^4]$. This results in the following

$$\begin{aligned} \delta_{im} &= \frac{E[(R_i - E(R_i))(R_m - E(R_m))^3]}{E[(R_m - E(R_m))^4]} = \alpha_{1i} \\ &+ \alpha_{2i} \frac{E[((R_m - R_f)^2 - E(R_m - R_f)^2)(R_m - E(R_m))^3]}{E[(R_m - E(R_m))^4]} \\ &+ \alpha_{3i} \frac{E[((R_m - R_f)^3 - E(R_m - R_f)^3)(R_m - E(R_m))^3]}{E[(R_m - E(R_m))^4]}. \end{aligned} \quad (\text{A.25})$$

Given the above, the definition of the systematic risk measures β_{im} , γ_{im} , and δ_{im} are consistent with the Four-Moment CAPM.

Throughout this paper, the parameters of the Cubic Market Model must be estimated from time series data of returns. Assuming that the parameters are constant over the whole time period being considered, then the excess return on asset i in period t ($R_{it} - R_{ft}$) might be assumed to be generated by the following model

$$R_{it} - R_{ft} = \kappa_i + \alpha_{1i}(R_{mt} - R_{ft}) + \alpha_{2i}(R_{mt} - R_{ft})^2 + \alpha_{3i}(R_{mt} - R_{ft})^3 + \varepsilon_{it}, \quad (\text{A.26})$$

where R_{it} , R_{ft} and R_{mt} are the returns on asset i , the risk-free rate and the market portfolio at time t ($t \in \{1, \dots, T\}$). The errors are assumed to be independent and identically distributed with $\varepsilon_{it} \sim N(0, \sigma^2)$. Here, using $\hat{\kappa}_i$, $\hat{\alpha}_{1i}$, $\hat{\alpha}_{2i}$, and $\hat{\alpha}_{3i}$ from the maximum likelihood estimation, and others, for example,

$E((R_m - R_f)^2) \approx \frac{1}{T} \sum_{t=1}^T (R_{mt} - R_{ft})^2$ then the systematic risk measures in equations (A.12), (A.13) and (A.14) can be estimated.

The Quadratic Market Model is consistent with Three-Moment CAPM

When $\delta_{im} = 0$ in equation (A.1), the equilibrium model of the Three-Moment CAPM can be represented as

$$E(R_i) - R_f = c_1\beta_{im} + c_2\gamma_{im}, \quad (\text{A.27})$$

where $E(R_i)$ and R_f are the expected asset returns on asset i and the risk-free rate. The systematic risk measures, β_{im} and γ_{im} , which are respectively the systematic

covariance (*beta*) and systematic skewness (*co-skewness*), respectively are defined as

$$\beta_{im} = \frac{E[(R_i - E(R_i))(R_m - E(R_m))]}{E[(R_m - E(R_m))^2]} = \frac{Cov(R_i, R_m)}{E[(R_m - E(R_m))^2]}, \quad (\text{A.28})$$

$$\gamma_{im} = \frac{E[(R_i - E(R_i))(R_m - E(R_m))^2]}{E[(R_m - E(R_m))^3]} = \frac{Cos(R_i, R_m)}{E[(R_m - E(R_m))^3]}. \quad (\text{A.29})$$

and c_1 , and c_2 are the market prices (or risk premiums) for the systematic covariance (β_{im}) and the systematic skewness (γ_{im}), respectively. These are given as follows

$$c_1 = \frac{dE(R_i)}{d\sigma(R_i)}\sigma(R_m) \quad (\text{A.30})$$

$$c_2 = \frac{dE(R_i)}{dS(R_i)}S(R_m). \quad (\text{A.31})$$

Let $\sigma(\cdot)$ and $S(\cdot)$ be called the standard deviation (*volatility*) and skewness of the market portfolio returns, R_m , and the asset returns, R_i , respectively, which are defined in equation (A.8) and (A.9), respectively.

When $\alpha_{3i} = 0$ in equation (A.11), the Quadratic Market Model is given as follows

$$R_i - R_f = \kappa_i + \alpha_{1i}(R_m - R_f) + \alpha_{2i}(R_m - R_f)^2 + \varepsilon_i, \quad (\text{A.32})$$

This is a simple polynomial regression of order two of the response, $R_i - R_f$ on the covariate $(R_m - R_f)$. The errors are assumed to be independent and identically distributed with $E(\varepsilon_i) = 0$ and $Var(\varepsilon_i) = \sigma^2$.

To show the link between the Three-Moment CAPM and the Quadratic Market Model, the systematic risk measures (β_{im} and γ_{im}) can be expressed as

$$\begin{aligned} \beta_{im} &= \frac{E[(R_i - E(R_i))(R_m - E(R_m))]}{E[(R_m - E(R_m))^2]} = \alpha_{1i} \\ &+ \alpha_{2i} \frac{E[((R_m - R_f)^2 - E(R_m - R_f)^2)(R_m - E(R_m))]}{E[(R_m - E(R_m))^2]}, \end{aligned} \quad (\text{A.33})$$

$$\begin{aligned} \gamma_{im} &= \frac{E[(R_i - E(R_i))(R_m - E(R_m))^2]}{E[(R_m - E(R_m))^3]} = \alpha_{1i} \\ &+ \alpha_{2i} \frac{E[((R_m - R_f)^2 - E(R_m - R_f)^2)(R_m - E(R_m))^2]}{E[(R_m - E(R_m))^3]}. \end{aligned} \quad (\text{A.34})$$

Note that the Three-Moment CAPM could only be employed if the DGP was at least quadratic, that is, α_{2i} should be statistically significantly different from zero. If not, β_{im} and γ_{im} are equal.

Throughout this paper, the parameters of the Quadratic Market Model must be estimated from the time series data of returns. Assuming that the parameters are constant over the whole time period being considered, then the excess return on asset i in period t ($R_{it} - R_{ft}$) might be assumed to be generated by the following model

$$R_{it} - R_{ft} = \kappa_i + \alpha_{1i}(R_{mt} - R_{ft}) + \alpha_{2i}(R_{mt} - R_{ft})^2 + \varepsilon_{it}, \quad (\text{A.35})$$

where R_{it} , R_{ft} , and R_{mt} are the returns on asset i , the risk-free rate and the market portfolio at time t ($t \in \{1, \dots, T\}$). The errors are assumed to be independent and identically distributed with $\varepsilon_{it} \sim N(0, \sigma^2)$. Here, using $\hat{\kappa}_i$, $\hat{\alpha}_{1i}$, and $\hat{\alpha}_{2i}$ from the maximum likelihood estimation, and others, for example,

$E((R_m - R_f)^2) \approx \frac{1}{T} \sum_{t=1}^T (R_{mt} - R_{ft})^2$ then the systematic risk measures in equations (A.33) and (A.34) can be estimated.

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B Tables

Table 1: Country stock market and regional classification.

Market	Region	Market	Region
<i>Developed</i>		<i>Emerging</i>	
France	Europe & Middle East	Brazil	Americas
Germany	Europe & Middle East	Chile	Americas
Italy	Europe & Middle East	India	Asia
Japan	Pacific	Korea	Asia
Norway	Europe & Middle East	Malaysia	Asia
Sweden	Europe & Middle East	Mexico	Americas
Switzerland	Europe & Middle East	Poland	Europe, Middle East & Africa
UK	Europe & Middle East	Russia	Europe, Middle East & Africa
USA	Americas	South Africa	Europe, Middle East & Africa

Notes: Further details about country stock market and regional classifications which are maintained by the Morgan Stanley Capital International Incorporation (MSCI Inc.) are available at <http://www.msci.com/products/indices/>.

Table 2: Descriptive statistics of weekly returns entire data.

Market	Mean	Std.Dev. ^a	Skewness	Kurtosis	JB^b	$LB(23)^c$
World	0.0007	0.0264	-0.91	7.62	535.90*	200.08*
<i>Developed</i>						
France	0.0003	0.0376	-0.62	5.69	190.31*	195.38*
Germany	0.0007	0.0394	-0.84	6.09	268.87*	175.86*
Italy	-0.0007	0.0385	-0.53	5.73	186.32*	314.50*
Japan	0.0002	0.0281	-0.39	5.59	159.38*	138.98*
Norway	0.0017	0.0480	-1.04	8.06	650.17*	407.92*
Sweden	0.0018	0.0429	-0.73	6.08	253.39*	205.03*
Switzerland	0.0011	0.0285	-0.50	5.27	134.08*	156.26*
UK	0.0006	0.0317	-0.63	5.96	224.26*	335.73*
USA	0.0008	0.0258	-0.77	8.73	767.03*	162.09*
Average	0.0007	0.0356	-0.67	6.36		
<i>Emerging</i>						
Brazil	0.0033	0.0526	-1.68	13.50	2640.05*	47.37*
Chile	0.0032	0.0343	-1.80	19.08	5904.80*	17.54
India	0.0028	0.0431	-0.26	5.56	148.12*	228.78*
Korea	0.0017	0.0469	-0.42	10.09	1108.05*	521.15*
Malaysia	0.0018	0.0243	-0.16	5.46	134.40*	152.18*
Mexico	0.0027	0.0394	-1.99	19.05	5943.54*	45.60*
Poland	0.0015	0.0498	-1.08	7.07	461.36*	277.23*
Russia	0.0018	0.0587	-1.44	15.55	3604.33*	215.49*
South Africa	0.0025	0.0422	-0.89	7.61	530.74*	219.64*
Average	0.0024	0.0435	-1.08	11.44		
Risk-free rate	0.0004	0.0003	0.63	1.90	60.96*	10762.17*

Notes: The portfolio has 522 observations for the weekly returns for each of the 18 global markets. ^aStd.Dev. is the standard deviation. ^b JB is the Jarque-Bera test statistic for testing the normality. JB follows χ^2 with 2 degrees of freedom and the critical value at the 5% level is 5.99. ^c LB statistic follows χ^2 with 23 degrees of freedom so the critical value at the 5% level is 35.17. * means the appropriate null hypothesis is rejected at the 5% significance level.

Table 3: Descriptive statistics of weekly returns before and after October 2008.

	Before October 2008					
Market	Mean	Std.Dev. ^a	Skewness	Kurtosis	JB^b	$LB(18)^c$
World	0.0012	0.0210	-0.26	5.37	79.00*	30.61*
<i>Developed</i>						
France	0.0017	0.0312	-0.51	6.86	214.32*	64.87*
Germany	0.0019	0.0333	-0.76	6.93	238.58*	33.07*
Italy	0.0013	0.0264	-0.83	5.67	133.16*	25.44
Japan	0.0008	0.0260	-0.36	3.31	8.45	12.80
Norway	0.0031	0.0387	-1.11	5.84	175.13*	29.17*
Sweden	0.0023	0.0363	-0.72	7.02	244.57*	40.25*
Switzerland	0.0017	0.0268	-0.37	6.44	166.86*	54.56*
UK	0.0011	0.0259	-0.16	5.44	81.81*	46.58*
USA	0.0009	0.0209	0.17	6.15	134.94*	35.69*
Average	0.0016	0.0295	-0.52	5.96		
<i>Emerging</i>						
Brazil	0.0057	0.0483	-0.83	5.20	101.98*	19.49
Chile	0.0039	0.0289	-0.36	3.86	17.06*	13.52
India	0.0044	0.0396	-0.60	5.94	135.55*	37.64*
Korea	0.0020	0.0364	-0.22	2.86	2.78*	16.14
Malaysia	0.0013	0.0225	-0.29	4.02	18.55*	21.61
Mexico	0.0034	0.0328	-0.63	4.08	36.84*	18.36
Poland	0.0039	0.0380	-0.70	4.28	48.17*	23.36
Russia	0.0034	0.0470	-0.37	7.75	311.54*	6.27
South Africa	0.0029	0.0350	-0.65	3.67	28.79*	27.30
Average	0.0034	0.0365	-0.52	4.63		
Risk-free rate	0.0006	0.0003	0.12	1.44	33.54*	321.58*
	After October 2008					
Market	Mean	Std.Dev. ^a	Skewness	Kurtosis	JB^b	$LB(14)^c$
World	0.0012	0.0307	-0.66	4.65	35.98*	16.75
<i>Developed</i>						
France	-0.0006	0.0441	-0.39	3.84	10.59*	16.68
Germany	0.0002	0.0450	-0.63	4.66	34.93*	19.19*
Italy	-0.0027	0.0501	-0.17	3.60	3.90	19.18
Japan	-0.0003	0.0272	-0.36	4.27	17.23*	9.57
Norway	0.0016	0.0539	-1.02	6.91	156.59*	39.77*
Sweden	0.0025	0.0488	-0.67	4.80	40.48*	26.50*
Switzerland	0.0010	0.0301	-0.47	3.55	9.61*	9.94
UK	0.0010	0.0372	-0.64	4.93	43.02*	14.95
USA	0.0019	0.0297	-0.54	5.41	56.18*	16.17
Average	0.0004	0.0408	-0.56	4.48		
<i>Emerging</i>						
Brazil	0.0017	0.0484	-0.41	3.62	8.55*	40.32*
Chile	0.0034	0.0338	-0.24	4.55	21.22*	12.81
India	0.0020	0.0441	0.33	4.74	27.92*	32.48*
Korea	0.0029	0.0474	-0.40	4.67	27.63*	23.09
Malaysia	0.0034	0.0235	-0.36	3.83	9.69*	6.81
Mexico	0.0034	0.0387	-0.19	4.30	14.74*	22.33
Poland	-0.0003	0.0598	-0.71	4.87	44.38*	48.41*
Russia	0.0014	0.0607	-0.80	6.26	106.36*	16.12
South Africa	0.0034	0.0448	0.00	3.96	7.45*	26.07*
Average	0.0024	0.0446	-0.31	4.53		
Risk-free rate	0.0001	0.0001	2.43	9.05	485.66*	1251.6*

Notes: The portfolio has 323 (193) observations for the weekly returns for each of the 18 global markets for the period before (after) the October 2008 financial crisis. See *a*, *b* in Table 2. ^c LB statistic follows χ^2 with 18 (14) degrees of freedom so the critical value at the 5% level is 28.87 (23.68). * means the appropriate null hypothesis is rejected at the 5% significance level.

Table 4: MAE ($\times 10^2$) and MSE ($\times 10^4$) of in-sample model fit entire data.

	<i>MAE</i> ($\times 10^2$)						
Model	LMM	QMM	CMM	GAM	TvLMM	TvQMM	TvCMM
<i>Developed</i>							
France	1.027	1.021	1.020	1.013	0.785	0.785	0.786
Germany	1.196	1.196	1.186	1.182	0.796	0.899	0.897
Italy	1.402	1.404	1.401	1.371	1.046	1.047	1.057
Japan	1.643	1.638	1.630	1.597	1.247	1.250	1.255
Norway	2.105	2.094	2.087	2.033	1.709	1.789	1.794
Sweden	1.516	1.511	1.504	1.500	1.178	1.204	1.182
Switzerland	1.124	1.120	1.113	1.115	0.941	0.947	0.952
UK	0.947	0.947	0.946	0.929	0.789	0.795	0.802
USA	0.600	0.600	0.590	0.588	0.423	0.422	0.427
Mean	1.285	1.281	1.275	1.259	0.990	1.015	1.017
Median	1.196	1.196	1.186	1.182	0.941	0.947	0.952
<i>Emerging</i>							
Brazil	2.501	2.499	2.496	2.407	1.671	1.734	1.794
Chile	1.862	1.861	1.867	1.784	1.428	1.550	1.597
India	2.571	2.565	2.556	2.523	2.261	2.204	2.258
Korea	2.472	2.462	2.461	2.412	1.725	1.726	1.726
Malaysia	1.445	1.444	1.433	1.423	1.126	1.126	1.143
Mexico	1.730	1.724	1.724	1.672	1.187	1.195	1.202
Poland	2.578	2.577	2.551	2.535	1.845	1.835	1.861
Russia	2.993	2.919	2.900	2.855	2.177	2.198	2.195
SouthAfrica	2.108	2.101	2.101	2.076	1.619	1.626	1.631
Mean	2.251	2.239	2.232	2.188	1.671	1.688	1.712
Median	2.472	2.462	2.461	2.407	1.671	1.726	1.726
	<i>MSE</i> ($\times 10^4$)						
Model	LMM	QMM	CMM	GAM	TvLMM	TvQMM	TvCMM
<i>Developed</i>							
France	1.944	1.925	1.909	1.893	1.116	1.116	1.118
Germany	2.694	2.693	2.623	2.607	1.110	1.398	1.389
Italy	3.507	3.498	3.484	3.325	1.949	1.952	1.980
Japan	4.672	4.635	4.603	4.473	2.675	2.688	2.714
Norway	8.394	8.184	8.101	7.548	5.218	5.749	5.771
Sweden	4.112	4.103	4.036	3.916	2.404	2.501	2.414
Switzerland	2.184	2.179	2.149	2.140	1.505	1.519	1.537
UK	1.696	1.695	1.680	1.621	1.170	1.185	1.200
USA	0.647	0.647	0.627	0.617	0.307	0.306	0.314
Mean	3.317	3.284	3.246	3.127	1.939	2.046	2.049
Median	2.694	2.693	2.623	2.607	1.505	1.519	1.537
<i>Emerging</i>							
Brazil	13.702	12.467	12.425	11.579	4.901	5.210	5.568
Chile	6.640	6.429	6.314	5.624	3.551	4.073	4.321
India	11.647	11.633	11.519	11.168	8.780	8.449	8.794
Korea	12.694	12.608	12.606	12.284	5.596	5.599	5.608
Malaysia	3.906	3.904	3.808	3.714	2.306	2.306	2.371
Mexico	5.747	5.404	5.336	4.972	2.404	2.439	2.469
Poland	12.632	12.571	12.164	11.986	6.104	6.060	6.168
Russia	18.218	16.817	16.666	15.797	8.912	9.129	9.106
SouthAfrica	7.800	7.720	7.684	7.525	4.400	4.437	4.460
Mean	10.332	9.950	9.836	9.405	5.217	5.300	5.429
Median	11.647	11.633	11.519	11.168	4.901	5.210	5.568

Notes: **Bold** displays the best market pricing model for each market in terms of the lowest MAE and MSE.

Table 5: MAE ($\times 10^2$) and MSE ($\times 10^4$) of in-sample model fit before October 2008.

	<i>MAE</i> ($\times 10^2$)						
Model	LMM	QMM	CMM	GAM	TvLMM	TvQMM	TvCMM
<i>Developed</i>							
France	0.931	0.932	0.905	0.902	0.746	0.794	0.758
Germany	1.099	1.094	1.087	1.087	0.787	0.870	0.867
Italy	1.079	1.067	1.068	1.051	0.881	0.889	0.894
Japan	1.635	1.623	1.566	1.521	1.318	1.362	1.387
Norway	2.182	2.096	2.098	2.070	1.953	1.902	1.907
Sweden	1.379	1.381	1.373	1.365	1.172	1.188	1.249
Switzerland	1.075	1.074	1.079	1.073	0.933	0.938	0.981
UK	0.911	0.908	0.907	0.883	0.813	0.812	0.819
USA	0.570	0.558	0.555	0.556	0.444	0.516	0.466
Mean	1.207	1.192	1.182	1.168	1.005	1.030	1.036
Median	1.079	1.074	1.079	1.073	0.881	0.889	0.894
<i>Emerging</i>							
Brazil	2.729	2.655	2.594	2.554	1.660	1.745	1.729
Chile	1.788	1.772	1.731	1.729	1.554	1.556	1.616
India	2.565	2.533	2.509	2.488	1.987	2.012	1.948
Korea	2.320	2.306	2.228	2.190	1.992	2.029	2.105
Malaysia	1.480	1.453	1.445	1.444	1.140	1.168	1.244
Mexico	1.776	1.742	1.705	1.696	1.454	1.463	1.473
Poland	2.378	2.340	2.347	2.329	2.226	2.205	2.252
Russia	2.965	2.899	2.909	2.881	2.395	2.432	2.428
SouthAfrica	2.119	2.074	2.074	2.063	1.917	1.893	1.876
Mean	2.236	2.197	2.172	2.153	1.814	1.834	1.852
Median	2.320	2.306	2.228	2.190	1.917	1.893	1.876
	<i>MSE</i> ($\times 10^4$)						
Model	LMM	QMM	CMM	GAM	TvLMM	TvQMM	TvCMM
<i>Developed</i>							
France	1.657	1.645	1.568	1.558	1.070	1.193	1.100
Germany	2.369	2.320	2.307	2.304	1.116	1.342	1.339
Italy	2.024	1.987	1.985	1.914	1.336	1.357	1.368
Japan	4.403	4.316	4.024	3.814	2.851	3.042	3.152
Norway	8.461	7.910	7.908	7.680	6.746	6.413	6.450
Sweden	3.225	3.170	3.076	2.969	2.254	2.305	2.541
Switzerland	2.029	2.024	1.949	1.912	1.492	1.504	1.612
UK	1.575	1.573	1.568	1.454	1.232	1.232	1.238
USA	0.546	0.525	0.521	0.521	0.335	0.441	0.361
Mean	2.921	2.830	2.767	2.681	2.048	2.092	2.129
Median	2.029	2.024	1.985	1.914	1.336	1.357	1.368
<i>Emerging</i>							
Brazil	16.379	14.357	13.725	12.880	4.867	5.273	5.162
Chile	5.533	5.445	5.162	5.147	4.052	4.128	4.403
India	11.927	11.660	11.224	11.050	6.872	7.043	6.599
Korea	8.784	8.603	8.258	8.050	6.492	6.711	7.348
Malaysia	3.777	3.629	3.522	3.528	2.201	2.317	2.647
Mexico	5.386	5.150	4.919	4.861	3.440	3.530	3.575
Poland	9.701	9.382	9.269	9.173	8.430	8.248	8.394
Russia	16.987	16.176	16.115	15.599	10.287	10.659	10.631
SouthAfrica	7.208	6.812	6.741	6.669	5.847	5.623	5.497
Mean	9.520	9.024	8.771	8.551	5.832	5.948	6.028
Median	8.784	8.603	8.258	8.050	5.847	5.623	5.497

Notes: **Bold** displays the best market pricing model for each market in terms of the lowest MAE and MSE.

Table 6: MAE ($\times 10^2$) and MSE ($\times 10^4$) of in-sample model fit after October 2008.

	<i>MAE</i> ($\times 10^2$)						
Model	LMM	QMM	CMM	GAM	TvLMM	TvQMM	TvCMM
<i>Developed</i>							
France	1.145	1.130	1.129	1.130	0.815	0.819	0.816
Germany	1.275	1.275	1.272	1.275	0.825	0.826	0.825
Italy	1.650	1.634	1.626	1.634	1.419	1.438	1.437
Japan	1.507	1.511	1.493	1.511	1.235	1.245	1.279
Norway	1.763	1.767	1.765	1.767	1.313	1.360	1.384
Sweden	1.660	1.665	1.658	1.665	1.254	1.313	1.300
Switzerland	1.102	1.108	1.096	1.108	0.990	0.994	1.032
UK	0.970	0.975	0.973	0.975	0.764	0.766	0.763
USA	0.582	0.582	0.578	0.582	0.412	0.412	0.428
Mean	1.295	1.294	1.288	1.294	1.003	1.019	1.029
Median	1.275	1.275	1.272	1.275	0.990	0.994	1.032
<i>Emerging</i>							
Brazil	1.964	1.964	1.954	1.964	1.746	1.745	1.738
Chile	1.821	1.815	1.815	1.815	1.543	1.415	1.406
India	2.459	2.450	2.432	2.450	2.202	2.118	2.171
Korea	2.339	2.339	2.340	2.339	1.825	1.829	1.844
Malaysia	1.260	1.257	1.254	1.257	0.976	0.979	0.994
Mexico	1.463	1.473	1.476	1.473	1.090	1.102	1.100
Poland	2.592	2.590	2.582	2.590	1.715	1.719	1.718
Russia	2.533	2.517	2.494	2.517	2.011	2.068	2.081
SouthAfrica	1.928	1.913	1.910	1.913	1.685	1.661	1.662
Mean	2.040	2.035	2.029	2.035	1.644	1.626	1.635
Median	1.964	1.964	1.954	1.964	1.715	1.719	1.718
	<i>MSE</i> ($\times 10^4$)						
Model	LMM	QMM	CMM	GAM	TvLMM	TvQMM	TvCMM
<i>Developed</i>							
France	2.235	2.208	2.188	2.208	1.086	1.103	1.091
Germany	2.812	2.806	2.794	2.806	1.129	1.130	1.127
Italy	4.629	4.542	4.531	4.542	3.414	3.487	3.494
Japan	4.011	3.993	3.925	3.993	2.752	2.766	2.921
Norway	5.987	5.709	5.697	5.709	3.128	3.335	3.448
Sweden	5.134	5.131	5.056	5.131	2.893	3.138	3.073
Switzerland	2.144	2.139	2.080	2.139	1.712	1.712	1.864
UK	1.761	1.749	1.749	1.749	1.072	1.084	1.079
USA	0.619	0.619	0.599	0.619	0.299	0.299	0.317
Mean	3.259	3.211	3.180	3.211	1.943	2.006	2.046
Median	2.812	2.806	2.794	2.806	1.712	1.712	1.864
<i>Emerging</i>							
Brazil	6.164	6.164	6.124	6.164	4.830	4.829	4.821
Chile	6.006	5.903	5.886	5.903	3.956	3.430	3.374
India	9.924	9.848	9.704	9.848	7.938	7.382	7.758
Korea	9.703	9.685	9.608	9.685	5.805	5.828	5.867
Malaysia	2.948	2.938	2.883	2.938	1.701	1.710	1.751
Mexico	3.684	3.654	3.648	3.654	2.026	2.052	2.041
Poland	13.527	13.464	13.280	13.464	5.583	5.587	5.532
Russia	12.482	12.196	12.117	12.196	7.860	8.687	8.750
SouthAfrica	6.013	5.977	5.973	5.977	4.580	4.477	4.479
Mean	7.828	7.759	7.691	7.759	4.920	4.887	4.930
Median	6.164	6.164	6.124	6.164	4.830	4.829	4.821

Notes: **Bold** displays the best market pricing model for each market in terms of the lowest MAE and MSE.

Table 7: MAE ($\times 10^2$) and MSE ($\times 10^4$) of out-of-sample forecast entire data.

	<i>MAE</i> ($\times 10^2$)						
Model	LMM	QMM	CMM	GAM	TvLMM	TvQMM	TvCMM
<i>Developed</i>							
France	1.247	1.201	1.211	1.238	0.799	0.798	0.796
Germany	1.502	1.479	1.491	1.477	0.855	0.853	0.851
Italy	2.101	2.046	2.045	2.053	1.532	1.555	1.544
Japan	1.421	1.412	1.421	1.420	1.064	1.066	1.079
Norway	1.461	1.473	1.497	1.454	1.188	1.195	1.203
Sweden	1.378	1.313	1.339	1.382	1.039	1.040	1.051
Switzerland	0.889	0.883	0.897	0.910	0.763	0.767	0.771
UK	0.746	0.758	0.795	0.842	0.617	0.626	0.639
USA	0.528	0.518	0.510	0.502	0.328	0.332	0.328
Mean	1.253	1.232	1.245	1.253	0.909	0.915	0.918
Median	1.378	1.313	1.339	1.382	0.855	0.853	0.851
<i>Emerging</i>							
Brazil	1.747	1.781	1.848	1.604	1.357	1.383	1.397
Chile	1.751	1.770	1.837	1.676	1.245	1.253	1.283
India	2.672	2.695	2.670	2.667	2.133	2.165	2.134
Korea	2.244	2.230	2.260	2.245	1.555	1.564	1.573
Malaysia	1.254	1.274	1.250	1.254	1.018	1.019	1.034
Mexico	1.322	1.304	1.414	1.296	0.894	0.897	0.943
Poland	2.394	2.390	2.415	2.400	1.632	1.624	1.647
Russia	2.224	2.188	2.038	2.027	1.525	1.527	1.516
SouthAfrica	1.770	1.767	1.763	1.770	1.324	1.325	1.328
Mean	1.931	1.933	1.944	1.882	1.409	1.418	1.428
Median	1.770	1.781	1.848	1.770	1.357	1.383	1.397
	<i>MSE</i> ($\times 10^4$)						
Model	LMM	QMM	CMM	GAM	TvLMM	TvQMM	TvCMM
<i>Developed</i>							
France	2.522	2.292	2.335	2.494	1.022	1.026	1.020
Germany	3.650	3.567	3.629	3.590	1.315	1.316	1.306
Italy	6.555	6.126	6.097	6.194	3.961	4.028	3.978
Japan	2.995	2.939	3.009	2.989	1.736	1.744	1.782
Norway	3.027	3.091	3.242	2.995	2.238	2.239	2.230
Sweden	3.536	3.292	3.358	3.518	2.239	2.245	2.232
Switzerland	1.521	1.487	1.516	1.598	1.115	1.117	1.118
UK	0.945	0.950	1.025	1.145	0.695	0.715	0.716
USA	0.473	0.442	0.436	0.429	0.213	0.215	0.213
Mean	2.803	2.687	2.738	2.773	1.615	1.627	1.621
Median	2.995	2.939	3.009	2.989	1.315	1.316	1.306
<i>Emerging</i>							
Brazil	4.811	5.475	5.457	3.844	2.926	3.091	3.076
Chile	6.196	7.101	6.898	6.409	3.146	3.184	3.258
India	10.233	10.456	10.232	10.161	6.098	6.229	6.085
Korea	7.568	7.534	7.977	7.566	4.241	4.272	4.328
Malaysia	2.722	2.788	2.728	2.707	1.791	1.793	1.855
Mexico	3.065	3.373	3.404	2.755	1.568	1.580	1.619
Poland	9.454	9.503	9.455	9.267	4.828	4.779	4.794
Russia	8.097	8.907	7.907	8.287	4.923	4.965	4.992
SouthAfrica	4.568	4.553	4.539	4.567	3.019	3.025	3.022
Mean	6.302	6.632	6.511	6.174	3.616	3.658	3.670
Median	6.196	7.101	6.898	6.409	3.146	3.184	3.258

Notes: **Bold** displays the best market pricing model for each market in terms of the lowest MAE and MSE.

Table 8: MAE ($\times 10^2$) and MSE ($\times 10^4$) of out-of-sample forecast before October 2008.

	<i>MAE</i> ($\times 10^2$)						
Model	LMM	QMM	CMM	GAM	TvLMM	TvQMM	TvCMM
<i>Developed</i>							
France	1.041	1.029	1.017	0.916	0.585	0.659	0.603
Germany	1.126	1.131	1.142	1.158	0.525	0.524	0.526
Italy	1.382	1.370	1.333	1.241	0.905	1.193	0.920
Japan	1.538	1.528	1.537	1.513	1.346	1.305	1.307
Norway	2.981	2.923	2.960	2.830	2.519	2.920	2.706
Sweden	1.382	1.362	1.345	1.281	1.234	1.320	1.263
Switzerland	1.152	1.148	1.154	1.147	0.647	0.648	0.648
UK	1.031	0.990	0.970	1.002	0.698	0.724	0.716
USA	0.723	0.707	0.717	0.674	0.392	0.415	0.405
Mean	1.373	1.354	1.353	1.307	0.983	1.079	1.010
Median	1.152	1.148	1.154	1.158	0.698	0.724	0.716
<i>Emerging</i>							
Brazil	2.806	2.791	2.793	2.767	2.049	2.013	2.117
Chile	2.387	2.368	2.344	2.379	1.953	2.358	2.035
India	3.488	3.449	3.451	3.499	1.833	1.818	1.772
Korea	2.645	2.635	2.511	2.589	1.751	1.776	1.734
Malaysia	1.778	1.777	1.778	1.755	1.034	1.093	1.038
Mexico	1.953	1.919	1.877	1.826	1.190	1.125	1.126
Poland	2.497	2.432	2.393	2.403	1.929	1.850	1.874
Russia	3.270	3.196	3.106	2.904	2.159	2.252	2.220
SouthAfrica	2.262	2.223	2.202	2.225	1.903	2.156	2.035
Mean	2.565	2.532	2.495	2.483	1.756	1.827	1.772
Median	2.497	2.432	2.393	2.403	1.903	1.850	1.874
	<i>MSE</i> ($\times 10^4$)						
Model	LMM	QMM	CMM	GAM	TvLMM	TvQMM	TvCMM
<i>Developed</i>							
France	1.828	1.758	1.723	1.247	0.530	0.695	0.566
Germany	2.838	2.747	2.819	2.895	0.453	0.434	0.433
Italy	3.008	2.911	2.828	2.589	1.268	2.425	1.326
Japan	3.621	3.599	3.688	3.611	3.026	2.863	2.874
Norway	13.713	13.363	13.454	12.262	9.980	11.162	11.474
Sweden	2.518	2.466	2.425	2.109	1.993	2.279	2.091
Switzerland	2.212	2.157	2.090	2.135	0.785	0.791	0.788
UK	1.750	1.659	1.632	1.709	0.861	0.913	0.931
USA	0.866	0.831	0.862	0.755	0.277	0.310	0.297
Mean	3.595	3.499	3.502	3.257	2.130	2.430	2.309
Median	2.518	2.466	2.425	2.135	0.861	0.913	0.931
<i>Emerging</i>							
Brazil	12.469	12.034	12.019	11.769	6.628	6.346	6.880
Chile	8.568	8.391	8.257	8.539	5.809	8.352	6.223
India	19.157	18.976	19.404	19.367	6.347	6.287	6.155
Korea	13.140	13.011	12.860	12.833	6.753	6.352	6.218
Malaysia	5.528	5.353	5.289	5.405	1.857	2.103	1.930
Mexico	7.216	6.881	6.685	6.156	2.565	2.592	2.632
Poland	8.441	8.067	8.142	7.880	5.661	5.159	5.212
Russia	24.280	22.476	19.438	15.247	7.881	8.385	8.382
SouthAfrica	7.697	7.236	7.107	7.389	5.454	6.688	5.824
Mean	11.833	11.381	11.022	10.509	5.439	5.807	5.495
Median	8.568	8.391	8.257	8.539	5.809	6.346	6.155

Notes: **Bold** displays the best market pricing model for each market in terms of the lowest MAE and MSE.

Table 9: MAE ($\times 10^2$) and MSE ($\times 10^4$) of out-of-sample forecast after October 2008.

	<i>MAE</i> ($\times 10^2$)						
Model	LMM	QMM	CMM	GAM	TvLMM	TvQMM	TvCMM
<i>Developed</i>							
France	1.103	0.998	0.988	1.029	1.013	0.997	0.987
Germany	1.336	1.314	1.274	1.295	1.176	1.093	1.058
Italy	1.909	1.856	1.846	1.867	1.837	1.852	1.794
Japan	1.360	1.358	1.303	1.359	1.144	1.151	1.030
Norway	1.418	1.421	1.406	1.418	1.305	1.348	1.298
Sweden	1.348	1.252	1.219	1.253	1.045	1.101	1.076
Switzerland	0.878	0.866	0.829	0.877	0.781	0.782	0.773
UK	0.755	0.753	0.743	0.765	0.737	0.720	0.692
USA	0.511	0.498	0.465	0.495	0.435	0.451	0.438
Mean	1.180	1.146	1.119	1.151	1.053	1.055	1.016
Median	1.336	1.252	1.219	1.253	1.045	1.093	1.030
<i>Emerging</i>							
Brazil	1.658	1.634	1.572	1.657	1.563	1.562	1.508
Chile	1.711	1.725	1.692	1.697	1.598	1.441	1.422
India	2.596	2.552	2.489	2.588	2.313	2.190	2.160
Korea	2.254	2.208	2.176	2.205	2.093	2.120	1.930
Malaysia	1.223	1.210	1.193	1.223	1.144	1.135	1.055
Mexico	1.270	1.243	1.234	1.270	1.160	1.203	1.150
Poland	2.292	2.257	2.228	2.291	2.024	2.026	1.707
Russia	2.103	2.075	2.038	2.077	1.873	1.796	1.766
South Africa	1.689	1.688	1.674	1.687	1.665	1.668	1.632
Mean	1.866	1.844	1.811	1.855	1.715	1.682	1.592
Median	1.711	1.725	1.692	1.697	1.665	1.668	1.632
	<i>MSE</i> ($\times 10^4$)						
Model	LMM	QMM	CMM	GAM	TvLMM	TvQMM	TvCMM
<i>Developed</i>							
France	1.813	1.656	1.645	1.677	1.541	1.645	1.501
Germany	3.004	2.946	2.834	2.880	2.291	1.996	1.866
Italy	5.581	5.352	5.333	5.357	5.278	5.339	5.131
Japan	2.805	2.745	2.532	2.789	2.026	2.048	1.666
Norway	2.979	2.958	2.946	2.978	2.649	2.755	2.618
Sweden	3.399	3.037	2.944	3.012	2.245	2.477	2.374
Switzerland	1.472	1.426	1.347	1.471	1.197	1.209	1.171
UK	0.958	0.955	0.922	0.959	0.919	0.888	0.823
USA	0.447	0.406	0.371	0.406	0.328	0.343	0.325
Mean	2.495	2.387	2.319	2.392	2.053	2.078	1.942
Median	2.805	2.745	2.532	2.789	2.026	1.996	1.666
<i>Emerging</i>							
Brazil	4.019	3.957	3.757	4.019	3.685	3.686	3.526
Chile	6.229	5.871	5.681	5.841	5.516	4.309	4.133
India	9.348	9.193	8.522	9.317	7.213	6.453	6.275
Korea	7.708	7.560	7.470	7.434	6.836	7.116	5.768
Malaysia	2.668	2.635	2.592	2.668	2.354	2.261	1.972
Mexico	2.642	2.602	2.581	2.641	2.417	2.513	2.273
Poland	8.977	8.728	8.690	8.975	7.234	7.246	5.029
Russia	8.151	8.064	7.783	7.912	6.400	6.265	6.013
South Africa	4.535	4.476	4.455	4.528	4.433	4.385	4.256
Mean	6.031	5.899	5.726	5.926	5.121	4.915	4.361
Median	6.229	5.871	5.681	5.841	5.516	4.385	4.256

Notes: **Bold** displays the best market pricing model for each market in terms of the lowest MAE and MSE.

Table 10: The number of developed and emerging markets that reject the null hypothesis based on the MAE and MSE in-sample model fit entire data.

	<i>MAE</i>					
Model	QMM	CMM	GAM	TvLMM	TvQMM	TvCMM
<i>Developed</i>						
LMM	0	1	3	9	9	9
QMM		0	3	9	9	9
CMM			4	9	9	9
GAM				9	9	9
TvLMM					6	6
TvQMM						8
<i>Emerging</i>						
LMM	0	0	3	9	9	9
QMM		0	4	9	9	9
CMM			4	9	9	9
GAM				9	9	9
TvLMM					5	6
TvQMM						7
	<i>MSE</i>					
Model	QMM	CMM	GAM	TvLMM	TvQMM	TvCMM
<i>Developed</i>						
LMM	0	0	4	9	9	9
QMM		0	5	9	9	9
CMM			5	9	9	9
GAM				9	9	9
TvLMM					6	8
TvQMM						9
<i>Emerging</i>						
LMM	0	0	3	9	9	9
QMM		0	8	9	9	9
CMM			7	9	9	9
GAM				9	9	9
TvLMM					6	7
TvQMM						7

Notes: Table displays the number of developed and emerging markets that reject the null hypothesis is that there are no different levels of forecasting accuracy for the two different models when using on MAE and MSE as the measures based on the Diebold-Mariano Test at the 5% significance level.

Table 11: The number of developed and emerging markets that reject the null hypothesis based on the MAE and MSE in-sample model fit before October 2008.

	<i>MAE</i>					
Model	QMM	CMM	GAM	TvLMM	TvQMM	TvCMM
<i>Developed</i>						
LMM	1	3	5	9	9	9
QMM		1	2	9	9	9
CMM			1	9	9	9
GAM				9	9	9
TvLMM					7	6
TvQMM						5
<i>Emerging</i>						
LMM	0	3	4	9	9	9
QMM		1	2	9	9	9
CMM			1	9	9	9
GAM				9	9	9
TvLMM					5	6
TvQMM						5
	<i>MSE</i>					
Model	QMM	CMM	GAM	TvLMM	TvQMM	TvCMM
<i>Developed</i>						
LMM	0	1	4	9	9	9
QMM		1	4	9	9	9
CMM			3	9	9	9
GAM				9	9	9
TvLMM					7	7
TvQMM						5
<i>Emerging</i>						
LMM	0	3	3	9	9	9
QMM		1	3	9	9	9
CMM			2	9	9	9
GAM				9	9	9
TvLMM					5	6
TvQMM						6

Notes: Table displays the number of developed and emerging markets that reject the null hypothesis is that there are no different levels of forecasting accuracy for the two different models when using on MAE and MSE as the measures based on the Diebold-Mariano Test at the 5% significance level.

Table 12: The number of developed and emerging markets that reject the null hypothesis based on the MAE and MSE in-sample model fit after October 2008.

	<i>MAE</i>					
Model	QMM	CMM	GAM	TvLMM	TvQMM	TvCMM
<i>Developed</i>						
LMM	0	0	0	9	9	9
QMM		1	0	9	9	9
CMM			0	9	9	9
GAM				9	9	9
TvLMM					2	6
TvQMM						6
<i>Emerging</i>						
LMM	0	0	0	9	9	9
QMM		0	0	9	9	9
CMM			0	9	9	9
GAM				9	9	9
TvLMM					5	5
TvQMM						3
	<i>MSE</i>					
Model	QMM	CMM	GAM	TvLMM	TvQMM	TvCMM
<i>Developed</i>						
LMM	0	0	1	9	9	9
QMM		0	0	9	9	9
CMM			0	9	9	9
GAM				9	9	9
TvLMM					5	6
TvQMM						6
<i>Emerging</i>						
LMM	0	0	1	9	9	9
QMM		0	1	9	9	9
CMM			1	9	9	9
GAM				9	9	9
TvLMM					6	4
TvQMM						3

Notes: Table displays the number of developed and emerging markets that reject the null hypothesis is that there are no different levels of forecasting accuracy for the two different models when using on MAE and MSE as the measures based on the Diebold-Mariano Test at the 5% significance level.

Table 13: The number of developed and emerging markets that reject the null hypothesis based on the MAE and MSE out-of-sample forecast entire data.

	<i>MAE</i>					
Model	QMM	CMM	GAM	TvLMM	TvQMM	TvCMM
<i>Developed</i>						
LMM	2	2	8	9	9	9
QMM		0	8	9	9	9
CMM			8	9	9	9
GAM				8	8	8
TvLMM					2	1
TvQMM						1
<i>Emerging</i>						
LMM	0	1	5	9	9	9
QMM		0	5	9	9	9
CMM			4	9	9	9
GAM				7	7	7
TvLMM					1	2
TvQMM						2
	<i>MSE</i>					
Model	QMM	CMM	GAM	TvLMM	TvQMM	TvCMM
<i>Developed</i>						
LMM	3	2	8	9	9	9
QMM		0	8	9	9	9
CMM			8	9	9	9
GAM				8	8	8
TvLMM					1	0
TvQMM						0
<i>Emerging</i>						
LMM	0	0	3	9	9	9
QMM		0	3	8	7	7
CMM			3	9	9	9
GAM				6	6	6
TvLMM					1	1
TvQMM						1

Notes: Table displays the number of developed and emerging markets that reject the null hypothesis is that there are no different levels of forecasting accuracy for the two different models when using on MAE and MSE as the measures based on the Diebold-Mariano Test at the 5% significance level.

Table 14: The number of developed and emerging markets that reject the null hypothesis based on the MAE and MSE out-of-sample forecast before October 2008.

	<i>MAE</i>					
Model	QMM	CMM	GAM	TvLMM	TvQMM	TvCMM
<i>Developed</i>						
LMM	1	1	7	8	9	9
QMM		1	7	6	9	9
CMM			7	6	9	9
GAM				8	8	8
TvLMM					5	5
TvQMM						2
<i>Emerging</i>						
LMM	0	0	3	9	9	9
QMM		0	3	7	9	9
CMM			3	7	9	9
GAM				7	7	7
TvLMM					3	3
TvQMM						3
	<i>MSE</i>					
Model	QMM	CMM	GAM	TvLMM	TvQMM	TvCMM
<i>Developed</i>						
LMM	2	1	7	5	8	8
QMM		0	7	6	8	8
CMM			7	5	8	8
GAM				7	7	7
TvLMM					4	4
TvQMM						3
<i>Emerging</i>						
LMM	0	0	3	9	9	9
QMM		0	3	6	8	8
CMM			3	7	9	9
GAM				6	6	6
TvLMM					3	2
TvQMM						2

Notes: Table displays the number of developed and emerging markets that reject the null hypothesis is that there are no different levels of forecasting accuracy for the two different models when using on MAE and MSE as the measures based on the Diebold-Mariano Test at the 5% significance level.

Table 15: The number of developed and emerging markets that reject the null hypothesis based on the MAE and MSE out-of-sample forecast after October 2008.

	<i>MAE</i>					
Model	QMM	CMM	GAM	TvLMM	TvQMM	TvCMM
<i>Developed</i>						
LMM	1	2	8	9	8	8
QMM		1	8	6	7	8
CMM			8	3	4	7
GAM				8	8	8
TvLMM					3	3
TvQMM						5
<i>Emerging</i>						
LMM	0	2	6	9	7	8
QMM		2	6	7	7	9
CMM			6	4	5	8
GAM				5	5	5
TvLMM					1	5
TvQMM						5
	<i>MSE</i>					
Model	QMM	CMM	GAM	TvLMM	TvQMM	TvCMM
<i>Developed</i>						
LMM	0	2	8	9	6	8
QMM		0	8	7	7	8
CMM			8	5	4	7
GAM				8	8	8
TvLMM					2	3
TvQMM						6
<i>Emerging</i>						
LMM	0	2	4	8	6	8
QMM		2	4	6	5	9
CMM			4	4	4	7
GAM				5	5	6
TvLMM					2	6
TvQMM						6

Notes: Table displays the number of developed and emerging markets that reject the null hypothesis is that there are no different levels of forecasting accuracy for the two different models when using on MAE and MSE as the measures based on the Diebold-Mariano Test at the 5% significance level.

Table 16: TvLMM parameter estimates (*standard errors*) and diagnostic test statistics (*p-value*) entire data.

	Parameter estimates					Diagnostic test statistics		
Market	$\hat{Q}_{1i} \times 100$	$\hat{H}_i \times 100$	$\hat{\phi}_{1i}$	$\hat{\kappa}_i$	$\hat{\alpha}_{1it}$	JB	$Het(174)$	$LB(23)$
<i>Developed</i>								
France	8.081 (2.106)	0.014 (0.001)	0.223 (0.053)	0.000 (0.000)	1.337 (0.043)	261.89* (0.000)	0.72 (0.984)	24.49 (0.178)
Germany	16.920 (2.917)	0.015 (0.001)	0.029 (0.011)	0.000 (0.000)	1.386 (0.050)	100.54* (0.000)	0.69 (0.993)	17.50 (0.556)
Italy	17.249 (4.223)	0.024 (0.002)	0.119 (0.032)	-0.002 (0.000)	1.272 (0.054)	109.24* (0.000)	1.53* (0.003)	9.80 (0.958)
Japan	21.197 (5.111)	0.033 (0.003)	0.057 (0.028)	0.000 (0.000)	0.741 (0.036)	87.98* (0.000)	0.76 (0.963)	16.94 (0.594)
Norway	20.077 (12.042)	0.061 (0.005)	0.575 (0.339)	0.002 (0.000)	1.380 (0.104)	78.68* (0.000)	0.59 (0.999)	32.88* (0.025)
Sweden	16.739 (4.110)	0.029 (0.002)	0.000 (0.000)	0.001 (0.000)	1.456 (0.062)	93.02* (0.000)	0.84 (0.873)	34.37* (0.017)
Switzerland	5.873 (2.167)	0.017 (0.001)	0.278 (0.098)	0.000 (0.000)	0.944 (0.031)	33.14* (0.000)	0.73 (0.981)	29.93 (0.053)
UK	4.433 (1.810)	0.014 (0.001)	0.418 (0.135)	0.000 (0.000)	1.088 (0.034)	312.38* (0.000)	0.57 (0.999)	41.34* (0.002)
USA	3.914 (0.811)	0.004 (0.000)	0.020 (0.009)	0.000 (0.000)	0.910 (0.016)	44.33* (0.000)	0.77 (0.955)	31.17* (0.039)
Mean	12.733	0.023	0.187	0.000	1.149			
<i>Emerging</i>								
Brazil	70.647 (14.591)	0.066 (0.006)	0.436 (0.081)	0.004 (0.000)	1.487 (0.142)	66.93* (0.000)	0.46 (0.999)	19.73 (0.411)
Chile	26.008 (5.871)	0.043 (0.003)	0.000 (0.000)	0.003 (0.329)	0.862 (0.046)	55.92* (0.000)	0.72 (0.985)	21.15 (0.002)
India	5.204 (3.695)	0.095 (0.007)	0.927 (1.144)	0.002 (0.000)	1.120 (0.177)	105.81* (0.000)	1.06 (0.353)	34.11* (0.018)
Korea	79.346 (14.584)	0.075 (0.006)	0.000 (0.000)	0.001 (0.000)	1.199 (0.094)	488.12* (0.000)	0.63 (0.999)	18.70 (0.476)
Malaysia	17.703 (4.871)	0.028 (0.002)	0.000 (0.000)	0.002 (0.000)	0.582 (0.025)	304.55* (0.000)	0.64 (0.999)	24.29 (0.185)
Mexico	33.105 (6.431)	0.032 (0.003)	0.000 (0.000)	0.003 (0.000)	1.167 (0.060)	26.92* (0.000)	0.60 (0.999)	29.18 (0.063)
Poland	68.183 (15.238)	0.079 (0.007)	0.312 (0.065)	0.002 (0.000)	1.405 (0.125)	57.47* (0.000)	0.77 (0.953)	41.28* (0.002)
Russia	62.157 (19.726)	0.110 (0.009)	0.526 (0.160)	0.002 (0.000)	1.367 (0.152)	247.08* (0.000)	0.38 (0.999)	25.09 (0.158)
SouthAfrica	36.599 (8.902)	0.055 (0.005)	0.254 (0.063)	0.002 (0.000)	1.238 (0.083)	14.24* (0.001)	0.74 (0.976)	31.66* (0.034)
Mean	44.322	0.065	0.255	0.002	1.122			

Notes: *Italic* numbers in parentheses denote the standard errors of the TvLMM parameter estimates and denote the diagnostic test statistics of the TvLMM. JB , the Jarque-Bera test statistic for the null hypothesis of normally distributed standardised residuals, follows χ^2 with 2 degrees of freedom so the critical value at the 5% level is 5.99. $LB(23)$, the Ljung-Box test statistic for the null hypothesis of no autocorrelation in the standardised residuals up to order $\sqrt{522} \approx 23$, follows χ^2 with $23-(m-1)$ degrees of freedom where m is the total number of estimated parameters. $Het(174)$, the test statistic for the null hypothesis of no heteroskedasticity in the standardised residuals up to order $522/3 = 174$, follows $F_{(174,174)}$ distribution so the critical value at the 5% level is 1.28. * means the appropriate null hypothesis is rejected at the 5% significance level Neslihanoglu (2014).

Table 17: TvLMM parameter estimates (*standard errors*) and diagnostic test statistics (*p-value*) before October 2008.

	Parameter estimates					Diagnostic test statistics		
Market	$\hat{Q}_{1i} \times 100$	$\hat{H}_i \times 100$	$\hat{\phi}_{1i}$	$\hat{\kappa}_i$	$\hat{\alpha}_{1it}$	<i>JB</i>	<i>Het</i> (108)	<i>LB</i> (18)
<i>Developed</i>								
France	8.124 (3.039)	0.013 (0.001)	0.254 (0.087)	0.001 (0.000)	1.282 (0.059)	8.45* (0.015)	0.34 (0.999)	13.02 (0.525)
Germany	17.557 (4.047)	0.014 (0.001)	0.079 (0.027)	0.001 (0.000)	1.377 (0.071)	7.09* (0.029)	0.42 (0.999)	29.47* (0.009)
Italy	10.554 (3.854)	0.016 (0.002)	0.000 (0.000)	0.000 (0.000)	1.038 (0.049)	1.78 (0.410)	0.65 (0.986)	14.99 (0.379)
Japan	11.766 (9.278)	0.033 (0.003)	0.724 (0.599)	0.000 (0.000)	0.931 (0.092)	71.73* (0.000)	0.69 (0.972)	27.20* (0.018)
Norway	2.027 (3.300)	0.071 (0.006)	0.966 (4.611)	0.003 (0.000)	1.355 (0.413)	62.55* (0.000)	1.97* (0.000)	11.84 (0.619)
Sweden	13.501 (5.294)	0.026 (0.003)	0.000 (0.000)	0.001 (0.000)	1.462 (0.083)	109.01* (0.000)	0.73 (0.947)	15.74 (0.330)
Switzerland	7.182 (3.948)	0.017 (0.002)	0.252 (0.119)	0.001 (0.000)	1.022 (0.050)	13.63* (0.000)	0.57 (0.998)	23.15 (0.058)
UK	1.480 (1.074)	0.013 (0.001)	0.858 (0.801)	0.000 (0.000)	1.047 (0.066)	113.60* (0.000)	0.55 (0.999)	25.65* (0.029)
USA	2.815 (1.481)	0.004 (0.000)	0.424 (0.202)	-0.001 (0.000)	0.913 (0.026)	44.50* (0.000)	1.13 (0.268)	21.98 (0.079)
Mean	8.334	0.023	0.395	0.001	1.159			
<i>Emerging</i>								
Brazil	130.900 (32.235)	0.068 (0.008)	0.404 (0.084)	0.007 (0.000)	1.583 (0.244)	37.05* (0.000)	0.72 (0.954)	15.05 (0.375)
Chile	9.367 (6.544)	0.045 (0.004)	0.763 (0.511)	0.003 (0.000)	0.904 (0.095)	19.01* (0.000)	0.97 (0.552)	34.71* (0.002)
India	36.234 (36.836)	0.081 (0.012)	0.765 (0.697)	0.004 (0.000)	1.125 (0.210)	4.99 (0.082)	1.43* (0.033)	18.95 (0.167)
Korea	32.506 (14.454)	0.074 (0.007)	0.000 (0.000)	0.001 (0.000)	1.117 (0.107)	36.30* (0.000)	0.81 (0.869)	19.79 (0.137)
Malaysia	25.752 (9.172)	0.027 (0.003)	0.000 (0.000)	0.001 (0.000)	0.599 (0.039)	14.57* (0.001)	1.29 (0.095)	16.89 (0.262)
Mexico	7.913 (5.589)	0.039 (0.004)	0.878 (0.965)	0.002 (0.000)	1.324 (0.202)	12.25* (0.002)	1.02 (0.459)	14.86 (0.388)
Poland	1.680 (3.284)	0.087 (0.008)	0.958 (4.072)	0.003 (0.000)	1.290 (0.288)	52.83* (0.000)	0.79 (0.890)	38.23* (0.000)
Russia	43.442 (23.363)	0.119 (0.011)	0.732 (0.418)	0.003 (0.000)	1.189 (0.227)	351.51* (0.000)	0.55 (0.999)	32.21* (0.004)
SouthAfrica	0.912 (0.785)	0.061 (0.005)	0.982 (3.460)	0.001 (0.000)	1.365 (0.436)	0.58 (0.747)	1.13 (0.266)	23.94* (0.046)
Mean	32.079	0.067	0.609	0.003	1.166			

Notes: *Italic* numbers in parentheses denote the standard errors of the TvLMM parameter estimates and denote the p – value of the diagnostic test statistics of the TvLMM. *JB*, the Jarque-Bera test statistic for the null hypothesis of normally distributed standardised residuals follows χ^2 with 2 degrees of freedom so the critical value at the 5% level is 5.99. *LB*(18), the Ljung-Box test statistic for the null hypothesis of no autocorrelation in the standardised residuals up to order $\sqrt{323} \approx 18$, follows χ^2 with $18-(m-1)$ degrees of freedom where m is the total number of estimated parameters. *Het*(108), the test statistic for the null hypothesis of no heteroskedasticity in the standardised residuals up to order $323/3 \approx 108$, follows $F_{(108,108)}$ distribution so the critical value at the 5% level is 1.37. * means the appropriate null hypothesis is rejected at the 5% significance level Neslihanoglu (2014).

Table 18: TvLMM parameter estimates (*standard errors*) and diagnostic test statistics (*p-value*) after October 2008.

	Parameter estimates					Diagnostic test statistics		
Market	$\hat{Q}_{1i} \times 100$	$\hat{H}_i \times 100$	$\hat{\phi}_{1i}$	$\hat{\kappa}_i$	$\hat{\alpha}_{1it}$	JB	$Het(64)$	$LB(14)$
<i>Developed</i>								
France	8.544 (3.291)	0.014 (0.002)	0.240 (0.082)	-0.003 (0.000)	1.408 (0.069)	4.63 (0.099)	1.92* (0.005)	10.47 (0.400)
Germany	13.740 (4.019)	0.016 (0.002)	0.009 (0.010)	-0.001 (0.000)	1.387 (0.071)	7.75* (0.021)	2.30* (0.001)	14.23 (0.163)
Italy	8.073 (5.292)	0.039 (0.005)	0.163 (0.075)	-0.005 (0.000)	1.514 (0.095)	6.79* (0.034)	1.82* (0.009)	11.77 (0.300)
Japan	8.474 (4.923)	0.032 (0.004)	0.261 (0.134)	-0.001 (0.000)	0.610 (0.037)	96.23* (0.000)	0.91 (0.647)	18.14 (0.053)
Norway	19.975 (7.754)	0.040 (0.006)	0.000 (0.000)	0.001 (0.000)	1.518 (0.107)	46.17* (0.000)	0.49 (0.997)	7.70 (0.658)
Sweden	15.953 (6.209)	0.036 (0.005)	0.000 (0.000)	0.001 (0.000)	1.440 (0.094)	59.60* (0.000)	0.50 (0.997)	18.32* (0.049)
Switzerland	2.812 (2.182)	0.019 (0.002)	0.000 (0.000)	0.000 (0.000)	0.872 (0.035)	3.73* (0.000)	0.88 (0.693)	10.72 (0.380)
UK	5.232 (2.867)	0.013 (0.002)	0.152 (0.065)	0.000 (0.000)	1.124 (0.045)	68.00* (0.000)	0.43 (0.999)	24.29* (0.007)
USA	2.555 (0.908)	0.004 (0.001)	0.044 (0.022)	0.001 (0.000)	0.908 (0.022)	29.23* (0.000)	0.85 (0.740)	11.62 (0.311)
Mean	9.484	0.024	0.097	-0.001	1.198			
<i>Emerging</i>								
Brazil	3.162 (2.490)	0.053 (0.006)	0.846 (0.975)	0.000 (0.000)	1.363 (0.151)	13.11* (0.001)	0.54 (0.993)	22.75* (0.012)
Chile	7.823 (7.249)	0.046 (0.006)	0.705 (0.606)	0.002 (0.000)	0.855 (0.084)	13.25* (0.001)	0.73 (0.898)	15.81 (0.105)
India	2.417 (2.904)	0.085 (0.010)	0.932 (2.039)	0.000 (0.000)	1.031 (0.201)	27.03* (0.000)	0.80 (0.807)	14.46 (0.153)
Korea	30.076 (12.929)	0.072 (0.010)	0.000 (0.000)	0.002 (0.000)	1.205 (0.110)	23.46* (0.000)	0.81 (0.801)	21.27* (0.019)
Malaysia	9.898 (4.334)	0.021 (0.003)	0.000 (0.000)	0.003 (0.000)	0.555 (0.028)	6.91* (0.032)	1.04 (0.441)	10.99 (0.358)
Mexico	13.526 (5.769)	0.026 (0.004)	0.022 (0.020)	0.002 (0.000)	1.094 (0.064)	10.88* (0.004)	0.97 (0.555)	11.77 (0.300)
Poland	68.095 (21.440)	0.078 (0.012)	0.000 (0.000)	-0.001 (0.000)	1.617 (0.184)	33.96* (0.000)	0.70 (0.918)	25.29* (0.005)
Russia	30.635 (12.939)	0.094 (0.012)	0.000 (0.000)	0.000 (0.000)	1.573 (0.155)	351.19* (0.000)	0.50 (0.997)	21.98* (0.015)
SouthAfrica	9.741 (7.275)	0.052 (0.007)	0.000 (0.000)	0.002 (0.000)	1.237 (0.083)	2.02 (0.364)	0.75 (0.872)	18.32* (0.049)
Mean	19.486	0.058	0.278	0.001	1.170			

Notes: *Italic* numbers in parentheses denote the standard errors of the TvLMM parameter estimates and denote the p - *value* of the diagnostic test statistics of the TvLMM. JB , the Jarque-Bera test statistic for the null hypothesis of normally distributed standardised residuals follows χ^2 with 2 degrees of freedom so the critical value at the 5% level is 5.99. $LB(14)$, the Ljung-Box test statistic for the null hypothesis of no autocorrelation in the standardised residuals up to order $\sqrt{193} \approx 14$, follows χ^2 with $14-(m-1)$ degrees of freedom where m is the total number of estimated parameters. $Het(64)$, the test statistic for the null hypothesis of no heteroskedasticity in the standardised residuals up to order $193/3 \approx 64$, follows $F_{(64,64)}$ distribution so the critical value at the 5% level is 1.51. * means the appropriate null hypothesis is rejected at the 5% significance level Neslihanoglu (2014).

Figure 1: The scatter plots of 9 developed markets weekly excess returns.

Figure 2: The scatter plots of 9 emerging markets weekly excess returns.