

# Finite-energy, accelerating Bessel pulses

M. Clerici<sup>1,5</sup>, D. Faccio<sup>1,2,5,\*</sup>, A. Lotti<sup>1,5</sup>, E. Rubino<sup>1,5</sup>, O. Jedrkiewicz<sup>1,5</sup>, J. Biegert<sup>2,3</sup>, P. Di Trapani<sup>1,4,5</sup>

<sup>1</sup>*CNISM and Department of Physics and Mathematics, University of Insubria, Via Valleggio 11, IT-22100 Como, Italy*

<sup>2</sup>*ICFO-Institut de Ciències Fotòniques, Mediterranean Technology Park, 08860 Castelldefels, Barcelona, Spain*

<sup>3</sup>*ICREA Institució Catalana de Recerca i Estudis Avançats, 08010 Barcelona, Spain*

<sup>4</sup>*Department of Quantum Electronics, Vilnius University, Saulėtekio Ave. 9, bldg.3, LT-10222 Vilnius*

<sup>5</sup>*Virtual Institute for Nonlinear Optics, Centro di Cultura Scientifica Alessandro Volta, Villa Olmo, Via Simone Cantoni 1, 22100 Como, Italy*

Corresponding author: [daniele.faccio@uninsubria.it](mailto:daniele.faccio@uninsubria.it)

**Abstract:** We numerically investigate the possibility to generate freely accelerating or decelerating pulses. In particular it is shown that acceleration along the propagation direction  $z$  may be obtained by a purely spatial modulation of an input Gaussian pulse in the form of finite-energy Bessel pulses with a cone angle that varies along the radial coordinate. We discuss simple practical implementations of such accelerating Bessel beams.

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## 1. Introduction

Recently the existence of optical beams that may freely accelerate in the transverse spatial dimension has been reported [1, 2] and the possibility to observe a similar acceleration in the temporal dimension of pulses has also been predicted [3, 4] These pulses are called "Airy" pulses due to the Airy function that describes their shape.

Bessel beams are a particularly interesting class of light beams first investigated in detail by Durnin *et al.* [5] and currently at the attention of a number of research areas ranging from extreme nonlinear optics [6] to particle trapping [7] (see also [8] and references therein). The ideal Bessel beam will be non-diffractive for all  $z$ . However this requires infinite energy so that practical implementations rely on finite-energy Bessel beams with a spatial apodization of some kind, usually Gaussian (giving the so-called Bessel-Gauss beam). We may therefore call the propagation "sub-diffractive" in the sense that the central intensity peak diffracts at a much slower rate if compared to the diffraction of a Gaussian beam with the same width. A fundamental property of Bessel beams is the conical distribution of wave-vectors such that all of the beam wave-vectors form an identical angle,  $\theta$ , with respect to the propagation axis. This leads to a superluminal phase velocity  $v_\phi(\omega_0) = c/(n(\omega_0) \cos \theta)$ , where  $c$  is the velocity of light and  $n(\omega_0)$  is the medium refractive index at the carrier frequency  $\omega_0$ . In the pulsed regime we should consider also the group velocity along the propagation direction  $z$ ,  $v_g$ . This will depend, among other parameters, on the specific method by which the Bessel pulse is obtained. For example a Bessel pulse generated by an axicon (conical lens) will have  $v_g = v_G / \cos \theta$  (Bessel-X pulse) [9, 10] while a Bessel pulse generated by a hologram (Pulsed Bessel Beam) [11, 12] will have  $v_g = v_G \cos \theta$ , where  $v_G = 1/(dk/d\omega|_{\omega_0})$  is the Gaussian-pulse group velocity determined by the material first order dispersion evaluated at  $\omega_0$ . Different dependencies on the Bessel cone angle  $\theta$  or even negative group velocities may be obtained by modifying the pulse front tilt [13].

In this paper we report the possibility to access accelerating light pulses by pure spatial modulation of the pulse shape. We numerically consider the specific case of Bessel-X pulses and show that decelerating pulses are obtained as the result of spherical aberrations in lenses while accelerating pulses may be obtained by using a nonlinear axicon and the acceleration profile along  $z$  may be varied by choosing the appropriate axicon profile. Therefore these pulses exhibit arbitrary acceleration rates whilst maintaining the sub-diffractive nature of the Bessel beam.

Our numerical investigations are based on ray-tracing, used to describe the propagation through the optical element that reshapes the input pulse and on a linear non-paraxial spectral propagator, developed following the indication in [14], in order to reproduce the actual pulse spatio-temporal profile during propagation after the reshaping optical element. The approach followed is identical to that described in detail in [17] with the sole difference that homemade codes were used in the place of the commercial software.

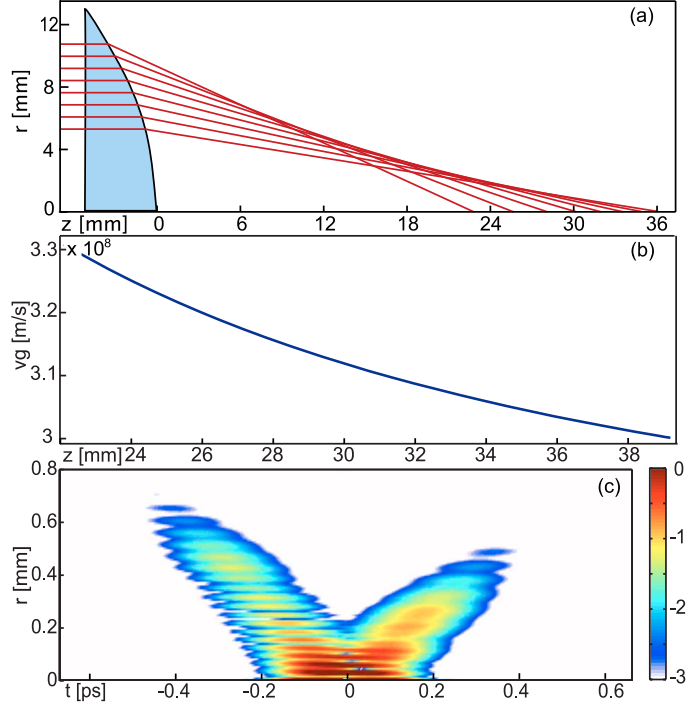


Fig. 1. (a) Ray-tracing of an annular-shaped input beam through a spherical lens. (b) On-axis group velocity of the pulse as a function of propagation distance for the propagation shown in (a). (c) Logarithmic spatio-temporal intensity profile evaluated at 3.2 cm from the lens output plane. The central wavelength of the pulse is 825 nm.

## 2. Decelerating Bessel pulses

We first consider the simple and well-known case of a spherical lens [15, 16, 17]. Due to spherical aberrations outer rays of the input beam will suffer a greater deviation than the paraxial (close to the optical axis) rays. These rays will focus on the optical axis for smaller  $z$  with a larger angle than the paraxial rays (see Fig. 1(a)) and this will lead to distortion of the spatio-temporal profile of the propagating pulse. In particular a leading Gaussian pulse will be formed, followed by a trailing Bessel-X pulse [15, 16, 17]. In the marginal focus the intensity of the Bessel-X pulse may be significantly larger than the leading Gaussian component, but will gradually decay and be re-absorbed in the main pulse at the paraxial focus. This immediately gives us a first example of a decelerating Bessel-X pulse: in the marginal focus the ray angles are large and  $v_g$  is largest. As the pulse moves toward the paraxial focus the angles decrease and the Bessel-X pulse gradually slows down until it reaches the group velocity  $v_G$  of the leading Gaussian component. Figure 1(b) shows the numerically evaluated  $v_g = v_G / \cos \theta$  of the Bessel-X pulse generated between the marginal and paraxial foci of an  $f = 43$  mm lens ( $\theta = \theta(z)$  is evaluated by ray-tracing and then substituted into the equation for the group velocity). Figure 1(c) shows the numerically evaluated spatio-temporal pulse profile at a distance  $z = 3.2$  cm after the lens when this is illuminated by a ring so as to eliminate the paraxial rays and thus create an isolated decelerating Bessel-X pulse. Indeed, by changing the input beam shape, i.e. by using a ring-shaped illumination and by changing the profile of the ring (e.g. apodization and radius) it is possible to select the illuminated portion of the propagation axis and therefore the properties, such as the maximum and minimum  $v_g$  values

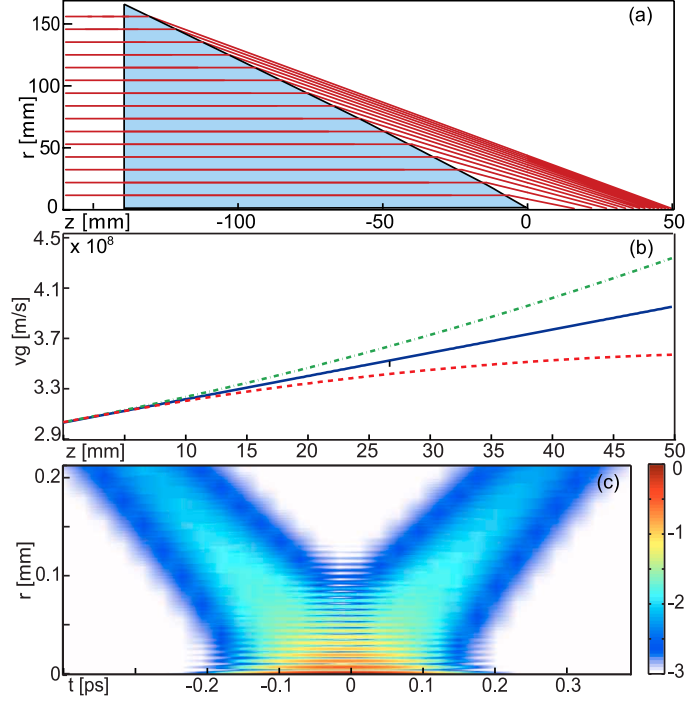


Fig. 2. (a) Ray-tracing for the nonlinear axicon described in the text. (b) Group velocity of the pulse as a function of propagation distance for the linear acceleration condition (solid line) and for quadratic positive and negative correction (dashed green and red curve respectively). (c) Logarithmic spatio-temporal intensity profile evaluated at 1.5 cm from the axicon tip. The central wavelength of the pulse is  $1.25 \mu\text{m}$ .

and the intensity evolution versus  $z$ , of the Bessel-X pulse. On the other hand, by changing the lens characteristics, e.g. focal length, it is possible to tune the deceleration rate versus  $z$ .

### 3. Accelerating Bessel pulses

We now turn our attention to the case of accelerating pulses. A simple lens as described above will always lead to a decelerating pulse. However, following the same reasoning applied to the lens, it is clear that a Bessel pulse characterized by wave-vector angles (with respect to the optical axis) that increase with propagation distance will lead to an acceleration. Such an accelerating Bessel-X pulse may be obtained for example by modifying the surface profile of a standard axicon. The axicon redirects the input wave-vectors by bending all of these toward the optical axis by the same angle. We may then modify the axicon profile by a nonlinear function so that moving away from the optical axis the rays are bent by a progressively increasing angle, as shown in the ray-tracing scheme in Fig. 2(a). Depending on the specific nonlinear function used, different acceleration rates may be obtained. Some examples are shown in Fig. 2(b). The pulse is propagating in vacuum and, as may be seen, extremely large accelerations may be obtained taking the pulse from  $c$  to  $\sim 4/3c$  in a few cms. In Fig. 2(c) we consider in detail the linearly accelerating Bessel-X pulse which is generated by illuminating with a 10 cm radius (at  $1/e$ ) Gaussian pulse, a nonlinear axicon approximated by a ninth-power polynomial,  $r = \sum a_i z^i$  with  $i = 0 \dots 9$  and  $a_0 = 1.598 \cdot 10^2$ ,  $a_1 = -1.128$ ,  $a_2 = 6.057 \cdot 10^{-3}$ ,  $a_3 = -4.528 \cdot 10^{-4}$ ,

$a_4 = 1.735 \cdot 10^{-5}$ ,  $a_5 = -3.804 \cdot 10^{-7}$ ,  $a_6 = 4.949 \cdot 10^{-9}$ ,  $a_7 = -3.774 \cdot 10^{-11}$ ,  $a_8 = 1.557 \cdot 10^{-13}$ ,  $a_9 = -2.686 \cdot 10^{-16}$ . The spatio-temporal profile is clearly that of a Bessel-X pulse and it maintains the same profile over the whole Bessel zone reaching a maximum group velocity of  $v_g \simeq 3.9 \times 10^8$  m/s. This is in stark contrast with the Airy pulse that substantially modifies its profile during propagation. However it is also true that the acceleration described here is fruit of the peculiar conical flux within the Bessel pulse and therefore requires at least two dimensions, i.e. the temporal dimension and at least one transverse spatial dimension, whereas the Airy pulse is possible in just a single dimension, e.g. in a fiber.

We note that the Bessel peak intensity increases during propagation and its width decreases. This is a direct consequence of the pulse acceleration: indeed the pulse width defined as the distance between the first zeros, is given by  $(2c/\omega) \times 2.4048/\sin\theta$ . So, for example, the accelerating Bessel pulse shown in Fig. 2(c) has an initial cone angle (at the axicon tip) of 8 deg and a beam peak diameter of 7  $\mu\text{m}$ . The final cone angle at the end of the 5 cm Bessel zone is 40 deg and the peak diameter is 1.5  $\mu\text{m}$ : this Bessel zone is huge if compared to the  $\sim 250 \mu\text{m}$  Rayleigh range of a Gaussian pulse with the same initial diameter.

Note also that the axicon size is defined by the desired acceleration rate and grows with the Bessel zone, so that, similar accelerations on a shorter scale may be obtained with smaller non-linear axicons. As an example the same acceleration described above could be obtained, on a 1 cm Bessel zone, with a  $\sim 30$  mm radius,  $\sim 28$  mm thick, nonlinear axicon. For smaller acceleration rates also graded index optics [18] could be considered.

We finally note that the outlined acceleration process manifests itself without any violation of the conservation laws. In fact, as for all accelerating pulses, the acceleration addresses the propagation feature of the intense peak of the pulse only, and not of the pulse as a whole. Indeed, the peak acceleration results from a progressive modification in the wave-packet spatial-temporal profile, which occurs along propagation. The fact that different angular portions of the spectrum construct different peaks at different  $z$  explains why the peak acceleration takes place.

#### 4. Conclusions

In conclusion we have shown that accelerating Bessel pulses may be obtained by a purely spatial modulation of an input Gaussian pulse. These pulses may be interesting for practical purposes considering (i) the relatively simple generation methods, (ii) the possibility to choose the specific acceleration rate, (iii) the robustness of the temporal profile, i.e. the acceleration of the pulse does not come at the expense of a progressive distortion and wash-out of the intensity peak as in the Airy pulse. Indeed, regarding this last point, the pulse temporal profile will spread in the same way as a Gaussian pulse due to material dispersion which, depending on the pulse duration and material, is often negligible. In the case of extremely short pulses or highly dispersive media, the concepts outlined in this work may possibly be extended also to the case of non-dispersive X-waves [19] or non-dispersive Bessel pulses [20], thus obtaining a weakly-localized accelerating light pulse that does not suffer spreading in any of its dimensions.

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