A noise simulator for eLISA: Migrating LISA Pathfinder knowledge to the eLISA mission

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Abstract. We present a new technical simulator for the eLISA mission, based on state space modeling techniques and developed in MATLAB. This simulator computes the coordinate and velocity over time of each body involved in the constellation, i.e. the spacecraft and its test masses, taking into account the different disturbances and actuations. This allows studying the contribution of instrumental noises and system imperfections on the residual acceleration applied on the TMs, the latter reflecting the performance of the achieved free-fall along the sensitive axis. A preliminary version of the results is presented.

1.   Introduction

The LISA Pathfinder [1] mission will be launched next year and will provide valuable information on the technology and performances that will be used for the future eLISA mission [2]. It is thus of a great importance to be able to migrate the knowledge acquired from LISA Pathfinder to this future mission, thus giving the physics community a tool to understand its performances and to suggest possible improvements.

The LISA Technology Package (LTP), which will be tested during the LISA Pathfinder mission, involves many subsystems as, for example, the micro-propulsion system, the interferometer readout or the controllers. Each of these subsystems might show some imperfections and could impact the final noise budget. These features of the system could affect the motion of the TMs, preventing them from achieving perfect free-fall, impacting the performances of eLISA, particularly at frequencies around and below 1 mHz. Evaluating the level of the residual acceleration is therefore of particular importance since its measurement will not be easily measurable in flight.

A plan for an end-to-end simulation is in preparation within the community and the simulator presented here will be one of the important building blocks of this effort.

This paper will first describe the mathematical approach used to model the system, in particular the LTI approximation and its justification. Then an overview of the simulator structure, showing which physical phenomena are considered and how they are modeled will be presented. Finally, preliminary results of this simulator, especially the influence of individual instrumental noises on the residual acceleration applied on the TMs along the sensitive axis will be presented.

2.   A description of the simulator and of the relevant reference frames

2.1. The eLISA constellation

The eLISA design consists of 3 S/C orbiting around the Sun, forming a constellation in a quasi-equilateral triangular shape. Each S/C contains actually several bodies, i.e. the S/C itself and one or two TMs. In the eLISA configuration, the "Mother" S/C contains 2 TMs (as well as 2 optical benches and 2 telescopes) and the two other "Daughters" contain only one TM. The arm length of this space interferometer is planned to be 1 million km.

To fully determine the dynamics of the system, one has to know the evolution of the 3x6 coordinates and their 3x6 corresponding derivatives per S/C over time. This includes the position of the center of mass (CoM) of the S/C and those of the TMs, their attitude, their linear and angular velocities. At the initial point of the orbits, the 3 bodies are assumed to be rigidly linked, i.e. the TMs are caged but released immediately. This implies that the bodies are in their nominal position, i.e. the S/C follows the programmed orbit, the TMs are centered inside their housing and the velocities of the 3 bodies are the same. Because the CoM positions of the 3 bodies are different, they will not naturally follow identical orbits: one of the tasks of the system on-board controllers will be to prevent any potential drift between the S/C and the TMs.

Great care has been taken in defining the reference frames w.r.t. which the different coordinates are expressed. The S/C position and attitude are expressed in a proper inertial reference frame, the *O-frame*, that will be introduced later. The TM coordinates are expressed in the housing frames. These frames are represented by an orthonormal basis whose origin is the center of the housing and whose unit vectors are perpendicular to the faces of the housing. They are fixed w.r.t. the S/C, i.e. their orientation are fixed in the reference frame B attached to the S/C. The set of reference frames used in this simulator is represented in the figure 1.
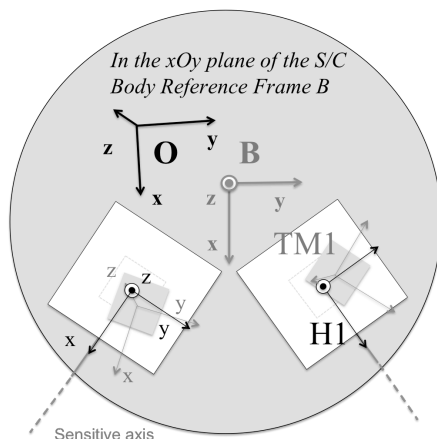

Figure 1. Reference frames involved in the simulator. The B frame is attached to the CoM of the S/C and the unit vectors, fixed w.r.t. the S/C, are used to describe the S/C orientation. H1 and H2 frames are attached to the housings and stay fixed w.r.t. the S/C. Their unit vectors are perpendicular to the faces of the housings. TM1 and TM2 are attached to the TMs CoM. Orientation of TM1 and TM2 w.r.t. H1 and H2 give the attitude of the TMs inside the S/C.

2.2. Equations of motion

The dynamics of the three bodies are constrained by 2 equations:

- Newton equations for translational motion. The corresponding solutions are the position vector of the center of mass of the bodies and their linear velocities.
- Euler equations for rotational motions. The solutions are the set of Euler angles able to describe the orientation of the bodies and their angular velocities.

These equations require that the coordinates be expressed in Galilean reference frames. If this is not the case, inertial terms have to be introduced to reflect the apparent inertia forces viewed from the point of view of the actual non-inertial reference frame. Such is the case for the position of the TMs expressed in the housing frame, which is a rotating frame w.r.t. the Galilean reference frame J attached to the Sun. As new rotating reference frame, called the orbital frame *O*, will be introduced, additional inertia terms will also appear in the dynamics of the S/C.

2.3. A Linear Time Invariant (LTI) State Space Model (SSM)

A convenient way to parameterize the dynamics is the so-called space state representation, see for example M.Weyrich [4]. It consists in expressing a set of equations in a matrix form whose solution is a vector (the State Vector) containing all the solutions of the individual equations. This vector contains all the coordinates involved in the dynamics.

A particular case occurs when the modeled system is linear and time-invariant (LTI model). This implies that the SSM matrices are constant and independent of the states. With this property, solving, assembling and converting (e.g. in frequency domain) the model reduces to handling this set of SSM matrices. Working in the frequency domain also requires this time-invariance.

The linearization process is simplified by the presence of system controllers. They allow the system to reach a stable equilibrium state. For instance, the controllers will force the TMs to be centered in their housing at all times. Thus, this equilibrium point becomes the natural point around which the linearization is made. The dynamics of the S/C presents a specific problem. If the attitude of the S/C is

expressed in the *EclipticJ2000-frame*[1] an equilibrium point cannot be defined. Note that the position of the S/C does not need to be linearized, only the attitude has to be. Expressing the attitude dynamics of The S/C w.r.t. the *EclipticJ2000-frame* will therefore not allow for a long-term linearization. To correct for this, a new reference frame is introduced in the next section.

2.4.   The orbital reference frame O

The strategy is to find a reference frame in which a linearization around an equilibrium point of the S/C dynamics can be performed. One can note that:

- The S/C follow, to a certain degree the geodesic determined by their initial (t=0) position and velocity. Instead of calculating their exact (non-geodesic!) trajectory as a function of time, one calculates the deviations from this initial geodesic.
- The controllers will maintain the proper attitude of the S/C such that their receiving telescopes and emitted lasers point to the opposite S/C.

For each S/C, the *O-frame* is then constructed in the following way:

- The origin of the *O-frame* follows the initial geodesic of the associated S/C.
- The orientation of the *O-frame* evolves in such as way that its X-axis points to the center of the line joining the two opposite S/C. This quantity is known, with sufficient precision, from the a-priori knowledge of the initial geodesic of the 3 S/C. Note that during the mission, the orientation of the S/C is controlled by sensing the waveform of the incoming lasers and that in the simulator the geometry of the emitting/receiving telescopes are mechanically fixed to the S/C body.

2.5.   The residual acceleration estimation

One of the main aims of the simulator is to calculate the residual acceleration along the sensitive axis (i.e. the laser link), thus testing the efficiency of the *drag-free* strategy. The simulator calculates, at all times, the position of the S/C and of the TMs with respect to the *O-frame*. Computing the acceleration of the TMs w.r.t. *O-frame* will provide the residual acceleration, with one caveat however. As the TMs do not follow the *O-frame* geodesic, a residual gravitational pull needs to be precisely calculated and accounted for. The final acceleration and its projection on the sensitive axis are then calculated. This quantity is therefore representative, to within a $\sqrt{2}$ factor[2], of the residual acceleration between two opposite TMs.

3.   Control strategy, Noises and imperfections

Many subsystems are involved in LTP to in order to allow the TMs to be in free-fall. Each of them has some degree of imperfections that may have some repercussions over the quality of the free-fall. Because the system requires a complex control strategy, every noise and imperfection may interact through the control loop, making the analysis of the noise contributions possibly counter-intuitive.

3.1.   The Control strategy

The goal of the control process is:

- To allow the telescopes of a given S/C to point towards the distant S/C, thus assuring that the laser link is maintained between them.
- To keep the TMs well centered and well oriented in their housing.

There are 15 coordinates to control, 3 for the S/C (attitude), and 6 for each TM (position and attitude). To ensure this, three types of control are defined:

- The first one is the *Drag-Free* control and corresponds to the actuation of the micro-propulsion system that positions the S/C w.r.t the TMs, assuring a control of the position of the TMs without

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[1] An inertial reference frame centred on the Sun whose main axis (x-axis) is on the ecliptic plane and is pointing towards the Earth position at the standard vernal equinox epoch (J2000). The z-axis is normal to the ecliptic plane and the y-axis completes the basis.
[2] We suppose here that the residual acceleration noise of distant TMs are gaussian and uncorrelated. Multiplying one of them by a $\sqrt{2}$ factor is statistically equivalent to adding them.

any force applied on it. This *Drag-Free* control will be used to constrain the x-coordinates of the TMs in their housing, i.e. along the sensitive axis, because we want to avoid as much as possible any applied force along the sensitive axis. An additional degree of freedom allows controlling the z-coordinate of TM1.

- The second one is the *Attitude* control which orients the S/C in order for it to point correctly towards the opposite S/C.

- The last one is the *Suspension* control. This corresponds to the actuation of the capacitive system that can exert electrostatic force on the TMs to correct their orientation and position on the Y-Z direction (except for the Z position of TM1).

One can emphasize here that in this context, the TM will not, strictly speaking, follow pure geodesics. However, on the sensitive axis, they will be as undisturbed as possible.

3.2. Noises and imperfections

In the eLISA system control loop, many subsystems come into play, as much for the actuation as for the measurements. In the simulator, all the subsystems are represented as symbolic blocks, all connected with links that represent inputs and outputs, see figure 2. Each block has a state space representation that models the subsystems properties, as for instance cross-talk matrices or time delays. A corresponding noise block is added to all actuations and measurement signals. Each noise is described by an amplitude and a frequency dependence by means of a transfer function.

To associate these blocks with their proper parameter, information from the

Figure 2. Block diagram of the simulator

LISAPathfinder SSM has been used. Relevant information has also been taken from F.Cirillo [5] and from valuable information provided by industrial support to ESA, in particular concerning the noise transfer functions. However, this information lies on specifications rather than on rigorous experimental characterizations. Since LPF will test this very kind of technology in space, these approximations will be replaced by the realistic values as measured during the mission.

4.   Preliminary results

The simulator has been run for a mission duration of about a couple of days (a few tens of minutes of CPU time), enough to evaluate, over the eLISA measurement band (10^{-5}-10^{-1} Hz), the behavior of the system. At the end, the simulator provides the evolution of the 18 coordinates of the system (and their derivatives) from which the residual acceleration can be computed and their associated Power Spectral Density (PSD). If necessary, calculations over the whole duration of the mission can also be performed in a very reasonable time.

Several runs have been made, activating only one source of noise at a time. A final calculation is done with the presence of all noises. Figure 3 shows, as a function of frequency, the total noise budget superimposed on the contribution of each component.

These results are very preliminary and will evolve as each noise contribution is actualized, taking into account the best present knowledge. They should not be taken as representative of the performance level of the future eLISA mission.
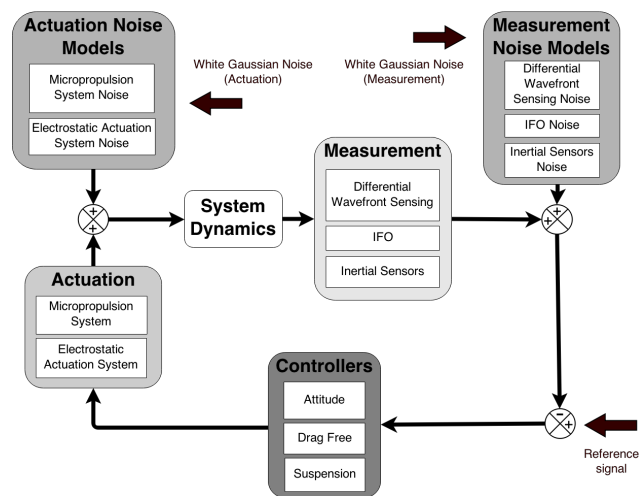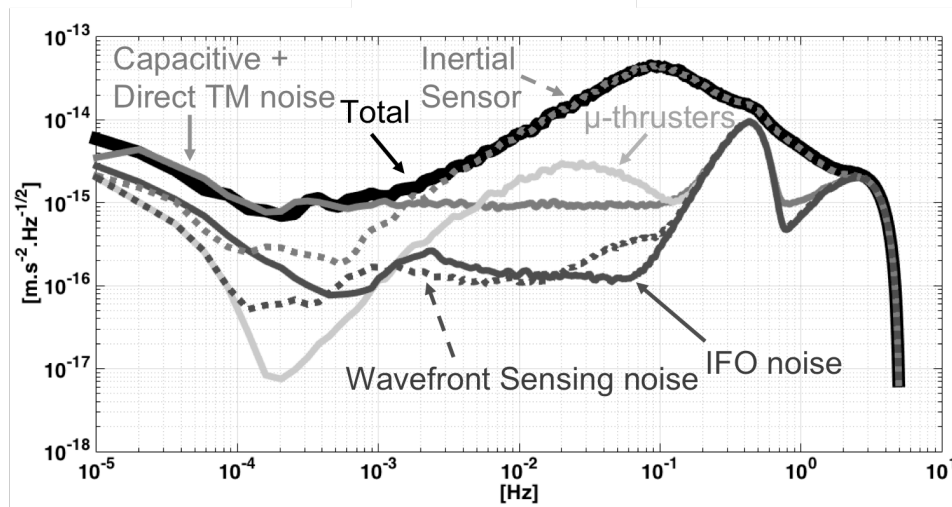
Figure 3. Decomposition of the contributions of individual noises to the PSD of the residual acceleration

5.   Conclusion

This work presents a first version of an eLISA simulator aimed at estimating the residual acceleration noise on the sensitive axis. It provides an efficient and very complete framework to evaluate and analyze the performances of this future mission. Because of its modularity, the impact of every system component (sensors, actuations, thrusters, controllers…) can be studied and adapted to the best present knowledge. The results of the LISAPathfinder mission will be essential to provide the simulator with realistic values and will show which subsystem has to be improved, and by what amount, in order for eLISA to satisfy its scientific requirements.

Without reaching the complexity level of an industrial simulator, it is complete enough to study its main components. Its ease of use should provide the scientific community with a tool to optimize the scientific performances of the mission.

The simulator is also constructed in such a way that the configuration of the system can easily evolve. For example, testing configurations with one test mass per S/C, spherical test masses, improved sensors and actuators can be studied. A quantitative comparison between a two-arm configuration (eLISA) and three-arm one (LISA) is also easy to implement.

Once the impact of each subsystem has been adapted to the level of the best knowledge available, the residual acceleration achieved for each system configuration can be transferred to data analysis programs, such as LISACode [3], in order to quantify the scientific performances of the future mission.

References
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