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# Optimal fiscal and monetary policy action in a closed economy\*

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## Abstract

We study optimized monetary and fiscal feedback policy rules. The setup is a New Keynesian DSGE model of a closed economy which is solved numerically using common parameter values and fiscal data from the euro area. Our aim is to welfare rank alternative tax-spending policy instruments used for shock stabilization and debt consolidation when, at the same time, the monetary authorities can follow a Taylor rule for the nominal interest rate.

Keywords: Feedback policy rules, New Keynesian.

JEL: E6, F3, H6.

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# 1 Introduction

Policymakers use their instruments to react to economic conditions. For instance, central banks may respond to inflation, the fiscal authorities to the state of public finances, and both of them to real economic activity. It is nevertheless believed that the use of fiscal policy is more complex and controversial than the use of monetary policy (see e.g. Leeper, 2010). The debate over the use of fiscal policy has been intensified since 2009 when most European governments embarked on the difficult task of reducing their public debts at a time of stagnant or negative growth.<sup>1</sup> What is the best policy reaction to economic conditions within this environment?

In this paper, we search for the best mix of monetary and fiscal policy actions in a closed economy, when the policy role is twofold: to stabilize the economy against shocks and to improve resource allocation by gradually reducing the public debt burden over time. In order to do so, we welfare rank various fiscal policy instruments used jointly with nominal interest rate policy.

Following most of the related literature (see below), we work with feedback policy rules. In particular, we specify feedback rules for public spending, the tax rate on labor income, the tax rate on capital income and the tax rate on consumption, when these fiscal policy instruments are allowed to respond to a number of macroeconomic variables used as indicators, while, at the same time, monetary policy can be used in a standard Taylor-type fashion. We optimally choose the magnitude of feedback policy reaction to those indicators. The welfare criterion is household's expected discounted lifetime utility. This type of policy is known as "optimized" feedback policy rules (see Schmitt-Grohé and Uribe, 2005, 2007, and many others). This enables us to welfare rank alternative policies in a stochastic setup, without our results - and, in particular, our welfare ranking of alternative policies - being driven by ad hoc differences in feedback policy coefficients, as it happens in most of the related literature on debt consolidation (see below). We compare two policy environments. In the first, used as a benchmark, the authorities just stabilize the economy from exogenous shocks. In the second, the fiscal authorities also aim at reducing the ratio of public debt to output gradually over time, which means that now we combine shock stabilization with resource allocation policies.

The setup is a rather standard New Keynesian DSGE model of a closed economy featuring

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<sup>1</sup>Even in normal times, fiscal imbalances can jeopardize the stability of the whole euro area. Hence the arguments for the restrictions of the Stability and Growth Pact. After the global financial crisis of 2007 and the sharp deterioration of public finances (in the euro area, the ratio of public debt to output was around 69% in 2008 and increased to around 95% in 2013), in view of growing concerns about fiscal sustainability, many countries have been forced to initiate substantial fiscal adjustments. However, the resulting spending cuts, and in particular the reliance on tax revenue measures, are believed to cause a significant burden on real activity. See e.g. European Commission (2013) and CESifo (2014).

imperfect competition, Calvo-type price fixities and wage rigidities. The model is solved numerically employing commonly used parameter values and fiscal data from the euro area. The steady state solution of this model (called the status quo) serves as a point of departure to study the implications of various policies aiming at shock stabilization and debt consolidation. To solve for optimized policy rules, we adopt the methodology of Schmitt-Grohé and Uribe (2004, 2007), in the sense that we take a second-order approximation to both the equilibrium conditions and the welfare criterion and, in turn, we compute the welfare-maximizing values of feedback policy rules, and the associated social welfare, under various policy scenarios.

Our main results are as follows. First, in all cases studied, the monetary authorities should aggressively react to inflation and the fiscal authorities should react to public debt. Also, in all cases studied, interest rate reaction to the output gap should be smaller in magnitude than reaction to inflation (this happens even if the policy target for output is the so-called natural level of output). In other words, the main concern of monetary authorities should be inflation stability rather than the real economy. On the other hand, the degree of fiscal reaction to the output gap (the so-called fiscal activism) relative to reaction to public debt, and hence what should be the main concern of fiscal authorities, depends crucially on the distorting effects of each fiscal instrument used and the degree of rigidities in the labor market. The more distorting a fiscal policy instrument is, the less it should be used for debt consolidation and the more it should be used to support the real economy. This applies in particular to labor taxes all the time and to capital taxes in the medium and long term. Rigidities in the labor market provide further arguments for fiscal activism. All this means that, under optimized rules, the final, or net, change in fiscal policy instruments is determined by the reconciliation of two typically opposing aims: to reduce public debt and to stimulate the economy. The final, or net, effect is a quantitative matter (see our fourth result below).

Second, when we focus on lifetime utility only, welfare differences between debt consolidation and no debt consolidation look to be small. However, this happens only because short-term effects work in opposite direction from medium- and long-term effects, so that the net, or lifetime, effects are small. In particular, the comparison of outcomes with consolidation to outcomes without consolidation implies that, in most cases, consolidation is costly in the short run and that these costs are not trivial. By contrast, in the medium- and long-term, debt consolidation becomes superior across all cases and this more than offsets its short-term costs, so that eventually lifetime, or net, utility is higher with debt consolidation.

Third, in the case of debt consolidation, the choice of fiscal policy instrument matters for how quickly public debt should be brought down. For instance, in our baseline experiments,

public debt reduction from 85%, which is its average value in the recent euro area data, to the 60% target level, which is the reference level of the Maastricht Treaty, should be achieved within 5 to 12.5 years depending on how distorting the fiscal instrument is (5 years if we use public spending or consumption taxes, and 12.5 years if we use capital taxes). This pace should be slower if there are labor market rigidities since, in the presence of such rigidities, fiscal policy should be mainly concerned about the real economy. On the other hand, if we use labor taxes, which are a particularly distorting instrument at any time, the pace of public debt reduction should be very slow, following an almost unit root process, and this is irrespectively of the degree of labour market rigidities.

Fourth, the choice of fiscal policy instrument matters for welfare too. If there are no rigidities in the labor market, the concern for public debt should dominate the concern for output and, in this case, it is better to use public spending along with interest rate policy. Practically, this means that, if there are no labor market rigidities, the best fiscal policy action is to cut public spending initially so as to bring public debt down. On the other hand, if there are rigidities in the labor market, the concern for output should dominate the concern for public debt and, in this case, it is better to use income (labor or capital) taxes on the side of fiscal policy. Practically, this means that, if there are labor market rigidities, the best fiscal policy action is to cut labor and capital taxes initially so as to stimulate the real economy and only in turn raise them to bring public debt down gradually over time.<sup>2</sup>

How does our work differ? Although there has been a rich literature on the interaction between fiscal and monetary policy,<sup>3</sup> as well as on public debt consolidation,<sup>4</sup> there has not been a welfare comparison of the main tax-spending policy instruments in a unified framework of a closed economy, and how this comparison depends on policy goals (shock stabilization only, or shock stabilization plus debt consolidation) as well as on the presence of labor market rigidities. Also, as said above, our results are based on optimized policy rules.

The rest of the paper is organized as follows. Section 2 presents the model. Section 3 presents the data, parameter values and the steady state solution. Section 4 explains how we work. The main results are in Sections 5 and 6. Robustness checks are in Section 7. Section 8 closes the paper. Details are in an Appendix.

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<sup>2</sup>These results should be contrasted to those in an open economy facing sovereign risk premia and a non-zero probability of default, where fiscal policy instruments should be earmarked to debt consolidation almost in all cases (see Philippopoulos et al., 2013).

<sup>3</sup>See e.g. Leeper (1991), Schmitt-Grohé and Uribe (2005 and 2007), Leith and Wren-Lewis (2008) and Leeper et al. (2009). For reviews, see e.g. Kirsanova et al. (2009), Wren-Lewis (2010) and Leeper (2010).

<sup>4</sup>See e.g. Coenen et al. (2008), Forni et al. (2010a, 2010b), Bi et al. (2012), Cantore et al. (2012), Cogan et al. (2013), Erceg and Lindé (2013) and Philippopoulos et al. (2013, 2014).

## 2 Model

The model is a standard New Keynesian model featuring imperfect competition and Calvo-type nominal rigidities, which is extended to include a relatively rich menu of state-contingent policy rules.

### 2.1 Households

There are  $i = 1, 2, \dots, N$  identical households. Households act competitively. The objective of each  $i$  is to maximize expected discounted lifetime utility:

$$E_0 \sum_{t=0}^{\infty} \beta^t U(c_{i,t}, n_{i,t}, m_{i,t}, g_t) \quad (1)$$

where  $c_{i,t}$  is  $i$ 's consumption bundle (defined below),  $n_{i,t}$  is  $i$ 's hours of work,  $m_{i,t}$  is  $i$ 's real money balances,  $g_t$  is per capita public spending,  $0 < \beta < 1$  is the time preference rate and  $E_0$  is the rational expectations operator.

In our numerical solutions, we use a utility function of the form (see also e.g. Gali, 2008):

$$u_{i,t}(c_{i,t}, n_{i,t}, m_{i,t}, g_t) = \frac{c_{i,t}^{1-\sigma}}{1-\sigma} - \chi_n \frac{n_{i,t}^{1+\eta}}{1+\eta} + \chi_m \frac{m_{i,t}^{1-\mu}}{1-\mu} + \chi_g \frac{g_t^{1-\zeta}}{1-\zeta} \quad (2)$$

where  $\chi_n, \chi_m, \chi_g, \sigma, \eta, \mu, \zeta$  are standard preference parameters.

The budget constraint of each household  $i$  (written in real terms) is:

$$\begin{aligned} (1 + \tau_t^c) c_{i,t} + x_{i,t} + b_{i,t} + m_{i,t} &= (1 - \tau_t^k) (r_t^k k_{i,t-1} + d_{i,t}) + \\ &+ (1 - \tau_t^n) w_t n_{i,t} + R_{t-1} \frac{P_{t-1}}{P_t} b_{i,t-1} + \frac{P_{t-1}}{P_t} m_{i,t-1} - \tau_{i,t}^l \end{aligned} \quad (3)$$

where  $P_t$  is the price index and small letters denote real variables, e.g.  $b_{i,t} \equiv \frac{B_{i,t}}{P_t}$ ,  $m_{i,t} \equiv \frac{M_{i,t}}{P_t}$ ,  $w_t \equiv \frac{W_t}{P_t}$ ,  $d_{i,t} \equiv \frac{D_{i,t}}{P_t}$ ,  $\tau_{i,t}^l \equiv \frac{T_{i,t}^l}{P_t}$ . Here,  $x_{i,t}$  is  $i$ 's real investment at  $t$ ,  $B_{i,t}$  is  $i$ 's end-of-period nominal government bonds,  $M_{i,t}$  is  $i$ 's end-of period nominal money holdings,  $r_t^k$  is the real return to inherited capital  $k_{i,t-1}$ ,  $D_{i,t}$  is  $i$ 's nominal dividends paid by firms,  $W_t$  is the nominal wage rate,  $R_{t-1} \geq 1$  is the gross nominal return to government bonds between  $t-1$  and  $t$ ,  $\tau_{i,t}^l$  is real lump-sum taxes/transfers made to each  $i$  from the government, and  $0 \leq \tau_t^c$ ,  $\tau_t^k$ ,  $\tau_t^n < 1$  are respectively the tax rates on consumption, capital income and labour income.

The motion of physical capital for each household  $i$  is:

$$k_{i,t} = (1 - \delta)k_{i,t-1} + x_{i,t} \quad (4)$$

where  $0 < \delta < 1$  is the depreciation rate of capital.

The quantity of each variety  $h$ , produced monopolistically by firm  $h$ , and consumed by household  $i$ , is denoted as  $c_{i,t}(h)$ . The composite of goods consumed by household  $i$  is given by:<sup>5</sup>

$$c_{i,t} = \left[ \sum_{h=1}^N \lambda [c_{i,t}(h)]^{\frac{\phi-1}{\phi}} \right]^{\frac{\phi}{\phi-1}} \quad (5)$$

where  $\phi > 0$  is the elasticity of substitution across goods produced and  $\sum_{h=1}^N \lambda = 1$  are weights (to avoid scale effects, we set  $\lambda = 1/N$  in equilibrium). Household  $i$ 's total consumption expenditure is:

$$P_t c_{i,t} = \sum_{h=1}^N \lambda P_t(h) c_{i,t}(h) \quad (6)$$

where  $P_t(h)$  is the price of variety  $h$ .

Details of the above problem and its solution are in Appendix 1.

## 2.2 Firms

There are  $h = 1, 2, \dots, N$  firms. Each firm  $h$  produces a differentiated good of variety  $h$  under monopolistic competition facing Calvo-type nominal fixities. The nominal profit of firm  $h$  is defined as:

$$D_t(h) = P_t(h)y_t(h) - P_t r_t^k k_{t-1}(h) - W_t n_t(h) \quad (7)$$

All firms use the same technology represented by the production function:

$$y_t(h) = A_t [k_{t-1}(h)]^\alpha [n_t(h)]^{1-\alpha} \quad (8)$$

where  $A_t$  is an exogenous stochastic TFP process whose motion is defined below.

Profit maximization by firm  $h$  is also subject to the demand for its product (see Appendix 2 for details):

$$y_t(h) = c_t(h) + x_t(h) + g_t(h) = \left[ \frac{P_t(h)}{P_t} \right]^{-\phi} y_t \quad (9)$$

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<sup>5</sup>As in e.g. Blanchard and Giavazzi (2003), we work with summations rather than with integrals.

which says that demand for firm  $h$ 's product,  $y_t(h)$ , comes from households' consumption and investment,  $c_t(h)$  and  $x_t(h)$ , where  $c_t(h) \equiv \sum_{i=1}^N c_{i,t}(h) = \left[\frac{P_t(h)}{P_t}\right]^{-\phi} c_t$  and  $x_t(h) \equiv \sum_{i=1}^N x_{i,t}(h) = \left[\frac{P_t(h)}{P_t}\right]^{-\phi} x_t$ , as well as from the government,  $g_t(h) = \left[\frac{P_t(h)}{P_t}\right]^{-\phi} g_t$ .

In addition, following Calvo (1983), firms choose their prices facing a nominal fixity. In each period, firm  $h$  faces an exogenous probability  $\theta$  of not being able to reset its price. A firm  $h$ , which is able to reset its price, chooses its price  $P_t^\#(h)$  to maximize the sum of discounted expected nominal profits for the next  $k$  periods in which it may have to keep its price fixed. This objective is given by:

$$E_t \sum_{k=0}^{\infty} \theta^k \Xi_{t,t+k} D_{t+k}(h) = E_t \sum_{k=0}^{\infty} \theta^k \Xi_{t,t+k} \left\{ P_t^\#(h) y_{t+k}(h) - \Psi_{t+k}(y_{t+k}(h)) \right\} \quad (10)$$

where  $\Xi_{t,t+k}$  is a discount factor taken as given by the firm,  $y_{t+k}(h) = \left[\frac{P_t^\#(h)}{P_{t+k}}\right]^{-\phi} y_{t+k}$  and  $\Psi_t(\cdot)$  denotes the minimum nominal cost function for producing  $y_t^H(h)$  at  $t$  so that  $\Psi_t'(\cdot)$  is the associated nominal marginal cost.

Details of the above problem and its solution are in Appendix 2.

### 2.3 Government budget constraint

The budget constraint of the consolidated government sector expressed in real terms and aggregate quantities is:

$$\begin{aligned} b_t + m_t &= R_{t-1} \frac{P_{t-1}}{P_t} b_{t-1} + \frac{P_{t-1}}{P_t} m_{t-1} + g_t - \\ &- \tau_t^c c_t - \tau_t^k (r_t^k k_{t-1} + d_t) - \tau_t^n w_t n_t - \tau_t^l \end{aligned} \quad (11)$$

where  $b_t$  is the end-of-period total stock of real public debt and  $m_t$  is the end-of-period total stock of real money balances. Note that  $c_t \equiv \sum_{i=1}^N c_{i,t}$ ,  $k_{t-1} \equiv \sum_{i=1}^N k_{i,t-1}$ ,  $D_t \equiv \sum_{i=1}^N D_{i,t}$ ,  $n_t \equiv \sum_{i=1}^N n_{i,t}$ ,  $B_{t-1} \equiv \sum_{i=1}^N B_{i,t-1}$  and  $T_t^l \equiv \sum_{i=1}^N T_{i,t}^l$ , and all other variables have been defined above. As above, small letters denote real variables.

In each period, one of the fiscal policy instruments  $\tau_t^c$ ,  $\tau_t^k$ ,  $\tau_t^n$ ,  $g_t$ ,  $\tau_t^l$ ,  $b_t$  has to follow residually to satisfy the government budget constraint (see below).

### 2.4 Decentralized equilibrium (given policy)

We now combine all the above to solve for a Decentralized Equilibrium (DE) for any feasible monetary and fiscal policy. The DE is defined to be a sequence of allocations, prices and policies such that: (i) all households maximize utility; (ii) a fraction  $(1 - \theta)$  of firms maximize

profits by choosing an identical price  $P_t^\#$ , while the rest,  $\theta$ , set their previous period prices; (iii) all constraints, including the government budget constraint, are satisfied; (iv) all markets clear.

To proceed with the solution, we need to define the policy regime. Regarding monetary policy, we assume, as is usually the case, that the nominal interest rate,  $R_t$ , is used as a policy instrument, while money balances are endogenously determined. Regarding fiscal policy, we assume that, in the transition, tax rates and public spending,  $\tau_t^c$ ,  $\tau_t^k$ ,  $\tau_t^n$ ,  $g_t$ ,  $\tau_t^l$ , are set exogenously, while the end-of-period public debt,  $b_t$ , follows residually from the government budget constraint (see Section 4 below for a discussion of public financing cases).

Appendix 3 presents the dynamic DE system. It consists of 14 equations in 14 variables  $\{y_t, c_t, n_t, x_t, k_t, m_t, b_t, P_t, P_t^\#, \tilde{P}_t, w_t, mc_t, d_t, r_t^k\}_{t=0}^\infty$ . This is given the independently set policy instruments,  $\{R_t, \tau_t^c, \tau_t^k, \tau_t^n, g_t, \tau_t^l\}_{t=0}^\infty$ , technology  $\{A_t\}_{t=0}^\infty$ , and initial conditions for the state variables. All these variables have been defined above, except from  $\tilde{P}_t$  and  $mc_t$ , where  $\tilde{P}_t \equiv \left(\sum_{h=1}^N [P_t(h)]^{-\phi}\right)^{-\frac{1}{\phi}}$  and  $mc_t$  is the firm's marginal cost as defined in Appendix 2.

Before we specify the processes of policy instruments and exogenous variables in the next two subsections, and by following the related literature, we transform the above equilibrium conditions. In particular, we express price levels in inflation rates, rewrite the firm's optimality condition in recursive form and introduce a new equation that helps us to compute household's expected discounted lifetime utility. Appendix 4 presents details and the resulting transformed DE system consisting of 17 equations in 17 variables.

## 2.5 Policy rules

Following the related literature, we focus on simple rules for the exogenously set policy instruments, which means that the monetary and fiscal authorities react to a small number of macroeconomic indicators. In particular, we allow the nominal interest rate,  $R_t$ , to follow a standard Taylor rule, meaning that it can react to inflation and output as deviations from a policy target, while we allow the distorting fiscal policy instruments, namely, government spending as a share of output,  $s_t^g \equiv \frac{g_t}{y_t}$ , and the tax rates on consumption, capital income and labor income,  $\tau_t^c$ ,  $\tau_t^k$  and  $\tau_t^n$ , to react to public debt and output, again as deviations from a policy target. The target values are defined below.

In particular, following e.g. Schmitt-Grohé and Uribe (2007), we use policy rules of the functional form:

$$\log\left(\frac{R_t}{R}\right) = \phi_\pi \log\left(\frac{\Pi_t}{\Pi}\right) + \phi_y \log\left(\frac{y_t}{y}\right) \quad (12)$$

$$s_t^g - s^g = -\gamma_l^g (l_{t-1} - l) - \gamma_y^g (y_t - y) \quad (13)$$

$$\tau_t^c - \tau^c = \gamma_l^c (l_{t-1} - l) + \gamma_y^c (y_t - y) \quad (14)$$

$$\tau_t^k - \tau^k = \gamma_l^k (l_{t-1} - l) + \gamma_y^k (y_t - y) \quad (15)$$

$$\tau_t^n - \tau^n = \gamma_l^n (l_{t-1} - l) + \gamma_y^n (y_t - y) \quad (16)$$

where variables without time subscripts denote target values, and  $\phi_\pi, \phi_y, \gamma_l^q, \gamma_y^q \geq 0$ , for  $q \equiv (g, c, k, n)$ , are feedback policy coefficients, and where:

$$l_{t-1} \equiv \frac{R_{t-1} b_{t-1}}{y_{t-1}} \quad (17)$$

denotes the beginning-of-period public debt burden as share of GDP.

## 2.6 Exogenous stochastic variables

We now define the processes of exogenous stochastic variables. For notational simplicity, we include shocks to TFP only (as we report below, the main results do not change if we add other shocks). In particular, we assume that the TFP follows an  $AR(1)$  process:

$$\log A_t = (1 - \rho^A) \log(A) + \rho^A \log A_{t-1} + \varepsilon_t^A \quad (18)$$

where  $0 \leq \rho^A \leq 1$  is a persistence parameter and  $\varepsilon_t^A \sim N(0, \sigma_A^2)$ .

## 2.7 Final equilibrium system (given feedback policy coefficients)

The final equilibrium system consists of the 17 equations of the transformed DE presented at the end of Appendix 4, and the 5 feedback policy rules as well as the definition of  $l_t$  presented in subsection 2.5. We thus end up with 23 equations in 23 variables  $\{y_t, c_t, n_t, x_t, k_t, m_t, b_t, \Pi_t, \Theta_t, \Delta_t, w_t, mc_t, d_t, r_t^k, z_t^1, z_t^2, V_t, R_t, s_t^g, \tau_t^c, \tau_t^k, \tau_t^n, l_t\}_{t=0}^\infty$ . Among them, there are 17 non-predetermined or jump variables,  $\{y_t, c_t, n_t, x_t, \Pi_t, \Theta_t, w_t, mc_t, d_t, r_t^k, z_t^1, z_t^2, V_t, s_t^g, \tau_t^c, \tau_t^k, \tau_t^n\}_{t=0}^\infty$ , and 6 predetermined or state variables,  $\{R_t, k_t, b_t, m_t, \Delta_t, l_t\}_{t=0}^\infty$ . This is given the process of TFP in (18), initial conditions for the state variables and the values of feedback policy coefficients in (12)-(16) which are specified below.

To solve this non-linear difference equation system, we take a second-order approximation around steady state. We will work as follows. We first solve the model numerically employing common parameter values and data from the euro area. The next section (Section 3) presents the steady state solution of this economy, or what we call the status quo. In turn, the next sections will study the optimal choice of feedback policy coefficients, and the resulting transition dynamics, under various policy scenaria when we depart from this status quo.

### 3 Data, parameterization and steady state

As said above, this section solves numerically for the steady state of the above model economy by using conventional parameter values and data from the euro area. In steady state, the gross inflation rate is set at one meaning zero price inflation (see Section 7 below for other cases). Also, since policy instruments react to deviations of macroeconomic indicators from their long-run values, feedback policy coefficients play no role in steady state.

#### 3.1 Data and parameterization

The fiscal data are from OECD Economic Outlook no. 89. The time unit is meant to be a quarter. Our baseline parameter values are summarized in Table 1 (see Section 7 below for other parameter values and various robustness checks).

Using the Euler equation for bonds, the value of time preference rate,  $\beta$ , follows so as to be consistent with the average value of the real interest rate in the data, 0.0075 quarterly or 0.03 annually. The real money balances elasticity,  $\mu$ , is taken from Pappa and Neiss (2005), who estimate this value using UK data; this implies an interest-rate semi elasticity of money demand equal to -0.29 which is a common value in this literature. The elasticity of intertemporal substitution,  $\sigma$ , the inverse of Frisch labour elasticity,  $\eta$ , and the price elasticity of demand,  $\phi$ , are set as in Andrès and Doménech (2006) and Galí (2008) in related studies. Regarding preference parameters in the utility function,  $\chi_m$  is chosen so as to obtain a value of real money balances as share of output equal to 1.97 quarterly, or 0.49 annually, which is close to the data (when we use the *M1* measure, the average value in the annual data is around 0.5),  $\chi_n$  is chosen so as to obtain steady-state labour hours equal to 0.28, while  $\chi_g$  is arbitrarily set at 0.1 which is a common valuation of public goods in related utility functions. Other parameters, like  $\theta$  measuring Calvo-type nominal fixities, are also set as in related studies of the euro area (see e.g. Galí et al., 2001). As reported below in Section 7, our results are robust to changes in parameter values.

Concerning the exogenous stochastic variables, we start by setting  $\rho^A = 0.8$  and  $\sigma_A = 0.017$

for the persistence parameter and the standard deviation respectively of TFP in equation (18) (the value of  $\rho^A$  is similar to that in Andrès and Doménech, 2006, while the value of  $\sigma_a$  is close to that in Bi, 2010, and Bi and Kumhof, 2009).

The steady state values of the exogenously set policy instruments are either set at their data averages, or are calibrated to deliver data-consistent long-run values for the endogenous variables. In particular,  $\tau^c$ ,  $\tau^k$ ,  $\tau^n$  are the averages of the effective tax rates in the data. Lump-sum taxes,  $s^l$ , follow residually in the long run, when total public spending as share of output,  $-s^l + s^g$ , is set at 0.43 and when the public debt-to-output ratio is set at 3.4 quarterly (or 0.85 annually), which are the values in the average data over 2008-2011.

Table 1: Parameter values

Parameter	Value	Description
$a$	0.33	share of capital
$\beta$	0.9926	time preference rate
$\mu$	3.42	parameter related to money demand elasticity
$\delta$	0.021	capital depreciation rate (quarterly)
$\phi$	6	price elasticity of demand
$\eta$	1	inverse of Frisch labour supply elasticity
$\sigma$	1	elasticity of intertemporal substitution
$\zeta$	1	elasticity of public consumption in utility
$\theta$	2/3	price rigidity parameter
$\chi_m$	0.05	preference parameter related to real money balances
$\chi_n$	6	preference parameter related to work effort
$\chi_g$	0.1	preference parameter related to public spending
$\rho^A$	0.8	serial correlation of TFP shock
$\sigma_A$	0.017	standard deviation of innovation to TFP shock
$R$	1.0075	long-run nominal interest rate
$\tau^c$	0.19	consumption tax rate
$\tau^k$	0.28	capital tax rate
$\tau^n$	0.38	labour tax rate
$s^g$	0.23	government spending as share of output
$-s^l$	0.2	government transfers as share of output

### 3.2 Steady state solution or the "status quo"

Table 2 reports the steady state solution of the model economy when we use the parameter values and the policy instruments in Table 1. The solution makes sense and the resulting great ratios are close to their values in the actual data (recall that, since the time unit is meant to be a quarter, stock variables - like public debt, capital or money balances - need to be divided by 4 to give annual values). This is what we call the "status quo". In what follows, we will depart from this solution to study the implications of various policy experiments.

Table 2: Steady state solution or the "status quo"

Variables	Solution	Variables	Solution	Data
$y$	0.7381	$d$	0.1230	-
$c$	0.4594	$r^k$	0.0395	-
$n$	0.2784	$z^1$	2.1820	-
$x$	0.1089	$z^2$	1.8183	-
$k$	5.1877	$u$	0.8040	-
$m$	1.4651	$\frac{c}{y}$	0.62	0.57
$\Pi$	1	$\frac{b}{y}$	3.4	3.4
$\Theta$	1	$\frac{x}{y}$	0.15	0.18
$\Delta$	1	$\frac{m}{y}$	1.97	-
$w$	1.4729	$\frac{k}{y}$	7	-
$mc$	0.8333	-	-	-

## 4 How we model policy

In this section, we explain the policy experiments we focus on (subsection 4.1), the motivation for debt consolidation (subsection 4.2), how we model debt consolidation (subsection 4.3) and how we compute optimized feedback policy rules (subsection 4.4). Recall that, along the transition path, nominal rigidities imply that money is not neutral so that interest rate policy matter to the real economy. Also, recall that, along the transition path, different counter-cyclical policy rules, and hence different values of feedback policy coefficients, can have different implications.

### 4.1 Types of policy action

We will study two environments regarding policy action. In the first, used as a benchmark, the role of policy is only to stabilize the economy against temporary shocks. For instance, an

adverse temporary TFP shock, as defined in equation (18) above, produces a contraction in output and a rise in the public debt to output ratio. Then, the policy questions are which policy instrument to use, and how strong the reaction of policy instruments to deviations from targets should be, when these targets are given by the status quo long-run solution. Technically speaking, in this case, we depart from, and end up, at the same steady state, which is the status quo in subsection 3.2 above, while transition dynamics are driven by temporary shocks only.

The second environment is richer. Now the role of policy is twofold: to stabilize the economy against the same shocks as above and, at the same time, to improve resource allocation by gradually reducing the public debt ratio over time. The policy questions are as above except that now the policy targets are given by the long-run solution of the reformed economy. Technically speaking, in this case, we depart from the status quo solution, but we end up at a new reformed long-run with lower public debt. Thus, now there are two sources of transition dynamics: temporary shocks and the difference between the initial and the new reformed steady state.

## 4.2 Is public debt bad?

Before we study the implications of different debt consolidation strategies, it is natural to ask "Is public debt bad?". Although it is widely recognized that we lack a theory of the optimal level of public debt, it is also well documented that a "high" level of public debt hurts the macroeconomy (see e.g. Wren-Lewis, 2010, for a review of the literature). Since the study of such issues is beyond the scope of the current paper, here we just report that, in our DSGE New Keynesian model, a public debt lower than in the recent average data (85% of GDP) is beneficial to the economy and this happens mainly through two channels. First, focusing on the steady state, a lower public debt creates fiscal space and this can be used, for instance, to cut distorting taxes (see subsection 4.3 below for public financing details). Second, focusing on the transition, our simulations show that, if we arbitrarily assume that the economy is subject to an initial debt shock (see also Cantore et al., 2012), this leads to a fall in output (this happens irrespectively of the fiscal instrument used to react to the public debt gap) and a fall in private investment (this happens when we use income taxes to react to the public debt gap). Results and details are available upon request from the authors.

This can justify the study of different debt consolidation strategies below.

### 4.3 How we model debt consolidation

We assume that the government aims to reduce the share of public debt from 85% (which was the average value in the data over 2008-2011 and was also our status quo solution above) to the target value of 60%. We choose the target value of 60% simply because it has been the reference rate of the Maastricht Treaty (we report however that our main results are not sensitive to the value of the debt target assumed). Obviously, debt reductions have to be accommodated by adjustments in the tax-spending policy instruments, which, in our model, are the output share of public spending, and the tax rates on capital income, labour income and consumption.

It is widely recognized that the implications of debt consolidation depend heavily on the public financing policy instrument used, namely, which policy instrument adjusts endogenously to accommodate the exogenous changes in fiscal policy (see e.g. Leeper et al., 2009, and Leeper, 2010). Therefore, to understand the logic of our results, we will use one fiscal instrument at a time. This means that, along the early costly phase, we allow one of the fiscal policy instruments to react to public debt imbalances, so as to stabilize debt around its new target value of 0.6 and, at the same time, it is the same fiscal policy instrument that adjusts residually in the long-run to close the government budget. Thus, we assume that the same policy instrument bears the short-term cost of, and reaps the medium- and long-term benefit from, debt consolidation. In our experiments, the policy rules for these instruments are as in subsection 2.5 above except that now the targetted values are those of the reformed long-run equilibrium. All other fiscal policy instruments, except the one used for shock stabilization and debt consolidation, are assumed to remain unchanged over time and equal to their pre-reform status quo values.

In particular, we work as follows. We first solve and compare the steady state equilibria with and without debt consolidation. In turn, setting, as initial conditions for the state variables, their long-run values from the solution of the economy without debt consolidation (this is the status quo in subsection 3.2), we compute the equilibrium transition path of each reformed economy under optimized policy rules and in turn compute the associated conditional expected discounted lifetime utility of the household. This is for each method of public financing used. The feedback policy coefficients along the transition path are chosen optimally. This is explained in the next subsection.

### 4.4 How we compute optimized feedback policy rules

Irrespectively of the policy experiments studied, to make the comparison of different policies meaningful, we compute optimized policy rules, so that results do not depend on ad hoc

differences in feedback policy coefficients across different policy rules. The welfare criterion is household's expected discounted lifetime utility, as defined in equation (67) in the Appendix.

We work in two steps. In the first, preliminary, step, we search for the ranges of feedback policy coefficients, as defined in equations (12-16) above, which allow us to get a locally determinate equilibrium (this is what Schmitt-Grohé and Uribe, 2007, call implementable rules). If necessary, these ranges will be further restricted in order to give economically meaningful solutions for the policy instruments; in particular, to give non-negative nominal interest rates, as well as tax rates and public spending ratios between zero and one (see e.g. subsection 5.2 below for numerical results).<sup>6</sup> In our search for determinacy and well-defined policy values, we experiment with one, or more, policy instruments and one, or more, operating targets at a time.

In the second step, within the ranges found as explained above, we compute the welfare-maximizing values of feedback policy coefficients (this is what Schmitt-Grohé and Uribe, 2005 and 2007, call optimized policy rules). The welfare criterion is to maximize conditional welfare,  $E_0V_0$ , as defined in equation (67), where conditionality refers to the initial conditions chosen; the latter are given by the status quo long-run solution. To this end, following e.g. Schmitt-Grohé and Uribe (2004), we take a second-order approximation to both the equilibrium conditions and the welfare criterion. As is well known, this is consistent with risk-averse behavior on the part of economic agents and can also help us to avoid possible spurious welfare results that may arise when one takes a second-order approximation to the welfare criterion combined with a first-order approximation to the equilibrium conditions (see e.g. Galí, 2008, Malley et al., 2009, and Benigno and Woodford, 2012).

In particular, we first compute a second-order accurate approximation of conditional welfare, and the associated decentralized equilibrium, as functions of feedback policy coefficients by using the perturbation method of Schmitt-Grohé and Uribe (2004) and, in turn, we use a matlab function (`fminsearch.m`) to compute the values of the feedback policy coefficients that maximize the second-order accurate approximation of conditional welfare (our matlab routines are available upon request). In this exercise, as said above, the feedback policy coefficients are restricted to be within some prespecified ranges delivering determinacy as well as mean-

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<sup>6</sup>Thus, we implement the zero lower bound (ZLB) for the nominal interest rate simply by restricting the feedback policy coefficients in the Taylor rule, (12). In the same way, since we study optimized fiscal action which may result in large deviations of fiscal instruments from their data averages, we also need to restrict the feedback policy coefficients in the fiscal policy rules, (13)-(16). Keep in mind that these restrictions (on monetary and fiscal feedback policy coefficients) may change when the specification of the model changes; we will thus report ranges in each case studied. The way we work is practically the same as in e.g. Schmitt-Grohé and Uribe (2007) who "approximate the zero lower bound constraint by requiring a low volatility of the nominal interest rate relative to its target value". However, see the criticism of such linearizations and higher-order perturbation techniques in Fernandez-Villaverde et al. (2012).

ingful values for the policy instruments. We work in this way both without, and with, debt consolidation.

## 5 Main results

This section presents the main numerical solutions. We start by presenting the steady state solution of the reformed economy with debt consolidation.

### 5.1 Steady state utility and output with debt consolidation

The new reformed steady state with debt consolidation is as defined in subsection 4.3 above. In other words, thanks to the fiscal space created by debt reduction, public spending can rise, or a tax rate can be reduced, residually.

Table 3 reports steady-state utility and output under alternative public financing cases in the reformed economy. For instance, in the first row of Table 3, the assumption is that it is public spending that takes advantage of debt reduction, in the sense that, once the debt burden has been reduced, public spending can increase relative to its value in the status quo solution. In the other rows, the fiscal space is used to finance cuts in one of the three tax rates. Table 3 reveals that the highest utility and output are achieved when the fiscal space is used to finance a cut in capital tax rates (this is further discussed below).

Table 3: Steady-state utility and output in the reformed economy

residual fiscal instrument	steady-state utility	steady-state output
$s^{*g}$	0.799820	0.7418
$\tau^{*c}$	0.807246	0.7418
$\tau^{*k}$	0.812269	0.7479
$\tau^{*n}$	0.810851	0.7460

### 5.2 Ranges of feedback policy coefficients

Moving to transition, we first check for local determinacy. As is well known, the latter may depend crucially on the values of feedback policy coefficients. In our model, (monetary and fiscal) policy guarantees determinacy, when the nominal interest rate reacts aggressively to inflation with  $\phi_\pi > 1.1$ , that is, when the Taylor principle is satisfied, and, at the same time, the fiscal policy instruments,  $s_t^g$ ,  $\tau_t^c$ ,  $\tau_t^k$ ,  $\tau_t^n$  react to public liabilities above a critical minimum value,  $\gamma_l^q > \underline{\gamma}_l^q > 0$ , where critical minimum values differ across different policy instruments (in

particular,  $\underline{\gamma}_l^g = 0.0069$ ,  $\underline{\gamma}_l^c = 0.0345$ ,  $\underline{\gamma}_l^k = 0.017$  and  $\underline{\gamma}_l^n = 0.0138$ ).<sup>7</sup> By contrast, the values of  $\phi_y$  and  $\gamma_y^q$ , measuring respectively the reaction of interest rate policy and fiscal policy to the output gap, are not found to be critical to determinacy.<sup>8</sup>

Nevertheless, as said in subsection 4.4 above, the feedback fiscal policy coefficients on public debt,  $\gamma_l^q$ , where  $q \equiv (g, c, k, n)$ , may need to be further restricted in order to get meaningful solutions for the fiscal instruments used, i.e. in order to get  $0 \leq s_t^g, \tau_t^c, \tau_t^k, \tau_t^n < 1$ . In particular, our computations imply that we need to work within the ranges  $\gamma_l^k \in (0.017, 0.15)$  for the capital tax rate and  $\gamma_l^c \in (0.0345, 0.2)$  for the consumption tax rate, which are narrower than those required for determinacy only. This makes sense. When debt consolidation is among the policy aims, the fiscal authorities may find it optimal to increase tax rates, and/or reduce public spending, beyond meaningful or historical ranges. Our simulations imply that this applies in particular to the capital tax rate,  $\tau_t^k$ , which, if it is left free, it can easily rise above 100% in the short run due to the high value of  $\gamma_l^k$  chosen (this is consistent with the Ramsey-Chamley result that, since capital is inelastic in the very short, the fiscal authorities may find it optimal to confiscate it). To avoid such problems, we restrict ourselves within the above ranges for  $\gamma_l^k$  and  $\gamma_l^c$ . Similarly, since in some experiments monetary policy may find it optimal to increase the feedback policy coefficient on inflation,  $\phi_\pi$ , to very high values, we restrict its value within the range  $1.1 < \phi_\pi \leq 3$  (see e.g. Schmitt-Grohé and Uribe, 2007, for a similar upper boundary). Within this range, the resulting equilibrium nominal interest rate is above the zero lower bound in all solutions reported below and this happens irrespectively of the fiscal policy instrument used jointly with interest rate policy. It is worth reporting that our main results regarding the welfare ranking of policy instruments do not depend on those policy restrictions (results are available upon request). Also, note that such practice (namely, imposing restrictions on policy instruments) is usual both in the policy literature (see e.g. Cantore et al., 2012), as well as in the theoretical literature on optimal taxation (see e.g. Chamley, 1986).

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<sup>7</sup>Actually, we can distinguish two regions of determinacy. In addition to the one discussed above, there is another region in which fiscal policy does not react to public liabilities, i.e.  $\gamma_l^q = 0$  for all fiscal instruments, while monetary policy reacts to inflation mildly with  $\phi_\pi \leq 1.1$ . This region is welfare inferior to the region discussed above. It also contains some sub-areas where determinacy breaks down. Several other papers on the fiscal theory of the price level have distinguished between the same two areas of determinacy (e.g. Leeper, 1991, and Schmitt-Grohé and Uribe, 2007).

<sup>8</sup>On the other hand, if we had included a tax on income from government bonds, we would have had to assume a narrower, more restrictive, range of feedback monetary policy reaction to inflation. For instance, if the tax rate is 0.2, monetary policy can guarantee determinacy when the nominal interest rate reacts to inflation with  $\phi_\pi > 1.29$  when fiscal policy uses government spending or consumption taxes to react to debt, with  $\phi_\pi > 1.43$  when fiscal policy uses capital taxes to react to debt and with  $\phi_\pi > 1.57$  when fiscal policy uses labour taxes to react to debt. As the tax rate on income from government bonds increases, the critical minimum value of  $\phi_\pi$  required for determinacy also increases (results are available upon request). This is similar qualitatively to the results in Edge and Rudd (2007).

Thus, the general message is that monetary and fiscal policy need to interact with each other in a specific way for policy to guarantee determinacy or, as Leeper (2010) puts it, there is a "dirty little secret": for monetary policy to control inflation, fiscal policy must behave in a particular manner. Benhabib et al. (2001) have also shown that the fiscal-monetary regime affects determinacy.

### 5.3 Optimized policy rules and welfare with debt consolidation

Within the above specified ranges for the feedback policy coefficients, we now compute optimized policy rules. Recall, as explained in subsection 4.1 above, that there are two sources of transition dynamics in this economy: temporary TFP shocks (whose moments were defined in subsection 3.1) and debt consolidation reforms (as defined in subsection 4.3).

Results for the case with debt consolidation are reported in Table 4. The first column lists the pair of policy instruments used (one monetary and one fiscal), the second column reports the optimal reaction of the interest rate to inflation and output, and the third column reports the optimal reaction of each fiscal policy instrument to debt and output. Expected discounted lifetime utility,  $E_0V_0$ , is reported in the last column.

There are three messages from Table 4. First, regarding monetary policy, the interest rate should react aggressively to inflation, while, reaction to the output gap is negligible. The idea behind the very small value of  $\phi_y$  is that stabilization of output around its steady state value is accompanied by increased volatility in inflation and nominal interest rates and this is not desirable.<sup>9</sup> The optimal reaction to the output gap, from the point of view of monetary authorities, is revisited in Section 7 below. Second, regarding fiscal policy, when we use public spending, or consumption taxes, or capital taxes, fiscal reaction to the output gap is smaller in magnitude than fiscal reaction to public debt. This means that public spending should fall, while consumption taxes and capital taxes should rise, to address the public debt problem. By contrast, when we use labor taxes, reaction to output is clearly stronger than reaction to debt. This means that the labor tax rate should be reduced at impact to help the real economy. In other words, a particularly distorting policy instrument, like labor taxes, should be used to address output imbalances rather than to reduce the debt burden. All this is confirmed by the impulse response functions shown below. Third, when we rank policy instruments according to expected discounted lifetime utility,  $E_0V_0$ , the best possible mix is  $R_t$  and  $s_t^g$ . Notice that, to the extent that feedback policy coefficients are chosen optimally, welfare differences, at least in

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<sup>9</sup>This is as in e.g. Schmitt-Grohé and Uribe (2007). The desirability of not responding to the output gap, at least when the output target is the steady state value of output, is confirmed when we keep  $\phi_\pi$  at its optimal value and arbitrarily increase  $\phi_y$ ; then, the volatility of both inflation and output rises. The same happens when we arbitrarily increase  $\phi_y$  and decrease  $\phi_\pi$ .

terms of lifetime utility, across different policy mixes are very small.<sup>10</sup> Keep in mind however that this welfare ranking is in terms of lifetime utility only; shorter time horizons may imply different things (see below).

Table 4 : Optimal monetary reaction to inflation and output  
and optimal fiscal reaction to debt and output

Policy instruments	Optimal interest-rate reaction to inflation and output	Optimal fiscal reaction to debt and output	Lifetime utility $E_0V_0$
$R_t \quad s_t^g$	$\phi_\pi = 1.1$ $\phi_y = 0.0011$	$\gamma_l^g = 0.1927$ $\gamma_y^g = 0.1134$	109.2534
$R_t \quad \tau_t^c$	$\phi_\pi = 2.7983$ $\phi_y = 0.0098$	$\gamma_l^c = 0.1943$ $\gamma_y^c = 0.0036$	109.0918
$R_t \quad \tau_t^k$	$\phi_\pi = 2.9323$ $\phi_y = 0.0037$	$\gamma_l^k = 0.1499$ $\gamma_y^k = 0.0007$	109.0754
$R_t \quad \tau_t^n$	$\phi_\pi = 2.983$ $\phi_y = 0.0728$	$\gamma_l^n = 0.0129$ $\gamma_y^n = 0.1138$	108.7748

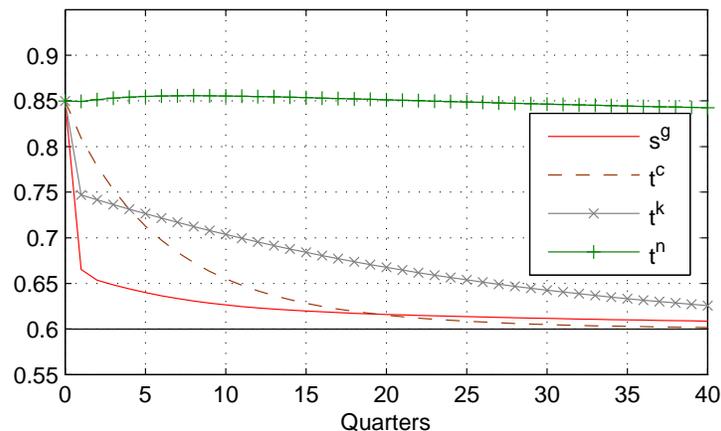
The optimized policy rules shape the motion of public debt over time. Figure 1 shows the resulting path of the public debt-to-GDP ratio. The duration of the debt consolidation phase, or equivalently the speed of debt reduction, depends heavily on the fiscal policy instrument used. In particular, more than 95% of debt consolidation should be achieved within 20 time periods (5 years), if we use the public spending ratio,  $s_t^g$ ; within 20 time periods (5 years), if we use the consumption tax rate,  $\tau_t^c$ ; and within 50 time periods (12.5 years), if we use the capital tax rate,  $\tau_t^k$ . On the other hand, if we use the labor tax rate,  $\tau_t^n$ , the debt-to-output ratio should converge very slowly to its 60% target looking like a unit-root process. The general idea is that the more distorting the policy instrument is, the slower the debt adjustment should be.

#### 5.4 Welfare over various time horizons with and without debt consolidation

We now study what happens to welfare over various time horizons. This is important because, for several (political-economy) reasons, economic agents' behavior can be short sighted. Setting

<sup>10</sup>When we express welfare differences using consumption equivalents, as in e.g. Lucas (1990), the results are the same. In particular, in Table 4, when the reference regime is  $R_t$  and  $s_t^g$ , the welfare losses expressed in consumption equivalents are -0.0012 when we use consumption taxes, -0.0013 when we use capital taxes and -0.0035 when we use labour taxes.

Figure 1: The path of public debt as share of output



the feedback policy coefficients as in Table 4 above, the expected discounted utility at various time horizons is reported in Table 5. In the same Table, we also report results without debt consolidation other things equal (these are the numbers in parentheses). As said in subsection 4.1, without debt consolidation, we again compute optimized policy rules but now the economy starts from, and returns to, the status quo solution.

There are two messages from Table 5. First, when we focus on lifetime utility, welfare differences between debt consolidation and no debt consolidation look to be small.<sup>11</sup> But, a more careful inspection of the results in Table 5 reveals that this happens only because short-term effects work in opposite direction from medium- and long-term effects, so that the net, or lifetime, effects are small. In particular, the comparison of outcomes with consolidation to outcomes without consolidation implies that, in most cases, consolidation is costly in the short run and that these costs are not trivial.<sup>12</sup> By contrast, after the first 60 periods or 15 years, debt consolidation becomes superior across all cases and this more than offsets its short-term costs, so that, eventually, the lifetime, or net, utility becomes higher with debt consolidation. Second, without debt consolidation, and to the extent that feedback policy coefficients are optimally chosen, the choice of the fiscal policy instrument used is trivial. Welfare differences appear after the second decimal point across all time horizons (these are the numbers in parentheses). On the other hand, with debt consolidation, the choice of the policy instrument matters more (these are the numbers without parentheses). Now, except from the case in which we care only

<sup>11</sup>When expressed in consumption equivalents, the welfare gains of debt consolidation are 0.0036 when we use government spending, 0.0025 when we use consumption taxes, 0.0024 when we use capital taxes and 0.00024 when we use labor taxes. In each case, the reference regime is the case without debt consolidation.

<sup>12</sup>The exception is when we use the capital tax rate in which case debt consolidation is clearly superior even in the short term. This is consistent with the Chamley-Judd result. Namely, in the very short run, capital taxes act as a capital levy on predetermined wealth so that the best fiscal policy is to bring public debt down by higher capital taxes. Recall that here policy is conducted by rules.

about the short run, the best policy mix is  $R_t - s_t^g$ . In the short run, by contrast, the best mix is  $R_t - \tau_t^k$  for the reasons explained above.

Table 5 : Welfare over different time horizons  
with, and without, debt consolidation

		2 periods	4 periods	10 periods	60 periods	Lifetime utility $E_0V_0$
$R_t$	$s_t^g$	1.5655 (1.6029)	3.1990 (3.1822)	7.9406 (7.7822)	39.7726 (39.0939)	109.2534 (108.7662)
$R_t$	$\tau_t^c$	1.4188 (1.6024)	2.8775 (3.1818)	7.3441 (7.7809)	39.1210 (39.0883)	109.0918 (108.749)
$R_t$	$\tau_t^k$	1.6667 (1.6030)	3.2929 (3.1825)	7.9569 (7.7827)	39.8577 (39.0870)	109.0754 (108.7452)
$R_t$	$\tau_t^n$	1.6016 (1.6026)	3.1796 (3.1818)	7.7736 (7.7814)	39.1113 (39.0756)	108.7748 (108.7420)

Notes: Results without debt consolidation in parentheses. Periods denote quarters.

## 6 Adding labor market rigidities

The previous analysis has assumed away any rigidities in the labor market. This is questionable since it is widely believed that labor market rigidities are a key feature of the European economy (see e.g. Blanchard, 2004). In this subsection, we extend the model to allow for such rigidities.

To avoid further complicating the model, but to also help it replicate the stylized facts in Europe regarding inertia in wage adjustment, we follow the setup employed in Blanchard and Galí (2007), Malley et al. (2009) and many others. In particular, we start by assuming that the nominal wage rate at time  $t$  is a weighted average of the nominal wage in the previous period,  $t - 1$ , and the nominal wage that would arise in case the labor market worked perfectly. In turn, expressing variables in real terms, the real wage rate follows the rule:

$$w_t \equiv \left( w_{t-1} \frac{P_{t-1}}{P_t} \right)^\gamma (MRS_t)^{1-\gamma} \quad (19)$$

where  $0 \leq \gamma \leq 1$  measures the degree of wage sluggishness and  $MRS_t \equiv \frac{\chi_n (1+\tau_t^c) n_{i,t}^\eta}{(1-\tau_t^n) c_{i,t}^{-\sigma}}$  (see equation (30) in Appendix 1). The idea behind this partial adjustment model is that real wages respond only sluggishly to current conditions in the labor market. As pointed out

by Blanchard and Galí (2007), "this is a parsimonious way of modeling the slow adjustment of wages to labor market conditions, as found in a variety of models of real wage rigidities, without taking a stand on what is the "right" model". In other words, although *ad hoc*, this specification can be consistent with a number of possible sources of rigidity in European labor markets, e.g. institutional, legal and socio-political rigidities and safety nets, etc. Finally, notice that this modeling has the following advantages: (i) if  $\gamma = 0$ , the standard model is obtained; (ii) in the steady-state, i.e. when  $w_t = w_{t-1} = w$ , it follows that again  $w = MRS$ . If  $\gamma = 1$ , we have full persistence in wage setting. In our numerical solutions below, we set  $\gamma = 0.9$ , which is close to the value used by Malley et al. (2009) for a number of European economies.

## 6.1 Results with labor market rigidities

The model is resolved using the new specification in the labor market. Regarding ranges of feedback policy coefficients guaranteeing determinacy and meaningful solutions for the policy instruments, we report that these ranges practically remain as in subsection 5.2 above, except that now, when we use capital and labor taxes on the side of fiscal policy, the minimum boundaries of interest rate reaction to inflation are related to the minimum boundaries of fiscal policy reaction to public debt (namely, if we allow the fiscal reaction to debt to rise, the monetary reaction to inflation needs also to rise to guarantee determinacy).

Within these ranges, the new results are reported in Tables 6 and 7, which are like Tables 4 and 5 respectively. There are three messages. First, optimal fiscal reaction to the output gap is now much bigger than in the case without labor market rigidities. That is, now the fiscal authorities find it optimal to also react to the recession so that debt reduction is not their only concern. Actually, in the case of public spending, capital taxes and labor taxes, the coefficient on the output gap is bigger than the coefficient on the debt gap; this means that now public spending should rise, and capital and labor taxes should fall, in order to stimulate the economy, and only in turn be used to address the debt problem. Second, when there are rigidities in the labor market, it is better to use income tax rates along with the interest rate. That is, under debt consolidation, the mixes  $R_t - \tau_t^k$  and  $R_t - \tau_t^n$  score better than  $R_t - s_t^g$  and this is the case both in the very short run and in the long run. Intuitively, this follows naturally from the first result above: since the emphasis should be now given to the real economy, it is better to use fiscal instruments, like income taxes, which can more more effectively stimulate the economy and which are relatively close to the heart of the labor market imperfection. Third, monetary reaction to the output gap remains very small, as it was also the case in Table 4.

Table 6 : Optimal monetary reaction to inflation and output  
and optimal fiscal reaction to debt and output (with labor rigidities)

Policy instruments	Optimal interest-rate reaction to inflation and output	Optimal fiscal reaction to debt and output	Lifetime utility $E_0V_0$
$R_t \quad s_t^g$	$\phi_\pi = 2.9988$ $\phi_y = 0.0001$	$\gamma_l^g = 0.1368$ $\gamma_y^g = 0.3907$	109.067
$R_t \quad \tau_t^c$	$\phi_\pi = 2.9988$ $\phi_y = 0.0186$	$\gamma_l^c = 0.2598$ $\gamma_y^c = 0.1514$	109.0807
$R_t \quad \tau_t^k$	$\phi_\pi = 1.1$ $\phi_y = 0.0306$	$\gamma_l^k = 0.0119$ $\gamma_y^k = 0.424$	109.174
$R_t \quad \tau_t^n$	$\phi_\pi = 1.102$ $\phi_y = 0.0456$	$\gamma_l^n = 0.0143$ $\gamma_y^n = 0.8083$	109.1488

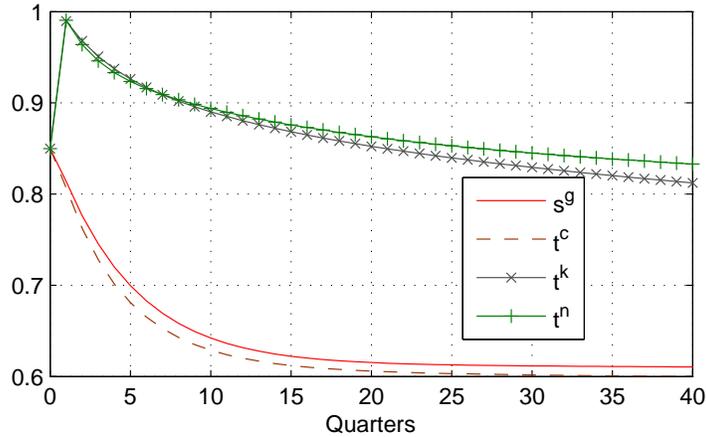
Table 7: Welfare over different time horizons  
with, and without, debt consolidation (with labor rigidities)

	2 periods	4 periods	10 periods	60 periods	Lifetime utility $E_0V_0$
$R_t \quad s_t^g$	1.5659 (1.6018)	3.1442 (3.1808)	7.8545 (7.7771)	39.6939 (39.0880)	109.067 (108.8049)
$R_t \quad \tau_t^c$	1.3337 (1.6005)	2.7513 (3.1783)	7.2392 (7.7786)	39.1385 (39.0736)	109.0807 (108.759)
$R_t \quad \tau_t^k$	1.5963 (1.6011)	3.1645 (3.1795)	7.7320 (7.7739)	39.1589 (39.0732)	109.174 (108.756)
$R_t \quad \tau_t^n$	1.6090 (1.6012)	3.1730 (3.1793)	7.7272 (7.7792)	39.0825 (39.0863)	109.1488 (108.7728)

Notes: Results without debt consolidation in parentheses. Periods denote quarters.

The resulting public debt dynamics are shown in Figure 2. The general message is that it is optimal to reduce public debt more gradually than in Figure 1. In particular, when we use public spending or consumption taxes, debt adjustment should take place at a slower pace during the first 5-10 periods or quarters. But the difference from Figure 1 becomes more striking when we use capital and labour income taxes. Now, it is clearly optimal to let the debt-to-output ratio further rise in the very short run, so as to help the real economy recover first, and only then decrease the debt gradually by following an almost unit root process to its 60% target.

Figure 2: The path of public debt as share of output

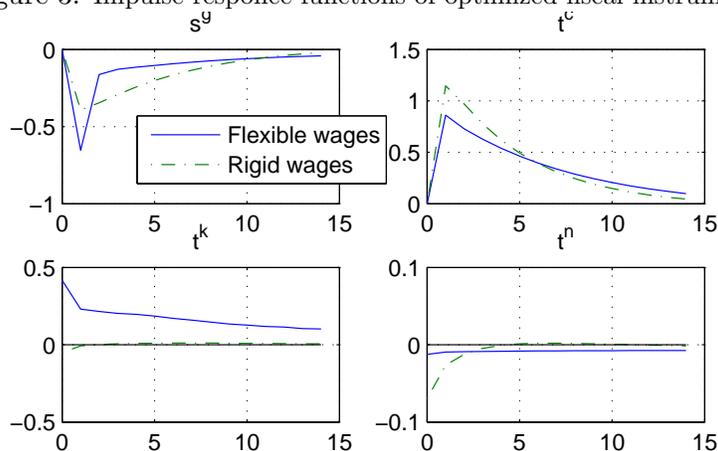


## 6.2 Impulse response functions of optimized fiscal instruments with and without labor rigidities

To make our results clearer, we also provide the impulse response functions of the optimized fiscal policy instruments studied above. This is shown in Figure 3. In this Figure, impulse response functions are computed as log-linear deviations from the status-quo solution. In these impulses, dynamics are driven by an adverse shock to TFP and by the debt consolidation reform. Solid lines correspond to the model without wage rigidities. Broken lines correspond to the model with wage rigidities.

As can be seen in Figure 3, consumption tax rates should rise, with or without wage rigidities. Public spending should clearly fall without wage rigidities, while this fall should be milder in the presence of wage rigidities (actually, recall from the values of feedback policy coefficients in Table 6, which were based on second-order approximations, that public spending should rise at impact in the presence of wage rigidities). The degree of wage rigidities also plays an important role when we use income taxes. If there are no such rigidities, capital tax rates should rise to bring public debt down. But, if there are wage rigidities, the change in capital taxes should be very mild to help the real economy recover first. The policy emphasis on the real activity becomes even more obvious when we use labor taxes which are particularly distorting. Now, labor tax rates should be cut so as to counter the recession and only later on they should be raised to address the debt problem; naturally, this gets more obvious in the presence of labor rigidities.

Figure 3: Impulse response functions of optimized fiscal instruments



## 7 Robustness

We finally conduct a sensitivity analysis. For instance, we check robustness to changes in parameter values; to different initial values for the state variables like public debt; to assuming a more volatile economy; and to two generalizations of the model that allow for policy reaction to the natural level of output and for trend inflation.

The bottom line will be that the main results (e.g. the welfare ranking of fiscal policy instruments, the comparison between debt consolidation and non debt consolidation, and how monetary and fiscal policy instruments should react to economic indicators) remain unaffected.

### 7.1 Alternative parameterizations of the model

We first report that our results are robust to changes in the magnitude of all key parameter values. For instance, we have experimented with changes in the values of the Calvo parameter in the firm's problem,  $\theta$ , and the preference parameter for public goods,  $\chi_g$ , whose values are relatively unknown. Our main results do not change within the ranges  $0.33 \leq \theta \leq 0.75$  and  $0 \leq \chi_g \leq 0.09$ . Our results are also robust to changes in the preference parameter for real money balances,  $\chi_m$ . In particular, all results remain unchanged within the range  $0.01 \leq \chi_m \leq 0.1$ .<sup>13</sup>

Results are available upon request.

<sup>13</sup>Actually, most results remain unchanged even within the wider range  $0.005 \leq \chi_m \leq 0.1$ . Note however that very low values of  $\chi_m$ , like 0.005, result in values for real money balances as share of output that are too low relative to the data (as said in subsection 3.1, the data average annual value is around 0.5). Thus, the range  $0.01 \leq \chi_m \leq 0.1$  gives values close to the data.

## 7.2 Different initial values of public debt

Our results are robust to changes in the initial value of public debt. Recall that so far the starting value of public debt as share of output has been 85% which was the average annual value over 2008-2011 in the data. Our results are not affected if instead we start with, say, 88% (data value in 2011) or 93% (data value in 2012).

Results are again available upon request.

## 7.3 Allowing for a more volatile economy

Our results are robust to assuming a more volatile economy. This can be captured by a higher standard deviation of the existing TFP shock (for instance, an arbitrary increase in standard deviation from 0.017, which has been its value so far, to 0.034), or by adding new shocks, like policy shocks in the feedback rules (12)-(16) as well as shocks to the time preference rate.<sup>14</sup>

It is worth pointing out however that, when extrinsic volatility rises (meaning a higher standard deviation and/or more sources of stochasticity), we need, in most cases, to further restrict the range of feedback policy coefficients in order to guarantee determinacy and well-defined values for the policy instruments employed for cyclical stabilization and debt consolidation.

Again, results are available upon request.

## 7.4 Policy reaction to the natural level of output

We now assume that, in each time period, policymakers react to the current natural level of output rather than to its steady state level. The natural level of output is defined as the level of output that would arise in the absence of nominal fixities. Technically, this means that the policy target for output in the feedback policy rules, (12)-(16), is now time-varying and its value follows from the fictional case in which, other things equal, the Calvo parameter is set at zero. Thus, we first solve for this fictional case (always computing welfare-maximizing feedback policy coefficients and the associated equilibrium values of the endogenous variables including output) and, in turn, we use the resulting time-path of the natural level of output as a time-varying target in the policy rules (12)-(16).

The new results, when policymakers react to the natural level of output, are reported in Table A.1 in Appendix 5, which is like Table 4 in the main text (since nominal fixities play no real role in the long run, the steady state solution remains the same as in Table 2). Comparison of these Tables implies that the main results do not change. On the other hand, there are some

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<sup>14</sup>We model shocks to the time preference rate by using an  $AR(1)$  process like that in equation (18) used for the TFP shock. Our main results do not change.

interesting quantitative differences. For instance, optimal interest rate reaction to the output gap, as measured by  $\phi_y$ , is much stronger in Table A.1 than it was in Table 4. This makes sense: while monetary policy should not respond to the output gap when the output target in the gap is its steady state level, reaction to this gap becomes more desirable when the output target accounts for the distortions in output caused by, for instance, nominal fixities (see also e.g. Kirsanova et al., 2009, and Sims, 2013, for which measure of output should be targeted by the central bank in interest rate rules).<sup>15</sup> Thus, although the feedback policy coefficient on inflation,  $\phi_\pi$ , is still higher than the coefficient on output,  $\phi_y$ , response to output is welfare enhancing relative to Table 4, where the interest rate responded solely to inflation. To put it differently, although inflation remains the main concern of monetary authorities, it is not the sole concern.

## 7.5 Allowing for trend inflation

We now allow for trend or steady state inflation. For instance, we allow the gross steady state rate of inflation,  $\Pi$ , to be 1.005 quarterly (or 1.02 annually) which is perceived to be the target value of the European Central Bank. This requires recalibration of the model. The new results are reported in Tables A.2 and A.3 in Appendix 6, which are respectively like Tables 2 and 4 in the main text. As is known (see e.g. Ascari, 2004), trend inflation can play a non-trivial role. For instance, steady state output falls with inflation (see page 650 in Ascari, 2004, and page 215 in Wickens, 2008, for interpretation of the output loss as trend inflation rises). Here, we get similar results: steady state output and utility are lower in Table A.2 with positive trend inflation than in Table 2 with zero trend inflation. Expected discounted lifetime utility is also lower with positive trend inflation irrespectively of the fiscal policy instrument used (compare Table A.3 to Table 4). Nevertheless, the main qualitative results do not change. For instance, public spending is again the least distorting fiscal policy instrument, and labor taxes the most distorting one, as it was the case in Table 4.

## 8 Concluding remarks and possible extensions

This paper studied the optimal mix of monetary and fiscal policy actions in a New Keynesian model of a closed economy. The aim was to welfare rank different fiscal (tax and spending)

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<sup>15</sup>It is useful to report the following experiment. We exogenously set  $\phi_\pi = 1.5$  and  $\phi_y = 0.35$  which are typical numbers in a Taylor rule for the nominal interest rate. Then, our computations, with such ad hoc policy coefficients, show that the standard deviation of both inflation and output are lower when the output target in the output gap measure is the current-period natural level of output than when the output target is its time-invariant steady state level.

policy instruments when, at the same time, the central bank followed a Taylor rule for the nominal interest rate. We did so when the policy task was to stabilize the economy against shocks and to reduce public debt over time.

Since the results have been listed in the Introduction, we close with some extensions. A first natural extension is to see what changes in an open economy setup. Actually, in two companion papers (Philippopoulos et al., 2013 and 2014), we have done so by examining respectively the case of a semi-small open economy facing sovereign risk premia when it borrows from abroad and the case of a multi-country model of a currency union with debtor and creditor country-members. We find that openness can change the results.

This type of work can be further extended in several ways. For instance, it would be interesting to study the implications of less conventional monetary policy instruments, like the case in which the central bank acts as a lender of last resort. Similarly, it would be interesting to use a more detailed decomposition of public spending (like spending on infrastructure, education services, social security, etc) and reexamine the attractiveness of each one of those spending categories as a tool for macroeconomic stabilization (but this would require a richer model where each public spending category functions a well-specified role). Finally, one could allow for a different policy reaction to economic fundamentals during booms and during recessions (although, as far as we know, policy regime switching is a challenging task computationally; see e.g. Foerster et al., 2014). We leave these extensions for future work.

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## 9 Appendices

### 9.1 Appendix 1: Households

This Appendix provides details and the solution of household's problem. There are  $i = 1, 2, \dots, N$  households. Each household  $i$  acts competitively to maximize expected lifetime utility.

#### 9.1.1 Household's problem

Household  $i$ 's expected lifetime utility is:

$$E_0 \sum_{t=0}^{\infty} \beta^t U(c_{i,t}, n_{i,t}, m_{i,t}, g_t) \quad (20)$$

where  $c_{i,t}$  is  $i$ 's consumption bundle (defined below),  $n_{i,t}$  is  $i$ 's hours of work,  $m_{i,t} \equiv \frac{M_{i,t}}{P_t}$  is  $i$ 's real money balances,  $g_t$  is per capita public spending,  $0 < \beta < 1$  is the time discount rate, and  $E_0$  is the rational expectations operator conditional on the current period information set.

We will use a utility function of the form:

$$u_{i,t}(c_{i,t}, n_{i,t}, m_{i,t}, g_t) = \frac{c_{i,t}^{1-\sigma}}{1-\sigma} - \chi_n \frac{n_{i,t}^{1+\eta}}{1+\eta} + \chi_m \frac{m_{i,t}^{1-\mu}}{1-\mu} + \chi_g \frac{g_t^{1-\zeta}}{1-\zeta} \quad (21)$$

where  $\chi_n, \chi_m, \chi_g, \sigma, \eta, \mu, \zeta$  are standard preference parameters.

The period budget constraint of each household  $i$  is in nominal terms:

$$\begin{aligned} (1 + \tau_t^c) P_t c_{i,t} + P_t x_{i,t} + B_{i,t} + M_{i,t} = \\ (1 - \tau_t^k) (r_t^k P_t k_{i,t-1} + D_{i,t}) + (1 - \tau_t^n) W_t n_{i,t} + R_{t-1} B_{i,t-1} + M_{i,t-1} - T_{i,t}^l \end{aligned} \quad (22)$$

where  $P_t$  is the price index,  $x_{i,t}$  is  $i$ 's real investment,  $B_{i,t}$  is  $i$ 's end-of-period nominal government bonds,  $M_{i,t}$  is  $i$ 's end-of period nominal money holdings,  $r_t^k$  is the real return to inherited capital,  $k_{i,t-1}$ ,  $D_{i,t}$  is  $i$ 's nominal dividends paid by firms,  $W_t$  is the nominal wage rate,  $R_{t-1}$  is the gross nominal return to government bonds between  $t-1$  and  $t$ ,  $T_{i,t}^l$  is nominal lump-sum taxes/transfers to each  $i$  from the government, and  $\tau_t^c, \tau_t^k, \tau_t^n$  are respectively tax rates on private consumption, capital income and labour income.

Dividing by  $P_t$ , the budget constraint of each  $i$  in real terms is:

$$(1 + \tau_t^c) c_{i,t} + x_{i,t} + b_{i,t} + m_{i,t} = (1 - \tau_t^k) (r_t^k k_{i,t-1} + d_{i,t}) + (1 - \tau_t^n) w_t n_{i,t} + R_{t-1} \frac{P_{t-1}}{P_t} b_{i,t-1} + \frac{P_{t-1}}{P_t} m_{i,t-1} - \tau_{i,t}^l \quad (23)$$

where small letters denote real variables, i.e.  $b_{i,t} \equiv \frac{B_{i,t}}{P_t}$ ,  $m_{i,t} \equiv \frac{M_{i,t}}{P_t}$ ,  $w_t \equiv \frac{W_t}{P_t}$ ,  $d_{i,t} \equiv \frac{D_{i,t}}{P_t}$ ,  $\tau_{i,t}^l \equiv \frac{T_{i,t}^l}{P_t}$ , at individual level.

The motion of physical capital for each household  $i$  is:

$$k_{i,t} = (1 - \delta) k_{i,t-1} + x_{i,t} \quad (24)$$

where  $0 < \delta < 1$  is the depreciation rate of capital.

Household  $i$ 's consumption bundle at  $t$ ,  $c_{i,t}$ , is a composite of  $h = 1, 2, \dots, N$  varieties of goods, denoted as  $c_{i,t}(h)$ , where each variety  $h$  is produced monopolistically by one firm  $h$ . Using the Dixit-Stiglitz aggregator, we define:

$$c_{i,t} = \left[ \sum_{h=1}^N \lambda [c_{i,t}(h)]^{\frac{\phi-1}{\phi}} \right]^{\frac{\phi}{\phi-1}} \quad (25)$$

where  $\phi > 0$  is the elasticity of substitution across goods produced and  $\sum_{h=1}^N \lambda = 1$  are weights (to avoid scale effects, we set  $\lambda = 1/N$ ).

Household  $i$ 's total consumption expenditure is:

$$P_t c_{i,t} = \sum_{h=1}^N \lambda P_t(h) c_{i,t}(h) \quad (26)$$

where  $P_t(h)$  is the price of variety  $h$ .

### 9.1.2 Household's optimality conditions

Each household  $i$  acts competitively taking prices and policy as given. Following the related literature, we first suppose that the household chooses its desired consumption of the composite good,  $c_{i,t}$ , and, in turn, chooses how to distribute its purchases of individual varieties,  $c_{i,t}(h)$ .

The first-order conditions include the budget constraint above and:

$$\frac{c_{i,t}^{-\sigma}}{(1 + \tau_t^c)} = \beta E_t \frac{c_{i,t+1}^{-\sigma}}{(1 + \tau_{t+1}^c)} \left[ (1 - \tau_{t+1}^k) r_{t+1}^k + (1 - \delta) \right] \quad (27)$$

$$\frac{c_{i,t}^{-\sigma}}{(1 + \tau_t^c)} = \beta E_t \frac{c_{i,t+1}^{-\sigma}}{(1 + \tau_{t+1}^c)} R_t \frac{P_t}{P_{t+1}} \quad (28)$$

$$\chi_m m_{i,t}^{-\mu} - \frac{c_{i,t}^{-\sigma}}{(1 + \tau_t^c)} + \beta E_t \frac{c_{i,t+1}^{-\sigma}}{(1 + \tau_{t+1}^c)} \frac{P_t}{P_{t+1}} = 0 \quad (29)$$

$$\chi_n \frac{n_{i,t}^\eta}{c_{i,t}^{-\sigma}} = \frac{(1 - \tau_t^n)}{(1 + \tau_t^c)} w_t \quad (30)$$

$$c_{i,t}(h) = \left[ \frac{P_t(h)}{P_t} \right]^{-\phi} c_{i,t} \quad (31)$$

Equations (27) and (28) are respectively the Euler equations for capital and bonds, (29) is the optimality condition for money balances, (30) is the optimality condition for work hours and (31) shows the optimal demand for each variety of goods.

Notice that equations (26) and (31) imply for the price index:

$$P_t = \left[ \sum_{h=1}^N \lambda [P_t(h)]^{1-\phi} \right]^{\frac{1}{1-\phi}} \quad (32)$$

## 9.2 Appendix 2: Firms

This Appendix provides details and the solution of firm's problem. There are  $h = 1, 2, \dots, N$  firms. Each firm  $h$  produces a differentiated good of variety  $h$  under monopolistic competition facing Calvo-type nominal fixities.

### 9.2.1 Demand for firm's product

Each firm  $h$  faces demand for its product,  $y_t(h)$ . The latter comes from households' consumption and investment,  $c_t(h)$  and  $x_t(h)$ , where  $c_t(h) \equiv \sum_{i=1}^N c_{i,t}(h)$  and  $x_t(h) \equiv \sum_{i=1}^N x_{i,t}(h)$ , and from the government,  $g_t(h)$ . Thus,

$$y_t(h) = c_t(h) + x_t(h) + g_t(h) \quad (33)$$

where we have from above:

$$c_t(h) = \left[ \frac{P_t(h)}{P_t} \right]^{-\phi} c_t \quad (34)$$

and similarly:

$$x_t(h) = \left[ \frac{P_t(h)}{P_t} \right]^{-\phi} x_t \quad (35)$$

$$g_t(h) = \left[ \frac{P_t(h)}{P_t} \right]^{-\phi} g_t \quad (36)$$

where  $c_t \equiv \sum_{i=1}^N c_{i,t}$  is total private consumption,  $x_t \equiv \sum_{i=1}^N x_{i,t}$  is total private investment and  $g_t$  is total public spending.

Since, at the economy level:

$$y_t = c_t + x_t + g_t \quad (37)$$

the above equations imply that the demand for each firm's product is:

$$y_t(h) = c_t(h) + x_t(h) + g_t(h) = \left[ \frac{P_t(h)}{P_t} \right]^{-\phi} y_t \quad (38)$$

### 9.2.2 Firm's problem

Nominal profit of each firm  $h$  is:

$$D_t(h) = P_t(h)y_t(h) - P_t r_t^k k_{t-1}(h) - W_t n_t(h) \quad (39)$$

All firms use the same technology represented by the production function:

$$y_t(h) = A_t [k_{t-1}(h)]^\alpha [n_t(h)]^{1-\alpha} \quad (40)$$

where  $A_t$  is an exogenous stochastic TFP process.

Under imperfect competition, profit maximization is subject to the demand function derived above, namely:

$$y_t(h) = \left[ \frac{P_t(h)}{P_t} \right]^{-\phi} y_t \quad (41)$$

In addition, following Calvo (1983), firms choose their prices facing a nominal fixity. In each period, firm  $h$  faces an exogenous probability  $\theta$  of not being able to reset its price. A firm  $h$ , which is able to reset its price, chooses its price  $P_t^\#(h)$  to maximize the sum of discounted expected nominal profits for the next  $k$  periods in which it may have to keep its price fixed.

### 9.2.3 Firm's optimality conditions

Following the related literature, we follow a two-step procedure. We first solve a cost minimization problem, where each firm  $h$  minimizes its cost by choosing factor inputs given technology and prices. The solution will give a minimum nominal cost function, which is a function of factor prices and output produced by the firm. In turn, given this cost function, each firm, which is able to reset its price, solves a maximization problem by choosing its price.

The solution to the cost minimization problem gives the input demand functions:

$$w_t = mc_t(1 - a)A_t[k_{t-1}(h)]^\alpha[n_t(h)]^{-\alpha} \quad (42)$$

$$r_t^k = mc_t a A_t [k_{t-1}(h)]^{\alpha-1} [n_t(h)]^{1-\alpha} \quad (43)$$

where  $mc_t = \Psi'_t(\cdot)$  is the marginal nominal cost with  $\Psi_t(\cdot)$  denoting the associated minimum nominal cost function for producing  $y_t(h)$  at  $t$ .

Then, the firm chooses its price,  $P_t^\#(h)$ , to maximize nominal profits:

$$E_t \sum_{k=0}^{\infty} \theta^k \Xi_{t,t+k} D_{t+k}(h) = E_t \sum_{k=0}^{\infty} \theta^k \Xi_{t,t+k} \left\{ P_t^\#(h) y_{t+k}(h) - \Psi_{t+k}(y_{t+k}(h)) \right\}$$

where  $\Xi_{t,t+k}$  is a discount factor taken as given by the firm and where  $y_{t+k}(h) = \left[ \frac{P_t^\#(h)}{P_{t+k}} \right]^{-\phi} y_{t+k}$ .

The first-order condition gives:

$$E_t \sum_{k=0}^{\infty} \theta^k \Xi_{t,t+k} \left[ \frac{P_t^\#(h)}{P_{t+k}} \right]^{-\phi} y_{t+k} \left\{ P_t^\#(h) - \frac{\phi}{\phi - 1} \Psi'_{t+k} \right\} = 0 \quad (44)$$

Dividing by the aggregate price index,  $P_t$ , we have:

$$E_t \sum_{k=0}^{\infty} \theta^k [\Xi_{t,t+k} \left[ \frac{P_t^\#(h)}{P_{t+k}} \right]^{-\phi} y_{t+k} \left\{ \frac{P_t^\#(h)}{P_t} - \frac{\phi}{\phi - 1} mc_{t+k} \frac{P_{t+k}}{P_t} \right\}] = 0 \quad (45)$$

Thus, the behaviour of firm  $h$ , which can reset its price, is summarized by (42), (43) and (45).

Each firm  $h$ , which can reset its price in period  $t$ , solves an identical problem, so  $P_t^\#(h) = P_t^\#$  is independent of  $h$ , and each firm  $h$ , which cannot reset its price, just sets its previous period price  $P_t(h) = P_{t-1}(h)$ . Then, it can be shown that the evolution of the aggregate price level is given by:

$$(P_t)^{1-\phi} = \theta (P_{t-1})^{1-\phi} + (1 - \theta) (P_t^\#)^{1-\phi} \quad (46)$$

### 9.3 Appendix 3: Decentralized equilibrium (given policy instruments)

We now combine the above to solve for a Decentralized Equilibrium (DE) for any feasible monetary and fiscal policy. In this DE, (i) all households maximize utility (ii) a fraction  $(1 - \theta)$  of firms maximize profits by choosing the identical price  $P_t^\#$ , while the rest,  $\theta$ , set their

previous period prices (iii) all constraints are satisfied and (iv) all markets clear.

The DE is summarized by the following conditions (quantities are in per capita terms):

$$\frac{c_t^{-\sigma}}{(1 + \tau_t^c)} = \beta E_t \frac{c_{t+1}^{-\sigma}}{(1 + \tau_{t+1}^c)} \left[ (1 - \tau_{t+1}^k) r_{t+1}^k + (1 - \delta) \right] \quad (47)$$

$$c_t^{-\sigma} \frac{1}{(1 + \tau_t^c)} = \beta E_t R_t \frac{c_{t+1}^{-\sigma}}{(1 + \tau_{t+1}^c)} \frac{P_t}{P_{t+1}} \quad (48)$$

$$\chi_m m_t^{-\mu} - \frac{c_t^{-\sigma}}{(1 + \tau_t^c)} + \beta E_t \frac{c_{t+1}^{-\sigma}}{(1 + \tau_{t+1}^c)} \frac{P_t}{P_{t+1}} = 0 \quad (49)$$

$$\chi_n \frac{n_t^\eta}{c_t^{-\sigma}} = \frac{(1 - \tau_t^n)}{(1 + \tau_t^c)} w_t \quad (50)$$

$$k_t = (1 - \delta) k_{t-1} + x_t \quad (51)$$

$$E_t \sum_{k=0}^{\infty} \theta^k \left\{ \Xi_{t,t+k} \left[ \frac{P_t^\#}{P_{t+k}} \right]^{-\phi} y_{t+k} \left( \frac{P_t^\#}{P_t} - \frac{\phi}{\phi - 1} m c_{t+k} \frac{P_{t+k}}{P_t} \right) \right\} = 0 \quad (52)$$

$$w_t = m c_t (1 - a) \frac{y_t}{n_t} \quad (53)$$

$$r_t^k = m c_t a \frac{y_t}{k_t} \quad (54)$$

$$d_t = y_t - w_t n_t - r_t^k k_{t-1} \quad (55)$$

$$y_t = \frac{1}{\left( \frac{\tilde{P}_t}{P_t} \right)^{-\phi}} A_t k_{t-1}^a n_t^{1-a} \quad (56)$$

$$b_t + m_t = R_{t-1} b_{t-1} \frac{P_{t-1}}{P_t} + m_{t-1} \frac{P_{t-1}}{P_t} + g_t - \tau_t^c c_t - \tau_t^n w_t n_t - \tau_t^k (r_t^k k_{t-1} + d_t) - \tau_t^l \quad (57)$$

$$y_t = c_t + x_t + g_t \quad (58)$$

$$(P_t)^{1-\phi} = \theta (P_{t-1})^{1-\phi} + (1 - \theta) (P_t^\#)^{1-\phi} \quad (59)$$

$$(\tilde{P}_t)^{-\phi} = \theta (\tilde{P}_{t-1})^{-\phi} + (1 - \theta) (P_t^\#)^{-\phi} \quad (60)$$

where  $\Xi_{t,t+k} \equiv \beta^k \frac{c_{t+k}^{-\sigma}}{c_t^{-\sigma}} \frac{P_t}{P_{t+k}} \frac{\tau_t^c}{\tau_{t+k}^c}$  and  $\tilde{P}_t \equiv \left( \sum_{h=1}^N [P_t(h)]^{-\phi} \right)^{-\frac{1}{\phi}}$ . Thus,  $\left( \frac{\tilde{P}_t}{P_t} \right)^{-\phi}$  is a measure of price dispersion.

We thus have 14 equilibrium conditions in 14 variables,  $\{y_t, c_t, n_t, x_t, k_t, m_t, b_t, P_t, P_t^\#, \tilde{P}_t, w_t, mc_t, d_t, r_t^k\}_{t=0}^\infty$ . This is given the independently set monetary and fiscal policy instruments,  $\{R_t, g_t, \tau_t^c, \tau_t^k, \tau_t^n, \tau_t^l\}_{t=0}^\infty$ , technology,  $\{A_t\}_{t=0}^\infty$ , and initial conditions for the state variables.

#### 9.4 Appendix 4: Decentralized equilibrium transformed (given policy instruments)

We now rewrite the above equilibrium conditions, first, by using inflation rates rather than price levels, second, by writing the firm's optimality condition (35) in recursive form and, third, by introducing a new equation that helps us to compute expected discounted lifetime utility.

##### 9.4.1 Variables expressed in ratios

We define three new endogenous variables, which are the gross inflation rate  $\Pi_t \equiv \frac{P_t}{P_{t-1}}$ , the auxiliary variable  $\Theta_t \equiv \frac{P_t^\#}{P_t}$ , and the price dispersion index  $\Delta_t \equiv \left[ \frac{\tilde{P}_t}{P_t} \right]^{-\phi}$ . We also find it convenient to express the two exogenous fiscal spending policy instruments as ratios of GDP,  $s_t^g \equiv \frac{g_t}{y_t}$  and  $s_t^l \equiv \frac{\tau_t^l}{y_t}$ .

Thus, we can use  $\Pi_t, \Theta_t, \Delta_t, s_t^g, s_t^l$  instead of  $P_t, P_t^\#, \tilde{P}_t, g_t, \tau_t^l$  respectively.

##### 9.4.2 Equation (52) expressed in recursive form

Following Schmitt-Grohé and Uribe (2007), we look for a recursive representation of (52):

$$E_t \sum_{k=0}^{\infty} \theta^k \Xi_{t,t+k} \left[ \frac{P_t^\#}{P_{t+k}} \right]^{-\phi} y_{t+k} \left\{ \frac{P_t^\#}{P_t} - \frac{\phi}{(\phi-1)} mc_{t+k} \frac{P_{t+k}}{P_t} \right\} = 0 \quad (61)$$

We define two auxiliary endogenous variables:

$$z_t^1 \equiv E_t \sum_{k=0}^{\infty} \theta^k \Xi_{t,t+k} \left[ \frac{P_t^\#}{P_{t+k}} \right]^{-\phi} y_{t+k} \frac{P_t^\#}{P_t} \quad (62)$$

$$z_t^2 \equiv E_t \sum_{k=0}^{\infty} \theta^k \Xi_{t,t+k} \left[ \frac{P_t^\#}{P_{t+k}} \right]^{-\phi} y_{t+k} mc_{t+k} \frac{P_{t+k}}{P_t} \quad (63)$$

Using these two auxiliary variables,  $z_t^1$  and  $z_t^2$ , we come up with two new equations which enter the dynamic system and allow a recursive representation of (61). Thus, we replace equation

(52) above with:

$$z_t^1 = \frac{\phi}{(\phi - 1)} z_t^2 \quad (64)$$

where (if we also use the definitions for  $\Pi_t$  and  $\Theta_t$ ):

$$z_t^1 = \Theta_t^{-\phi-1} y_t + \beta \theta E_t \frac{c_{t+1}^{-\sigma}}{c_t^{-\sigma}} \frac{1 + \tau_t^c}{1 + \tau_{t+1}^c} \left( \frac{\Theta_t}{\Theta_{t+1}} \right)^{-\phi-1} \left( \frac{1}{\Pi_{t+1}} \right)^{-\phi} z_{t+1}^1 \quad (65)$$

$$z_t^2 = \Theta_t^{-\phi} y_t m c_t + \beta \theta E_t \frac{c_{t+1}^{-\sigma}}{c_t^{-\sigma}} \frac{1 + \tau_t^c}{1 + \tau_{t+1}^c} \left( \frac{\Theta_t}{\Theta_{t+1}} \right)^{-\phi} \left( \frac{1}{\Pi_{t+1}} \right)^{1-\phi} z_{t+1}^2 \quad (66)$$

Thus, instead of (52), we can use (64), (65) and (66), and thus add two new endogenous variables,  $z_t^1$  and  $z_t^2$ .

### 9.4.3 Lifetime utility written as a first-order dynamic equation

To compute social welfare, we follow Schmitt-Grohé and Uribe (2007) by defining a new endogenous variable,  $V_t$ , whose motion is:

$$V_t = \frac{c_t^{1-\sigma}}{1-\sigma} - \chi_n \frac{n_t^{1+\phi}}{1+\phi} + \chi_m \frac{m_t^{1-\mu}}{1-\mu} + \chi_g \frac{(s_t^g y_t)^{1-\zeta}}{1-\zeta} + \beta E_t V_{t+1} \quad (67)$$

where  $V_t$  is the expected discounted lifetime utility of the household at any  $t$ .

Thus, we add equation (67), and a new endogenous variable  $V_t$ , to the equilibrium system.

### 9.4.4 Equations of transformed DE

Using the above, the final non-linear stochastic system is:

$$\frac{c_t^{-\sigma}}{(1 + \tau_t^c)} = \beta E_t \frac{c_{t+1}^{-\sigma}}{(1 + \tau_{t+1}^c)} \left[ (1 - \tau_{t+1}^k) r_{t+1}^k + (1 - \delta) \right] \quad (68)$$

$$\frac{c_t^{-\sigma}}{R_t} \frac{1}{(1 + \tau_t^c)} = \beta E_t \frac{c_{t+1}^{-\sigma}}{(1 + \tau_{t+1}^c)} \frac{1}{\Pi_{t+1}} \quad (69)$$

$$\chi_m m_t^{-\mu} - \frac{c_t^{-\sigma}}{(1 + \tau_t^c)} + \beta E_t \frac{c_{t+1}^{-\sigma}}{(1 + \tau_{t+1}^c)} \frac{1}{\Pi_{t+1}} = 0 \quad (70)$$

$$\chi_n \frac{n_t^\eta}{c_t^{-\sigma}} = \frac{(1 - \tau_t^n)}{(1 + \tau_t^c)} w_t \quad (71)$$

$$k_t = (1 - \delta) k_{t-1} + x_t \quad (72)$$

$$z_t^1 = \frac{\phi - 1}{\phi} z_t^2 \quad (73)$$

$$w_t = mc_t(1-a)\frac{y_t}{n_t} \quad (74)$$

$$r_t^k = mc_t a \frac{y_t}{k_{t-1}} \quad (75)$$

$$d_t = y_t - w_t n_t - r_t^k k_{t-1} \quad (76)$$

$$y_t = \frac{1}{\Delta_t} A_t k_{t-1}^a n_t^{1-a} \quad (77)$$

$$b_t + m_t = R_{t-1} b_{t-1} \frac{1}{\Pi_t} + m_{t-1} \frac{1}{\Pi_t} + s_t^g y_t - \tau_t^c c_t - \tau_t^n w_t n_t - \tau_t^k [r_t^k k_{t-1} + d_t] - s_t^l y_t \quad (78)$$

$$y_t = c_t + x_t + s_t^g y_t \quad (79)$$

$$\Pi_t^{1-\phi} = \theta + (1-\theta) [\Theta_t \Pi_t]^{1-\phi} \quad (80)$$

$$\Delta_t = (1-\theta) \Theta_t^{-\phi} + \theta \Pi_t^\phi \Delta_{t-1} \quad (81)$$

$$z_t^1 = \Theta_t^{-\phi-1} y_t + \beta \theta E_t \frac{c_{t+1}^{-\sigma}}{c_t^{-\sigma}} \frac{1 + \tau_t^c}{1 + \tau_{t+1}^c} \left( \frac{\Theta_t}{\Theta_{t+1}} \right)^{-\phi-1} \left( \frac{1}{\Pi_{t+1}} \right)^{-\phi} z_{t+1}^1 \quad (82)$$

$$z_t^2 = \Theta_t^{-\phi} y_t mc_t + \beta \theta E_t \frac{c_{t+1}^{-\sigma}}{c_t^{-\sigma}} \frac{1 + \tau_t^c}{1 + \tau_{t+1}^c} \left( \frac{\Theta_t}{\Theta_{t+1}} \right)^{-\phi} \left( \frac{1}{\Pi_{t+1}} \right)^{1-\phi} z_{t+1}^2 \quad (83)$$

$$V_t = \frac{c_t^{1-\sigma}}{1-\sigma} + \chi_m \frac{m_t^{1-\mu}}{1-\mu} - \chi_n \frac{n_t^{1+\phi}}{1+\phi} + \chi_g \frac{(s_t^g y_t)^{1-\zeta}}{1-\zeta} + \beta E_t V_{t+1} \quad (84)$$

There are 17 equations in 17 endogenous variables,  $\{y_t, c_t, n_t, x_t, k_t, m_t, b_t, \Pi_t, \Theta_t, \Delta_t, w_t, mc_t, d_t, r_t^k, z_t^1, z_t^2, V_t\}_{t=0}^\infty$ . This is given the independently set policy instruments,  $\{R_t, s_t^g, \tau_t^c, \tau_t^k, \tau_t^n, s_t^l\}_{t=0}^\infty$ , technology,  $\{A_t\}_{t=0}^\infty$ , and initial conditions for the state variables. Recall that  $\{R_t, s_t^g, \tau_t^c, \tau_t^k, \tau_t^n\}_{t=0}^\infty$  follow the feedback rules specified above, while  $\{s_t^l\}_{t=0}^\infty$  remains constant and equal to its average value in the data.

## 9.5 Appendix 5: The natural level of output as a policy target

In this Appendix, the policy target for output, in each period, is the level without nominal fixities. Then, working as explained in the text (see Section 7), Table 4 changes to:

Table A.1: Optimal monetary reaction to inflation and output  
and optimal fiscal reaction to debt and output  
with reaction to the natural level of output

Policy instruments	Optimal interest-rate reaction to inflation and output	Optimal fiscal reaction to debt and output	Lifetime utility $E_0V_0$
$R_t \quad s_t^g$	$\phi_\pi = 1.1$ $\phi_y = 0.0337$	$\gamma_l^g = 0.1922$ $\gamma_y^g = 0.0992$	109.2589
$R_t \quad \tau_t^c$	$\phi_\pi = 3$ $\phi_y = 0.1966$	$\gamma_l^c = 0.2$ $\gamma_y^c = 0.0499$	109.0948
$R_t \quad \tau_t^k$	$\phi_\pi = 3$ $\phi_y = 0.3759$	$\gamma_l^k = 0.15$ $\gamma_y^k = 0.1744$	109.0752
$R_t \quad \tau_t^n$	$\phi_\pi = 3$ $\phi_y = 0.3783$	$\gamma_l^n = 0.0092$ $\gamma_y^n = 0.0958$	108.7812

## 9.6 Appendix 6: Allowing for steady-state inflation

In this section, we allow for steady state, or trend, inflation (2% annually). Then, working as explained in the text (see Section 7), Table 2 changes to:

Table A.2: Steady state solution or "the status quo"  
with 2% long-run inflation annually

Variables	Solution	Variables	Solution	Data
$y$	0.737742	$d$	0.12281	-
$c$	0.459145	$r^k$	0.0395211	-
$n$	0.278446	$z^1$	2.15648	-
$x$	0.108917	$z^2$	1.79706	-
$k$	5.18651	$u$	0.799858	-
$m$	1.26165	$\frac{c}{y}$	0.62	0.57
$\Pi$	1.005	$\frac{b}{y}$	3.4	3.4
$\Theta$	1.01042	$\frac{x}{y}$	0.15	0.18
$\Delta$	1.00049	$\frac{m}{y}$	1.71	-
$w$	1.4723	$\frac{k}{y}$	7.03	-
$mc$	0.833534	-	-	-

while Table 4 changes to:

Table A.3: Optimal monetary reaction to inflation and output  
and optimal fiscal reaction to debt and output  
with 2% long-run inflation annually

Policy instruments	Optimal interest-rate reaction to inflation and output	Optimal fiscal reaction to debt and output	Lifetime utility $E_0V_0$
$R_t \quad s_t^g$	$\phi_\pi = 1.1$ $\phi_y = 0.0101$	$\gamma_l^g = 0.2251$ $\gamma_y^g = 0.4922$	108.7945
$R_t \quad \tau_t^c$	$\phi_\pi = 2.9030$ $\phi_y = 0.0001$	$\gamma_l^c = 0.2$ $\gamma_y^c = 0.0512$	108.5390
$R_t \quad \tau_t^k$	$\phi_\pi = 2.9811$ $\phi_y = 0.0001$	$\gamma_l^k = 0.1498$ $\gamma_y^k = 0$	108.5412
$R_t \quad \tau_t^n$	$\phi_\pi = 3$ $\phi_y = 0.0525$	$\gamma_l^n = 0.0128$ $\gamma_y^n = 0.0488$	108.1972