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# EFFECTS OF THERMOCOUPLE ELECTRICAL INSULATION ON THE MEASUREMENT OF SURFACE TEMPERATURE

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## Abstract

Analytical, numerical and experimental analyses have been performed to investigate the effects of thermocouple wire electrical insulation on the temperature measurement of a reference surface. Two diameters of type K thermocouple, 80 $\mu$ m and 200 $\mu$ m, with different exposed wire lengths (0 mm, 5mm, 10mm, 15mm and 20mm) were used to measure various surface temperatures (4°C, 8°C, 15°C, 25°C and 35°C). Measurements were made with the thermocouple in direct contact with the surface, with wires extending vertically and exposed to natural convection. Analytical results of the thermocouple wire with insulation confirm that there is no specific value for the critical radius and the rate of heat flux around the thermocouple wire continuously increases with the wire diameter even when this is larger than the critical radius. Numerical simulation using COMSOL Multiphysics software also confirms that there is negligible thermal effect from the electrical insulation. Moreover, the experimental results agree well with those obtained by both the analytical and numerical methods and further confirm that the diameter of the thermocouple has an impact on the temperature measurement.

**Keywords:** *Thermocouple insulation; conduction error; surface temperature, numerical simulation*

## 1 Introduction

When thermocouple wires are exposed to an environment with a temperature different to that of the object being measured, heat transfer occurs through the wires, which disturbs the system, alters the thermocouple junction temperature and causes an error in the temperature measurement.

Boelter and Lockhart[1] carried out experimental work to measure the temperature of a thick stainless steel plate. The plate was kept at constant temperature(s) by heating one side and cooling other side by hot and cold air flow respectively. Two types of thermocouple were tested (iron-constantan and Chromel-Alumel) with different wire sizes and thermocouples were attached to the cold air side during the measurement process. Moreover, they investigated the influence of vertical and horizontal thermocouple attachment methods on the surface temperature measurement. They suggested that using an inter-thermocouple wire inside a plate or extending the wires along the surface being measured for a length more than 50 times the wire diameter can potentially minimise any measurement error. Tarnopolsky and Seginer [2] performed experimental analysis to study the effects of wire diameter and electrical insulation on conduction error during temperature measurement of vegetable leaves. Small wire(s) size (AWG40) type T thermocouples were placed parallel to the surface while the probe was attached using special glue. Different surface contact lengths of insulated and uninsulated thermocouple wires were tested. They verified that a length of bare thermocouple wire glued to a surface requires only half of the contact length of an insulated wire to achieve a uniform temperature between its junction and the measured surface. He et al.[3]conducted a CFD analysis and compared results between the effects of vertical and parallel positions of thermocouples on surface temperature measurement. Thermocouple(s) were attached to the uniform heat generating surface exposed to air flow rate with different speeds. They showed that placing thermocouple wires parallel to the surface can reduce measurement error by half as compared with a vertical position.

Various thermocouple arrangements inside low conductivity materials exposed to high heat transfer were examined experimentally by Brewer[4] and Dow[5].They proved that relatively high error was produced when thermocouple wires passed through a low conductivity material parallel to the heat flow. Therefore, they recommended placing the wires at the same isothermal surface of the junction for several diameters to minimize the error. Singh and Dybbs [6] measured temperature variation inside the body by inserting thermocouples at different depths parallel as well as normal to temperature variation through the body. They

advised that the thermocouple wires and the junction should be at the same isothermal plane in order to reduce error. Consequently, if the experiment conditions don't allow, then the temperature reading should be corrected. However, correction is not appropriate if the measurement error of the thermocouple is larger than the error due to conduction.

Another strategy was adopted by Li and Wells [7] to measure surface temperature by pushing a thermocouple through a hole opposite to the surface. Surface temperature was measured during quenching process by a type K thermocouple which was inserted into the hole near the surface. Experimental and numerical study confirmed that the effect of the hole and thermocouple should be considered during the temperature measurement. Furthermore, Li and Wells [7] proved that an increase in the hole diameter caused larger effect on the temperature measurement. Two dimensional analysis by Bartkus [8] predicted that most of the error in thermocouple measurement within the body comes from the increase in thermal resistance between the thermocouple insulation and the surrounding materials. Attia et al. [9] experimental and numerical results consolidated the conclusions of Li and Wells [7] and Bartkus [8]. Moreover, Attia et al. [9] studied the effects of different thermocouple materials properties (E, J and T) and the surrounding material on temperature measurement inside the body. They showed an increase in thermocouple thermal conductivity augmented heat transfer and thus underestimated the temperature reading. Further, the existence of a thermocouple hole altered the temperature field around the thermocouple and caused a reading error [9].

Tarnopolsky and Seginer [2] observed that a thermocouple with lower thermal conductivity (type-K) needs 60% less contact length than one with a higher conductivity (type-T). Dow [5] pointed out that because of its high thermal conductivity, alumina tubes produce higher error in comparison with resin-glass insulation when used as an insulation material for thermocouple wires. While numerical results of Kidd [10] for skin temperature measurement confirmed that pairing chromel-constantan wires gave a lower conduction error in comparison to other materials used for thermocouple wires. Experimental results of Boelter and Lockhart [1] showed iron-constantan gives higher error in temperature measurement than Chromel-Alumel. Shaukatullah and Claassen [11] performed experimental results for the temperature measurement of a chip surface with different thermocouple sizes and attachment methods. They advised that using a small diameter of thermocouple with lower thermal conductivity can minimize thermocouple wires conduction error.

Boelter and Lockhart [1] confirmed that there is negligible effect from electrical insulation on temperature measurement when the thermocouple diameter (including the insulation) is less than the critical radius. Further, Mohun [12] discussed analytically the effect of electrical insulation for temperature measurement inside a solid wall. Mohun showed that the presence of electrical insulation over a critical length can only affect the thermocouple reading if the wires pass through a variable environment temperature. Tszeng and Zhou [13] used the finite element method to analyse conduction error through thermocouple wires when the probe was in direct contact with the surface. They showed that when the heat flux along thermocouple wires insulation surface is small and the thermocouple is fine, the effect of insulation on thermocouple probe temperature is negligible. Moreover, Tszeng and Zhou [13] recommended using bare wire with small diameter rather than a larger diameter thermocouple with insulation. Woolley [14] confirmed that alumina oxide  $Al_2O_3$  insulation causes higher measurement error in comparison to glass braid insulation during temperature measurement at the interface between aluminium and sand during a metal casting process. These results have been demonstrated for different sizes of thermocouples and for very high temperature difference (~ 1500K).

Experimental results presented by Perera et al. [15] studying the effect of different fixing methods of thermocouple on an LED lens for surface temperature measurement. They indicated that using thermal adhesive tape or silicone elastomer has an identical effect on measurement. Furthermore, fixing the thermocouple junction with a spot weld gave better results than soldering or condenser-discharge welding (Boelter and Lockhart [1]). Shaukatullah and Claassen [11] showed that using silver epoxy or silver epoxy with insulating epoxy to fix the thermocouple to the surface gave a good contact and consequently lower error in temperature measurement. Moreover, attaching the thermocouple to the surface with polyimide or aluminium tapes produced higher errors due to poor contact. He, Smith and Xiong [3] mentioned that an increase of the epoxy drop diameter from 2.5mm to 7.5 led to reduced measurement error but this increased again for a diameter of 10mm. Moreover, the results confirmed that the thermocouple error can be minimized when using high thermal conductivity silver filled epoxy instead of classic epoxy of low conductivity.

Another approach was followed by Robertson and Sterbutzel [16] who used two thermocouples and a heater which were attached to a probe. The first thermocouple is in direct contact with the surface, measured the disturbed temperature and the second, away from the surface measured the temperature of the probe itself. Both thermocouple outputs

were fed into a power controller which supplied a heater current proportional to the temperature difference. Consequently, the heater reduced the temperature difference between the thermocouples. When both thermocouples are at the same temperature there is no heat flux along the thermocouple wires and the first thermocouple accurately records surface temperature.

In the present work heating and cooling impact of different stripped lengths of thermocouple electrical insulation and surface temperatures on the thermocouple reading was recorded. During the measurement process the thermocouple junction was in direct contact with surface without any fixing glue while the wires were extended vertically and exposed to free convection from the outside environment. Moreover, analytical and numerical analyses investigated in detail the effect of thermal contact resistance between the thermocouple probe and the surface on temperature variation within the probe. According to the best knowledge of the authors, these are the first experimental, analytical and numerical works investigating in detail the effect of thermocouple insulation on surface temperature measurement while the thermocouple is in a vertical position with no fixing glue.

## **2 Experimental techniques**

### **2.1 Experimental setup**

The temperature controlled surface consisted of a conventional peltier device with one side attached to a large heat sink and the other side exposed to the environment with a small PT100 thermometer adhered using high thermal conductivity glue as shown in Figure 1(a). Figure 1(b) shows the electrical connections made to the Peltier plate consisting of two power supplies, peltier plate, switch (to reverse the current) and PT100 signal conditioning circuit feeding into a voltmeter.

The temperature of the peltier plate was controlled by changing the current supplied (magnitude and current direction). The PT100 thermometer was connected to the voltmeter and a TC08 pico log data acquisition system to independently record the peltier surface temperature.

## 2.2 Measurement procedure

Each thermocouple was fixed to a Z-positioning micrometer stage and pressed down against the peltier surface until the thermocouple reading became steady-state. The two thermocouples used were type-K with bare wire diameters of 80 $\mu\text{m}$  (250 $\mu\text{m}$  including PFA insulation) and 200 $\mu\text{m}$  (500 $\mu\text{m}$  including PTFE insulation), see Table 1 and \*manufacturer lab facility

Table 2. The aim of this study is to investigate the effect of thermocouple impact on temperature measurement, therefore two specific sizes and types of thermocouple have been used. Different types or sizes of thermocouple may also be used but their effect will vary based on the size and properties of wires as well as their insulation.

The average environment temperature was recorded while the peltier surface temperature (as measured using the PT100) was set to 4 $^{\circ}\text{C}$ , 8 $^{\circ}\text{C}$ , 15 $^{\circ}\text{C}$ , 25 $^{\circ}\text{C}$ , and 35 $^{\circ}\text{C}$  as the surface temperature (measured by the thermocouple) was recorded. The insulation on the thermocouple wires were stripped off to various lengths from the tip, 5mm, 10mm, 15mm, and 20mm respectively to investigate its effect on the temperature measurement. Five runs were performed for each stripped length to confirm the reproducibility in each of the experiments.

During the experiments the power supplied to the peltier unit was controlled by changing the current, keeping the peltier surface at a constant temperature as recorded by the PT100. This signal was input into the TC08 pico data acquisition system connected to the computer to allow continuous recording of the readings. Throughout this time the thermocouple was held in direct contact with the peltier surface while its signal was also continually recorded using the TC08 data recorder.

### 2.2.1 Thermocouple calibration

A thermocouple calibration process was performed by comparing the thermocouple reading when fully submerged in crushed ice and boiling water with the standard water freezing and boiling temperature respectively [17]. Freezing or water boiling standard temperature was considered (to 2d.p.) as those at standard atmospheric conditions (e.g. 1 atm) where water boils at 99.98 $^{\circ}\text{C}$ <sup>1</sup> and freezes at 0 $^{\circ}\text{C}$  [18].

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<sup>1</sup>The boiling point of 99.98 $^{\circ}\text{C}$  was used in accordance with the strict two point calibration of Vienna Standard Mean Ocean Water (VSMOW) and as used elsewhere in the literature, see e.g. R. Tillner-Roth and D. G. Friend, J. Phys. Chem. Ref. Data, vol. 27, No. 1, 199.

The TC08 was connected to a laptop via a USB cable to record the readings. Consequently, It was recommended by Pico Technology Technical Support to unplug the laptop from its power supply to avoid incorrect earthing that may impact on the TC08 resulting in inaccurate measurement [19]. One advantage of the TC08 is that it incorporates cold junction compensation, eliminating measurement variations caused by drifts in environment temperature during the experiments [20].

### 2.2.2 Experimental results

Figure 2 (a-b) shows the result of different exposed wire lengths for both sizes of thermocouple, 80 $\mu$ m and 200 $\mu$ m. The vertical axis indicates the difference between the thermocouple measured temperature and the peltier (PT100 measured) surface temperature while the horizontal axis represents the peltier surface temperature. The maximum temperature drop is equal to 2 $^{\circ}$ C and 4 $^{\circ}$ C for 80 $\mu$ m and 200 $\mu$ m respectively with an environment temperature of 13 $^{\circ}$ C and the peltier surface temperature of 35 $^{\circ}$ C.

Experimental working conditions (atmospheric: 13 $^{\circ}$ C and 1 bar) are essentially constant for each of the thermocouple sizes and every exposed wire length. During the experiments and due to the temperature difference between peltier surface and (4 $^{\circ}$ C-35 $^{\circ}$ C) and the environment heat will be conducted along the thermocouple wires. Figure 2(a-b) shows the error in temperature measurement by the thermocouple versus actual peltier temperature. The plots in Figure 2(a-b) should have zero slopes if there is no temperature measurement error; however this is clearly not the case. It can also be seen that the 200 $\mu$ m thermocouple has a higher conduction error than 80 $\mu$ m for different stripped lengths as shown in Figure 2a and Figure 2b. The reason for this is that the larger diameter provides a larger heat transfer area and consequently the heat flux to or from the thermocouple is higher.

During the experiment the thermocouple probe was pressed against the peltier surface in order to increase the contact area with the surface and minimize the thermal contact error [21]. The probability of getting the same contact area in each experiment for the size 200 $\mu$ m is greater than 80 $\mu$ m because of the probe size. Therefore, it can be seen in Figure 2(a-b) that the experimental error of the 80 $\mu$ m is larger than 200 $\mu$ m (consequently the error bar is larger). Moreover, change in the peltier surface temperature leads to change in the air circulation around the thermocouple probe. Consequently, the combined effect of the air circulation (due to the varying surface temperature) and the effect of the probe contact area with surface cause a different experimental error resulting in a different error bar length, as shown in Figure 2(a-b).

Generally, we can conclude from these results that the effect of the stripped insulation of any length on the temperature measurement is negligible or, more accurately the experimental error for a typical system is higher than the effect of the stripped insulation[6], see Figure 2(a-b). Besides, the temperature difference between the environment and the working range of the peltier surface 4°C-35°C is not high enough to have a strong effect on the temperature measurement, see Figure 13.

### 3 Mathematical modelling

#### 3.1 Thermocouple wire length

It can be assumed that each strand of thermocouple wire behaves as a one-dimensional very long fin exposed to free convection from the outside environment, see Figure. In this case, the following analysis can be used to provide an analytical prediction of the wire length over which the temperature becomes equal to that of the environment. Fin analysis considers heat transfer by conduction occurring along the thermocouple wire due to its high thermal conductivity in comparison with the surrounding insulation, as such it assumes that there is no radial temperature gradient across the metal wire [6], see Table 1.

We must consider that a thermocouple consists of contact between two dissimilar metals. Both wires have different physical and thermal properties and therefore should be considered to each have a different effect on the thermocouple junction temperature. To simplify this, a single equivalent wire model was adopted instead of the two-wire model see Figure 4[9, 22].The equivalent bare wire diameter is calculated from

$$D_{weq} = \sqrt{2}D_w \quad (1)$$

The equivalent insulation outer diameter is calculated by considering the average thickness around each wire, see Figure 4[22], which becomes

$$t_{eqins} = \frac{1}{2} \left[ \frac{2D_{ins} - 2D_w}{3} + \frac{D_{ins} - D_w}{2} \right] \quad (2)$$

The values of  $D_{ins}$  and  $D_w$  are listed in Table 2.

Equivalent thermal conductivity  $k_{weq}$  for a single wire model is calculated from [6, 23]

$$k_{weq} = \frac{k_{w1} + k_{w2}}{2} \quad (3)$$

$k_{w1}$  and  $k_{w2}$  are given in Table 1.

Heat lost by convection is calculated using Eq. (4)[21]:

$$dq_{conv} = \frac{2\pi(T-T_{inf})dx}{\frac{\ln(\frac{D_{eqins}}{D_{weq}})}{k_{ins}} + \frac{2}{D_{eqins}h}} \quad (4)$$

Applying Fourier's law through the elemental area:

$$q_x = -k_{weq}A_{weq} \frac{dT}{dx} \quad (5)$$

$$\frac{d^2T}{dx^2} - m^2(T - T_{inf}) = 0 \quad (6)$$

where  $m$  is calculated from

$$m = \sqrt{\frac{4}{k_{weq}D_w^2 \frac{\ln(\frac{D_{eqins}}{D_{weq}})}{k_{ins}} + \frac{2}{D_{eqins}h}}} \quad (7)$$

Applying the following boundary conditions for a very long fin with outside boundary conditions as shown in Figure 3(b),

$$T=T_0 \text{ at } x=0, \text{ and } T=T_{inf} \text{ when } x \rightarrow \infty \quad (8)$$

where,  $T_0$  is the fin's base temperature.

The temperature variation along the fin is calculated from

$$T=T_{inf} + (T_0-T_{inf})e^{-mx} \quad (9)$$

The best approximation considers the thermocouple wire as a vertical thin cylinder so the Nusselt number  $Nu_H$  can be calculated from [24]:

$$Nu_H = \frac{4}{3} \left[ \frac{7Ra_H Pr_a}{5(20+21Pr_a)} \right]^{1/4} + \frac{4(272+315Pr_a)H}{35(64+63Pr_a)D} \quad (10)$$

where  $Nu_H = hH/k_a$ ; Rayleigh number,  $Ra_H = g\beta_a \Delta TH^3 / \alpha_a \nu_a$ ; Prandtl number,  $Pr_a = \nu_a / \alpha_a$ , see Table 3 for air properties.

The heat transfer coefficient ( $h$ ) is then derived as

$$h = \frac{4k_a}{3H} \left[ \frac{7Ra_H Pr_a}{5(20+21Pr_a)} \right]^{1/4} + \frac{4(272+315Pr_a)k_a}{35(64+63Pr_a)D} \quad (11)$$

The maximum temperature difference between the environment and surface in our experiments was observed equal to ( $\Delta T=22^{\circ}\text{C}$ ), see section 2.2. Substituting this temperature difference, with assuming a value  $H$  into Eq.(11) to calculate  $h$ . Substituting these values into Eqs.(7)-(9) to calculate a new value of  $H$ . Repeat calculation above until  $H$  reaches a constant value which represents the calculated length of the thermocouple wire where it reaches the environment temperature is shown in Figure 5.

Figure 5(a-b) shows the required length for the wire end temperature to equal to that of the environment for each diameter of thermocouple with and without insulation. The increase in the diameter of the thermocouple leads to an increase in the area that is exposed to convective heat transfer with the outside environment. Moreover, a larger wire diameter means an increase in the cross-sectional area which allows more heat to be conducted through the wires. Consequently, a longer length is required for the 200 $\mu\text{m}$  diameter to reach the environment temperature in comparison to the 80  $\mu\text{m}$  diameter wires. It is also shown that there is no effect from the length of electrical insulation for these two thermocouples.

### 3.2 Thermocouple insulation effect

Heat transfer to a cylindrical shape with surrounding insulation depends on the ratio between the insulation thermal conductivity and the heat transfer coefficient with the outside environment [21]. This ratio is called the critical radius ( $r_{cr}$ ) and is defined as

$$r_{cr} = \frac{k_{ins}}{h} \quad (12)$$

The following two cases explain the effect of  $h$  on the heat transfer with the cylinder:

Case 1: Assuming constant  $h$  as calculated from Eq.(11) and solving Eq.(4), we can generate the results shown in Figure 6(a) where the heat flux continuously increases until the wire diameter becomes equal to the critical radius, at which point the heat flux starts to decrease. \*manufacturer lab facility

Table 2 shows the critical radius for each of the thermocouple sizes.

The increase in insulation thickness enlarges the outer surface area which in turn increases the heat flux rate continuously until the diameter becomes equal to the critical radius. This can be seen in Figure 6(b) where the heat flux starts to decrease beyond a given diameter [21]. This behaviour of heat flux is understood by the fact that the thermal resistance to convection heat transfer is minimum when the cylinder outer diameter is equal to  $r_{cr}$  as shown in Figure 7.

However, due to the small size of the thermocouple wire, the effect of wire curvature on the convection heat coefficient  $h$  around the wires should be considered [24]. This case has been discussed in detail in Case 2 below.

Case 2: The same calculation procedure as case 1 is repeated however this time we consider the effect of wire curvature on  $h$  [24]. The variation of  $h$  with the wire diameter can be calculated using Eq.(11). In this case there is no minimum value of thermal resistance as shown in Figure 7. Therefore, we see a constant increase in heat flux with no apparent critical radius as shown in Figure 6(b).

### 3.3 Thermocouple probe temperature distribution

With these insights, we undertook a more detailed investigation of the effect of electrical insulation on temperature as measured by the thermocouple probe.

#### 3.3.1 Stepped fin analysis

Thermocouple probe is the effective part which measures the surface temperature. Thermocouple wires interact with environment and transfer heat to (or from) the surface and alert the measured temperature [25]. Therefore, the probe measures the disturbed temperature not the actual surface temperature. Consequently, the temperature distribution along the thermocouple junction and wire can be analyzed by considering them as a stepped fin [26, 27], see Figure 8. The thermocouple wires represented by the single wire model (see section 3.1) were considered to be a very long fin while the thermocouple junction was considered to be a fin with prescribed end temperature [21]. Therefore, the junction temperature distribution can be calculated from Eq.(13) for the length  $0 \leq x \leq D_p$ :

$$\frac{\theta_p}{\theta_b} = \frac{\left(\frac{\theta_L}{\theta_b}\right) \sinh m_p + \sinh m_p (D_p - x)}{\sinh m_p D_p} \quad (13)$$

where  $m_p = \sqrt{h_p p_p k_p A_p}$ ,  $k_p = k_{weq}$  and is calculated from Eq.(3).

Thermocouple wire temperature can be calculated from Eq.(9) for the length ( $D_p \leq x \leq L_w$ ):

$$\frac{\theta_w}{\theta_L} = e^{-m_w(x-D_p)} \quad (14)$$

where  $m_w$  is calculated from Eq.(7). At  $x=D_p$ :  $T_L=T_w=T_p$  joint between the probe and the wires the following boundary condition is applied [26, 27]:

$$-A_p k_p \frac{dT_p}{dx} = -A_{weq} k_{weq} \frac{dT_w}{dx} + h(A_p - A_{weq})(T_L - T_{inf}) \quad (15)$$

Substituting Eqs.(13)and(14) into Eq.(15)

$$\theta_L = \frac{\frac{m_p}{\sinh m_p D_p}}{\frac{m_p \cosh m_p D_p}{\sinh m_p D_p} + \alpha m_w + \frac{(1-\alpha)h_p}{k}} \theta_b \quad (16)$$

where,  $\alpha = A_{weq}/A_p$ .

### 3.3.2 Electrical resistance analogy

An electrical resistance analogy of the thermocouple thermal interaction with the environment and surface is shown in Figure 9. The thermocouple measures the average temperature of  $T_L$  and  $T_b$  which represent top and bottom temperatures of the junction, see Figure 8. Figure 8 can be redrawn as electrical resistance as shown in Figure 9. If we assume  $R_m$ , is very small therefore and can be neglected[25]:

$$q = \frac{T_s - T_b}{R_c} = \frac{T_b - T_L}{R_p} = \frac{T_L - T_{inf}}{R_w} \quad (17)$$

Heat flux transfers through all resistance is equal to the heat flux transferred to the probe plus and the wires. Therefore,

$$q = q_p + q_w \quad (18)$$

Applying fin analysis for both the thermocouple wire and junction heat fluxes are calculated using Eqs.(13)and (14)

$$q_p = \frac{\cosh m_p - \frac{\theta_L}{\theta_b}}{\sinh m_p D_p} \sqrt{h_p p_p k_p A_p} \theta_b \quad (19)$$

$$q_w = k_{weq} m_w \frac{\pi}{4} D_{weq}^2 \theta_L \quad (20)$$

Substitute Eqs.(19) and (20) into Eqs.(18)and (17) then

$$\theta_b = \frac{\theta_s - R_c \theta_L (Z_w - Z_p)}{1 + Z_p R_c \cosh m_p D_p} \quad (21)$$

Substitute Eq.(21)into Eq.(16)then

$$\theta_L = \frac{\theta_s A_1}{1 + R_c Z_p \cosh m_p D_p + A_1 R_c (Z_w - Z_p)} \quad (22)$$

where

$$Z_w = k_w m_w \frac{\pi}{4} D_{weq}^2 \quad (23)$$

$$Z_p = \sqrt{h_p p_p k_p A_p / \sinh m_p D_p} \quad (24)$$

$$A_1 = \frac{\frac{m_p \cosh m_p (D_p - D_p)}{\sinh m_p D_p}}{\frac{m_p \cosh m_p D_p}{\sinh m_p D_p} + \alpha m_w + \frac{(1-\alpha) h_p}{k}} \quad (25)$$

$\theta_L$  was calculated from Eq.(22) and the results substituted into Eq.(21) to compute  $\theta_b$  while  $\theta_s$  is taken to be the value measured using the PT100 device, see section 2.

Thermocouple tip size on the 80 $\mu$ m and 200 $\mu$ m diameter thermocouples were measured using a microscope and were found to be 449 $\mu$ m and 635 $\mu$ m respectively. These values have been used with values of  $R_c$  were 0.000025, 0.00035, and 0.000045[m<sup>2</sup> K/W] for the size 80 $\mu$ m and between 0.000045, 0.0005, and 0.000055[m<sup>2</sup> K/W] for the size 200 $\mu$ m have been substituted into Eqs.(22) and (21) to calculate the results of the Figure 10 (a-b).

The experimental data of a stripped insulation length of 0mm and 20mm were chosen for comparison with analytical results. Figure 11 shows good agreement with the experimental results within 0.5°C for the values of  $R_c$  0.00035 [m<sup>2</sup> K/W] for the size 80 $\mu$ m and 0.000055[m<sup>2</sup> K/W] for the size 200 $\mu$ m. Furthermore, analytical results show that the effect of the insulation is negligible for totally insulated and uninsulated wire of length 20mm for zero thermal contact resistance  $R_c = 0$ . Consequently, the other stripped lengths (5mm, 10mm, and 15mm) should be already having negligible effect. The analysis presented above is one-dimensional and assumes the contact area between the probe and the surface is equal to the probe diameter. Thermal contact resistance depends on the shared area between the probe and the surface and in reality the probe geometry is irregular making it too complex to specify an actual contact area. Therefore, it is difficult to specify the actual value of  $R_c$ .

#### 4 Numerical modelling

A three dimensional model of the actual geometry of a thermocouple was created as shown in Figure 12. This model considered the actual size of the thermocouple wires and insulation but the geometry of the junction was represented as a cube with side length equal to the junction diameter. Consequently, the contact area is the bottom surface of this cube. In order to model

thermal contact resistance in COMSOL we need to define a thermal joint conductivity  $h_j$  [28]. From Fourier's law Eq.(5)

$$q = h_j \Delta T \left[ \frac{W}{m^2} \right] \quad (26)$$

where

$$h_j = \frac{k_{res}}{d_{res}} \left[ \frac{W}{m^2 K} \right] \quad (27)$$

where  $k_{res}$  and  $d_{res}$  represent thermal conductivity of the contact layer between the surface and the junction and its thickness respectively. It is seen that  $h_j$  is the reverse of  $R_c$ . COMSOL applies a *slit boundary condition*[28] to include the effect contact resistance:

$$\begin{aligned} -\mathbf{n} \cdot (-k_d \nabla T_d) &= -\frac{k_{res}}{d_{res}} (T_u - T_d) \\ -\mathbf{n} \cdot (-k_u \nabla T_u) &= -\frac{k_{res}}{d_{res}} (T_d - T_u) \end{aligned} \quad (28)$$

where subscript  $d$  and  $u$  refer to downside and upside of the slit.

boundary conditions of Eq.(28) can be defined in COMSOL by creating a *contact pair* between the thermocouple junction and the peltier surface. Moreover, the ratio  $k_{res}/d_{res}$  is equivalent to *thermal joint conductivity*  $h_j$  [28] which is equal to the inverse of  $R_c$ . The values of  $R_c$  0.000035[m<sup>2</sup> K/W] for 80μm size and 0.000055[m<sup>2</sup> K/W] for 200μm were selected from analytical analysis to substitute for  $h_j$  in the numerical analysis. These values were chosen as they gave good agreement with experimental results to within 0.5°C, see section 3.3.2. Using this model we investigated the effects of the two different exposed wire lengths: 0 mm and 20 mm on the junction temperature. Figure 10 (a-b) shows a comparison between experimental and numerical results for the 80μm and 200μm thermocouples. There is good agreement between the experimental and numerical analysis for size 80μm as shown in Figure 11.

Figure 11 shows the maximum divergence of about 7% while the highest temperature difference is 0.49°C in comparison with the experimental results. While the size 200μm deviates from the experimental results by 3.5% with about 1.1°C temperature difference when the surface temperature is higher than that of the environment. Furthermore, numerical results showed that the effect of insulation is negligible for totally insulated and uninsulated wire of length 20mm for zero thermal contact resistance  $R_c = 0$ .

In the case of one-dimensional analysis the contact area was assumed to be equal to the probe(s) diameter, see section 3.3 and Figure 8. In the three-dimensional numerical model a squared shape contact area with side equal to the probe diameter was used Figure 12. Both

analyses assumed that the contact area was larger than actual value seen in experiments due to the spherical geometry of the actual probe. Moreover, surface roughness leads to increased thermal resistance in experiments, an effect that was not considered in the analyses above. Accordingly, the analytical and numerical analyses underestimate thermal resistance and consequently the calculated temperature drop is less than the true value.

Furthermore, the percentage deviation of theory from experiment for both the numerical and analytical results is dissimilar as shown in Figure 11. The analytical analysis is a one-dimensional approach where both thermocouple wires are assumed to act as a single equivalent wire, see section 3.1. Consequently, the effect of ambient temperature is considered on a single wire with an equivalent diameter and thermal conductivity, this differs from the actual thermocouple wire properties and size, see Table 1 and Table 2. Whereas, in the numerical analysis the model is three-dimensional and the actual size and properties were used, see Figure 12.

Experimental working conditions were approximately the same for both sizes of thermocouples. By comparing experimental, analytical and numerical results we can conclude that the thermal contact resistance of the thermocouple is in the order of  $1 \times 10^{-5}$  [ $\text{m}^2 \text{ K/W}$ ] and  $1 \times 10^{-4}$  [ $\text{m}^2 \text{ K/W}$ ] for 80 $\mu\text{m}$  and 200 $\mu\text{m}$  respectively.

Figure 13 shows a calculation of the effect of the electrical insulation when the peltier temperature is above the range considered in our experiments. The insulation has a negligible effect for the 80 $\mu\text{m}$  thermocouple even when surface temperature reaches 800[ $^{\circ}\text{C}$ ]. However, there is a noticeable effect of insulation for the 200 $\mu\text{m}$  thermocouple when the temperature reaches 250[ $^{\circ}\text{C}$ ]. Therefore, for larger thermocouples the effect of insulation should be taken into consideration at elevated surface temperatures.

## 5 Conclusion

Analytical, numerical and experimental analyses have been performed to investigate the effects of different insulation lengths on thermocouple measurements of surface temperature. During the experimental work the thermocouple probe was in direct contact with the surface while the wires were exposed to natural convection from the outside environment. Two sizes of thermocouple (80 $\mu\text{m}$  and 200 $\mu\text{m}$ ) were used to measure a surface temperature. A satisfactory agreement was found between experimental, analytical, and numerical results within the range of surface temperatures measured (4 $^{\circ}\text{C}$ -35 $^{\circ}\text{C}$ ) and an average environment temperature 13 $^{\circ}\text{C}$ . From this the following can be concluded:

1. Stripping different lengths (0mm, 5mm, 10mm, 15, and 20mm) of insulation has a negligible effect on the heat transfer along the thermocouple wire and consequently on surface temperature measurement.
2. Both sizes of thermocouples considered, 80 $\mu$ m and 200 $\mu$ m have different insulation thickness, however, stripping different lengths has no impact on either measurement. Therefore, the effect of the stripped insulation is independent to the thermocouple size within the temperature measured range (4°C-35°C).
3. The effect of stripped insulation on the thermocouples with a wire diameter of 200 $\mu$ m becomes relevant when the peltier surface temperature reaches 250°C while for 80 $\mu$ m diameter wires insulation has negligible effect even for surfaces above 800°C.
4. Regardless of the stripped length of insulation, a larger diameter of thermocouple wire has a larger impact on surface temperature measurement than a smaller thermocouple.
5. The effect of the wire's curvature on heat transfer has been considered due to the small size of the wire. The impact of this curvature means that there is no specific critical diameter of the thermocouple wire(s) over which heat transfer to the wires decreases, see Figure 6 and Eq.(12).
6. If the experimental error in temperature measurement is higher than the impact of using the thermocouple (with or without insulation) the error is negligible. Therefore, any stripped length of electrical insulation can be said to have no impact on measurement accuracy.
7. The effect of the electrical insulation can be neglected until the surface temperature reaches 800°C for 80 $\mu$ m thermocouples while for the 200 $\mu$ m thermocouples the effect of the insulation must be considered when the surface temperature reaches 250°C.

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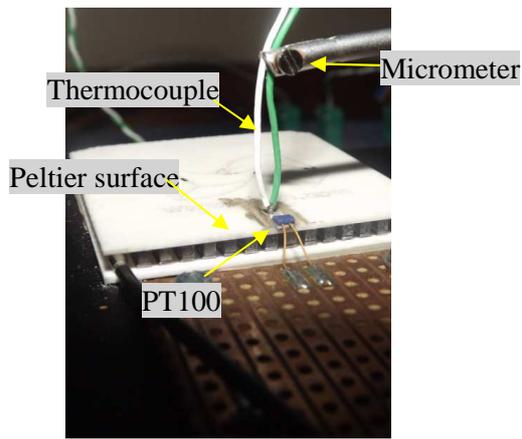
### **References**

- [1] L. M. K. Boelter and R. W. Lockhart, NACA Technical Note 2427, An Investigation of Aircraft Heaters: XXXV-Thermocouple Conduction Error Observed in Measuring Surface Temperatures, July 1951, University of California, Washinton.
- [2] M. Tarnopolsky and I. Seginer, Agricultural and Forest Meteorology, Leaf Temperature Error from Heat Conduction Along Thermocouple Wires, March 1999, vol. 93, pp. 185-194, DOI: 10.1016/S0168-1923(98)00123-3.

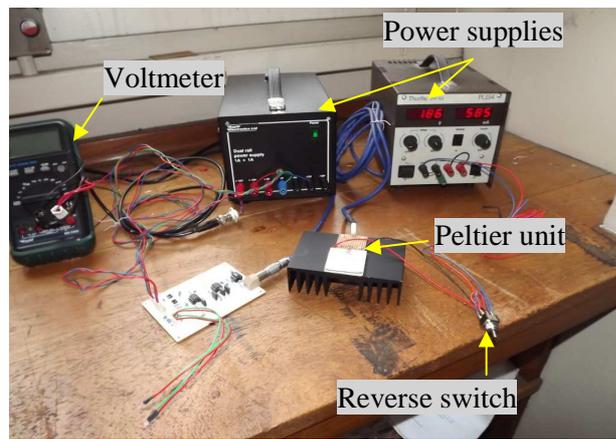
- [3] Q. He, S. Smith and G. Xiong, Semiconductor Thermal Measurement and Management Symposium (SEMI-THERM) - 27th Annual IEEE, Thermocouple Attachment Using Epoxy in Electronic System Thermal Measurements — A numerical Experiment, 20-24 March 2011, San Jose, CA, DOI: 10.1109/STHERM.2011.5767212.
- [4] W. D. Brewer, NASA Technical Note D-3812, Effect of Thermocouple Wire Size and Configuration on Internal Temperature Measurements in a Charring Ablator, March 1967, Langley Research Center, Langley Station, Hampton, Va, Washington, D.C.
- [5] M.B. Dow, NASA Technical Note D-2165, Comparison of Measurements of Internal Temperatures in Ablation Material by Various Thermocouple Configurations, November 1964, Washington, D.C.
- [6] B.S. Singh and A. Dybbs, Journal of Heat Transfer, Error in Temperature Measurements Due to Conduction Along the Sensor Leads, August 1976, vol. 98, no. 3, pp. 491-495, DOI: 10.1115/1.3450581.
- [7] D. LI and M.A. Wells, Metallurgical and Materials Transactions B, Effect of Subsurface Thermocouple Installation on the Discrepancy of the Measured Thermal History and Predicted Surface Heat Flux during a Quench Operation, June 2005, vol. 36B, no. 3, DOI: 10.1007/s11663-005-0064-6.
- [8] S. J. Bartkus, R. A. Dulinskas, R. Skema and V. Lapo, Heat Transfer-Soviet Research, Errors of Temperature Measurement within a Body with Internal Heat Sources, May-June 1987, vol. 19, no. 3.
- [9] M. H. Attia, A. Cameron and L. Kops, Journal of Manufacturing Science and Engineering, Distortion in Thermal Field Around Inserted Thermocouples in Experimental Interfacial Studies, Part 4: End Effect, February 2002, vol. 124 pp. 135-145 DOI: 10.1115/1.1419199.
- [10] C. T. Kidd, ISA Transactions, Thin-Skin Technique Heat-Transfer Measurement Errors Due to Heat Conduction into Thermocouple Wires, 1985 vol. 24, no. 2, pp. 1-9.

- [11] H. Shaukatullah and A. Claassen, Semiconductor Thermal Measurement and Management Symposium - Ninteenth Annual IEEE, Effect of Thermocouple Wire Size and Attachment Method on Measurement of Thermal Characteristics of Electronic Packages 2003, pp. 97-105, DOI: 10.1109/STHERM.2003.1194345.
- [12] W.A. Mohun, Canadian Journal of Research, Precision of Heat Transfer Measurements With thermocouples-Insulation Error, 1948, vol. 26, no. Sec. F, pp. 565-583.
- [13] T. C. Tszeng and G. F. Zhou, Journal of Heat Transfer, A Dual-Scale Computational Method for Correcting Surface Temperature Measurement Errors, August 2004, vol. 126, DOI: 10.1115/1.1773585.
- [14] J. W. Woolley and K. A. Woodbury, Heat Transfer Engineering, Thermocouple Data in the Inverse Heat Conduction Problem, October 2011, vol. 32, no. 9, DOI: 10.1080/01457632.2011.525468.
- [15] I. U. Perera, N. Narendran and Y. W. Liu, SPIE Proceedings - LED-based Illumination Systems, Accurate Measurement of LED Lens Surface Temperature, September 2013, San Diego, CA, DOI: 10.1117/12.2023091.
- [16] D. Roertson and G. Sterbutzel, IEEE Transaction on Industry and General Applications, An Accurate Surface Temperature Measuring System, January / February 1970, vol. IGA-6, no. 1, DOI: 10.1109/TIGA.1970.4181127.
- [17] American Society for Testing Materials, Manual on the Use of Thermocouples in Temperature Measurement, 1993, 4th ed.
- [18] D. R. Lide, CRC Handbook of Chemistry and Physics, Internet Version 2005, <<http://www.hbcpnetbase.com>>, CRC Press, Boca, Raton, FL, 2005,
- [19] Pico Technical Support, Noise rejection / Request Ticket Number: TS00065500, 13/10/2014 2014,
- [20] Pico Technology Limited, TC-08 User's Guide Thermocouple Logger, 2005-7.
- [21] J.P. Holman, Heat transfer, 1987, 6th ed., McGraw-Hill.

- [22] R.J. Moffat, Proceedings of the 5th annual ISA test measurement symposium, Temperature Measurement in Solids, October 28-31 1968, No. 68-514, New York.
- [23] M. H. Attia and L. Kops, International Journal of Advanced Manufacturing Technology, Thermometric Design Considerations for Temperature Monitoring in Machine Tools and CMM Structures, 1993, vol. 8, pp. 311-319.
- [24] Aderian Bejan, Convection Heat Transfer, 1995, 2nd ed., John Wiley & Sons, Incorporated.
- [25] L. Thierry, S. Toullier, D. Teyssieux and D. Briand, Journal of Heat Transfer, Thermal Contact Calibration between a Thermocouple Probe and a Microplate, September 2008, vol. 130, no. 9, pp. 091601-091601-7, DOI: 10.1115/1.2943306.
- [26] B. Kundu and A. Aziz, Journal of Heat Transfer, Performance of a Convectively Heated Rectangular Fin With a Step Change in Cross-Sectional Area and Losing Heat by Simultaneous Convection and Radiation (Step Fins Under Radiation Environment), October 2010, vol. 132, no. 10, pp. 104502-104502-6, DOI: 10.1115/1.4001928.
- [27] C. Arslanturk, Journal of Thermal Science and Technology, Optimization of Straight Fins with A step Change in Thickness and Variable Thermal Conductivity by Homotopy Perturbation Method, 2010, vol. 30, no. 2, pp. 09-19.
- [28] Comsol AB, Heat Transfer Model Library, 2008, Comsol Multiphysics.
- [29] K.M.B. Jansen, International Journal of Heat and Mass Transfer, Heat Transfer in Injection Moulding Systems with Insulation Layers and Heating Elements, January 1995, vol. 38, no. 2, pp. 309-316, DOI: 10.1016/0017-9310(95)90021-7.
- [30] M. D. Lechner, Springer Handbook of Condensed Matter and Materials Data, Polymers, 2005, Springer Berlin Heidelberg, Part3, pp. 477-522.

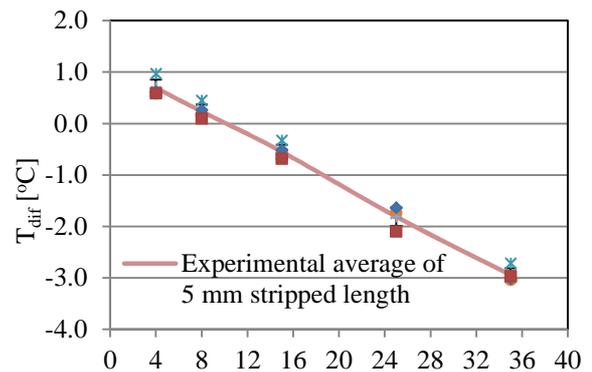
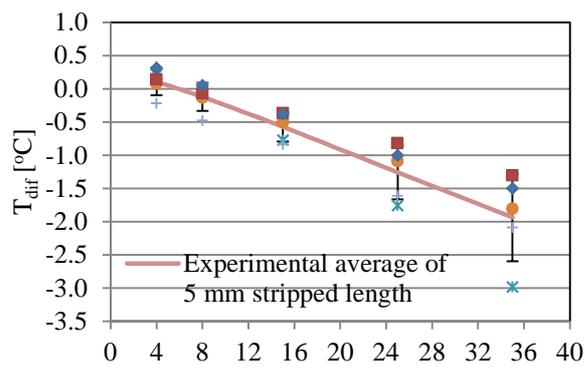
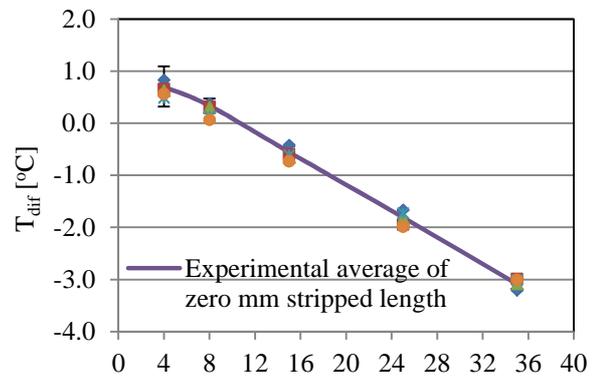
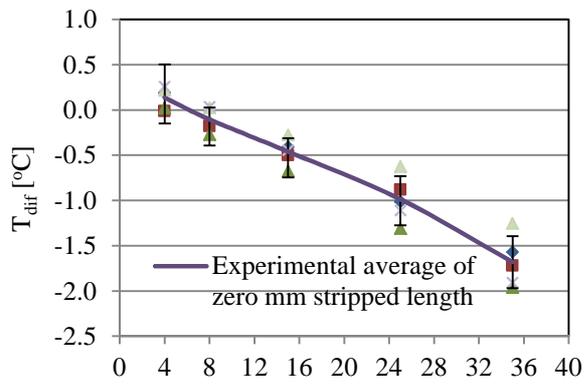


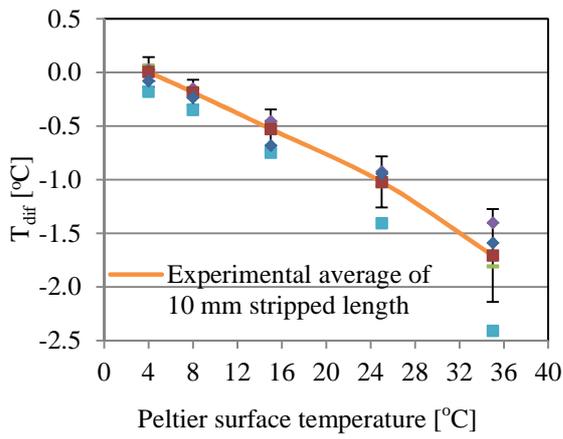
(a)



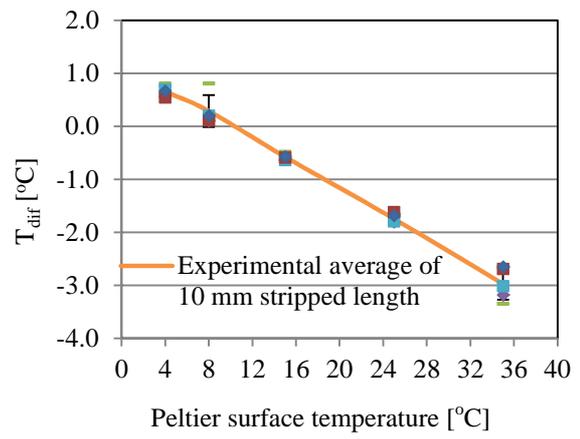
(b)

Figure 1(a)peltier unit, (b) peltier unit circuit.



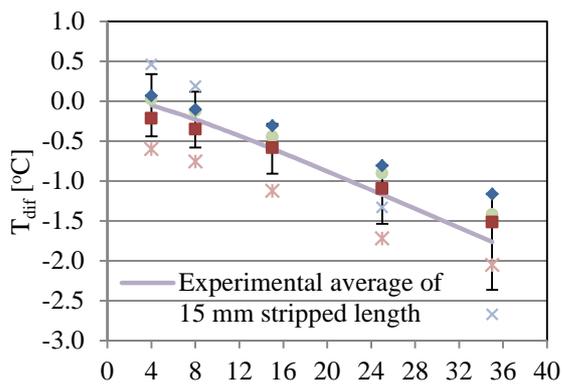


(a) 80µm

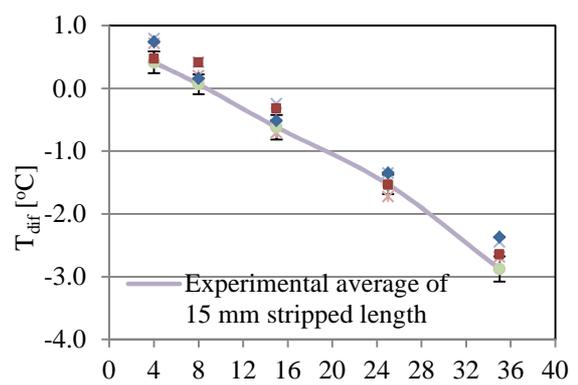


(b) 200 µm

Figure 2 Effect of variable distance of stripped electrical insulation of thermocouple on temperature measurement. The coloured markers represent different, repeat experimental runs.



(a) 80µm



(b) 200 µm

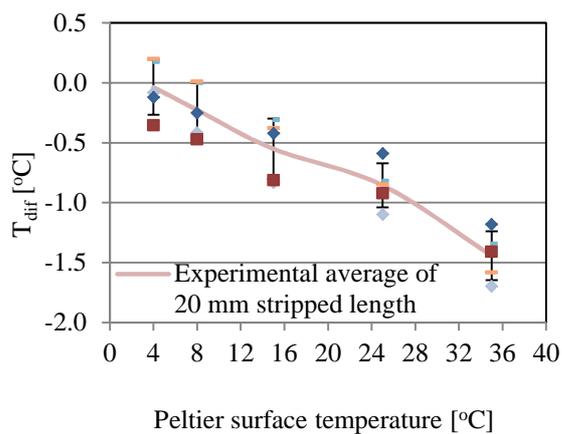
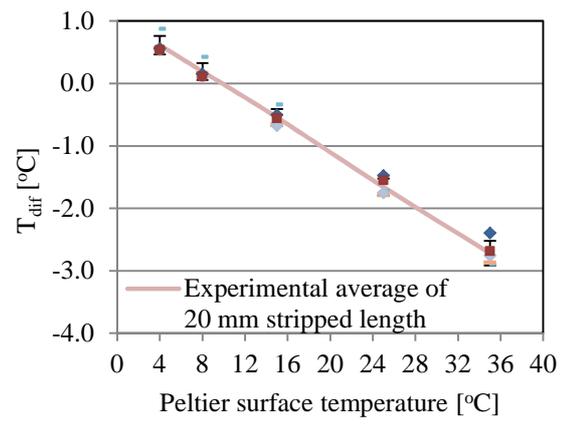


Figure 2 (continued)



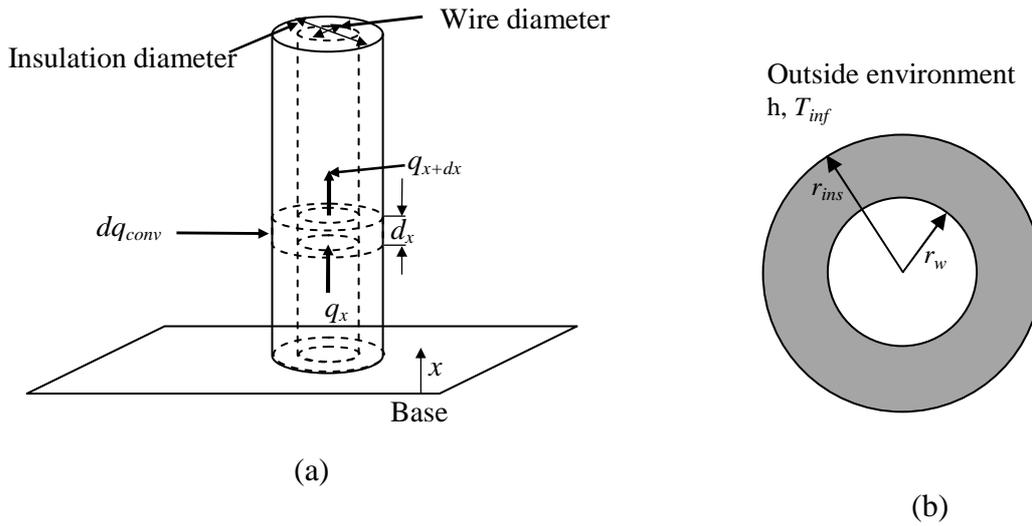


Figure 3(a) One-dimensional conduction and convection through fin with insulation, (b) Cylindrical cross-section of with insulation.

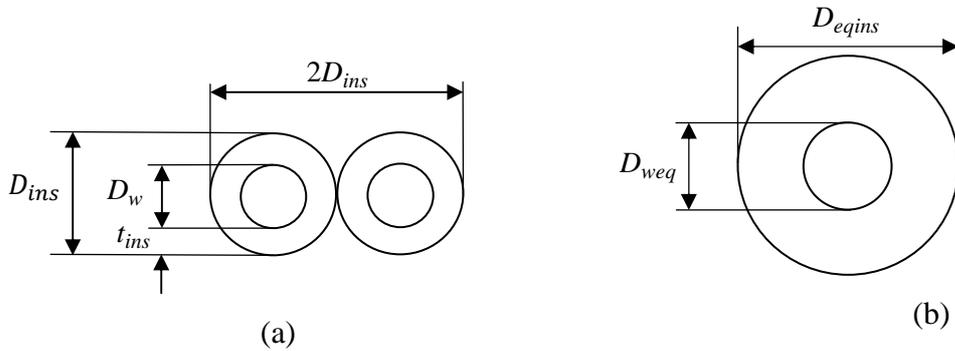


Figure 4 Cross sectional area of (a) two wires and (b) one equivalent wire.

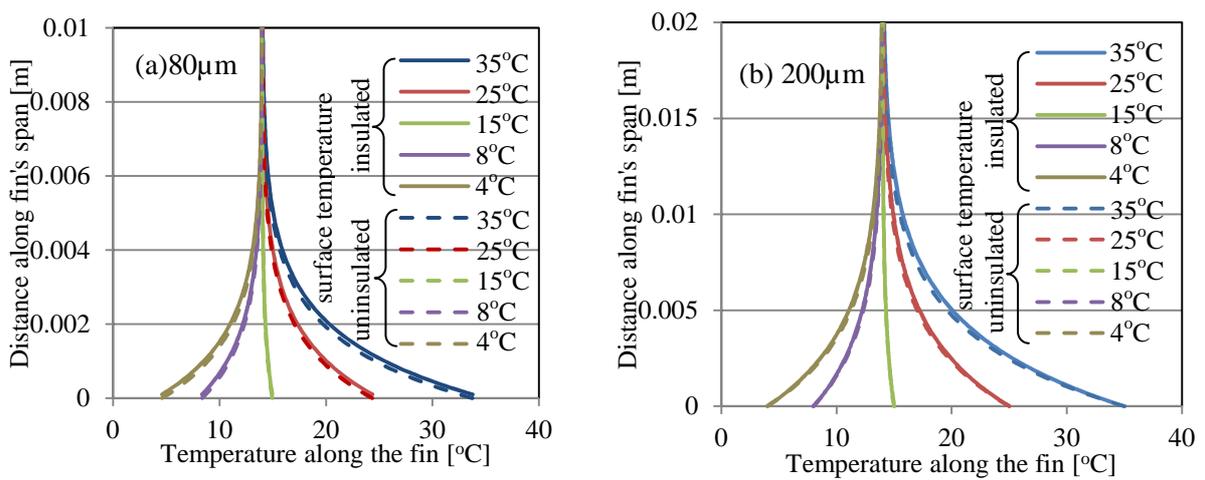


Figure 5 Fin's length required to reach the environment temperature where each curve starts with the peltier surface temperature with and without insulation.

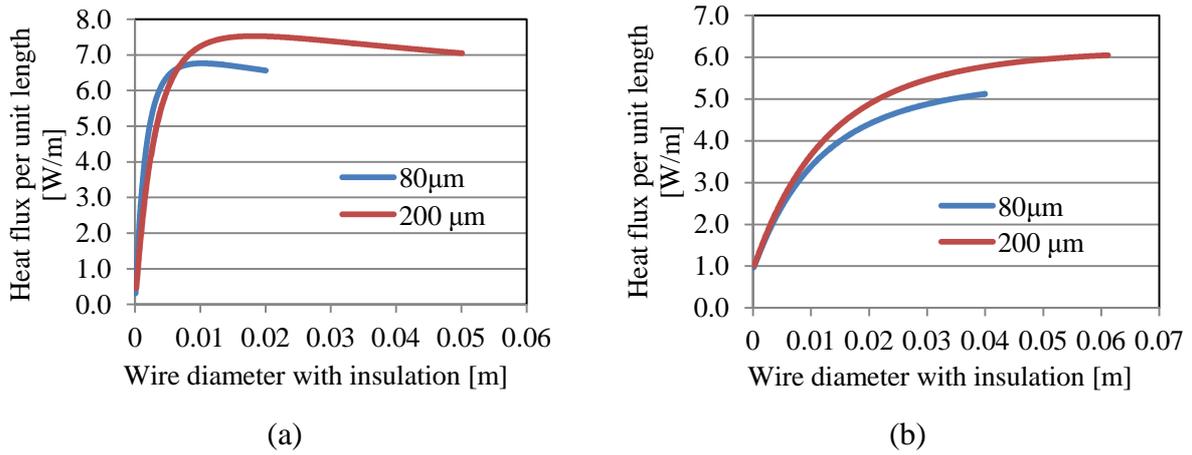


Figure 6 Variation of heat flux with wire diameter (a) constant  $h$ , (b) variable  $h$ .

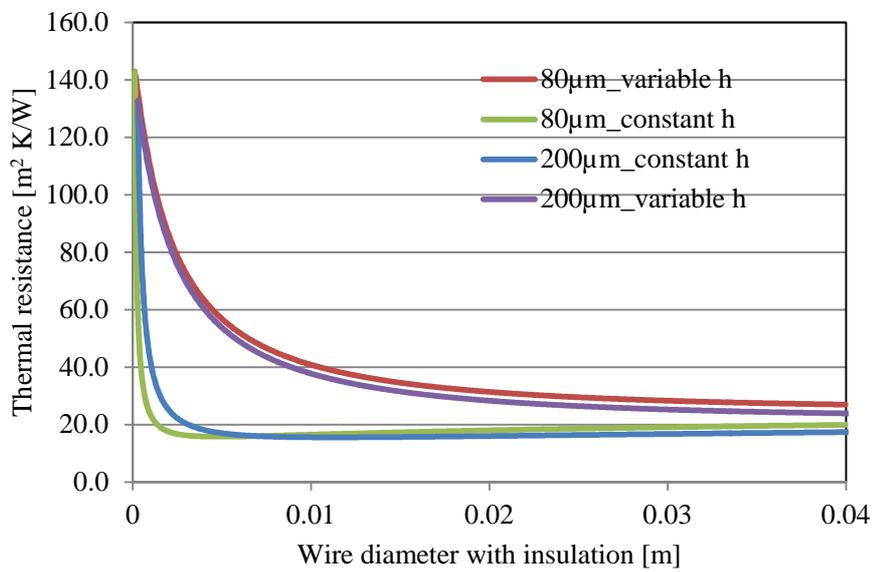


Figure 7 Variation of thermal resistance of the thermocouple wires with insulation of size  $80\mu\text{m}$  with constant and variable  $h$ .

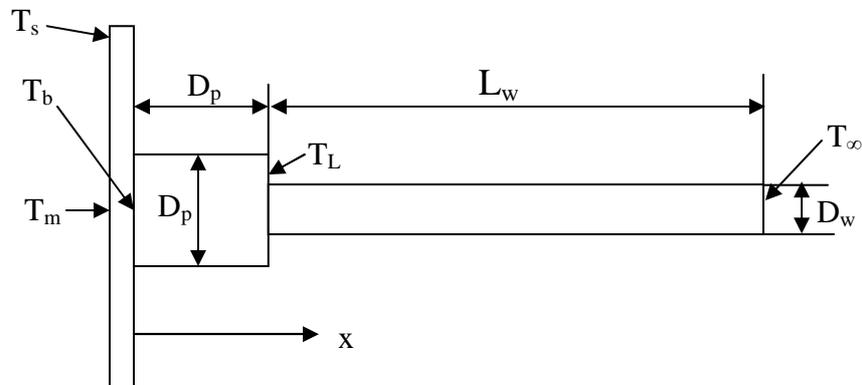


Figure 8 Analogy of thermocouple with probe geometry as a stepped fin.

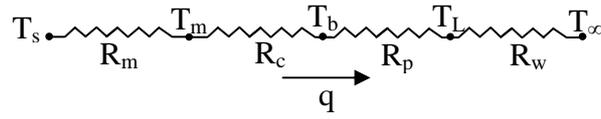
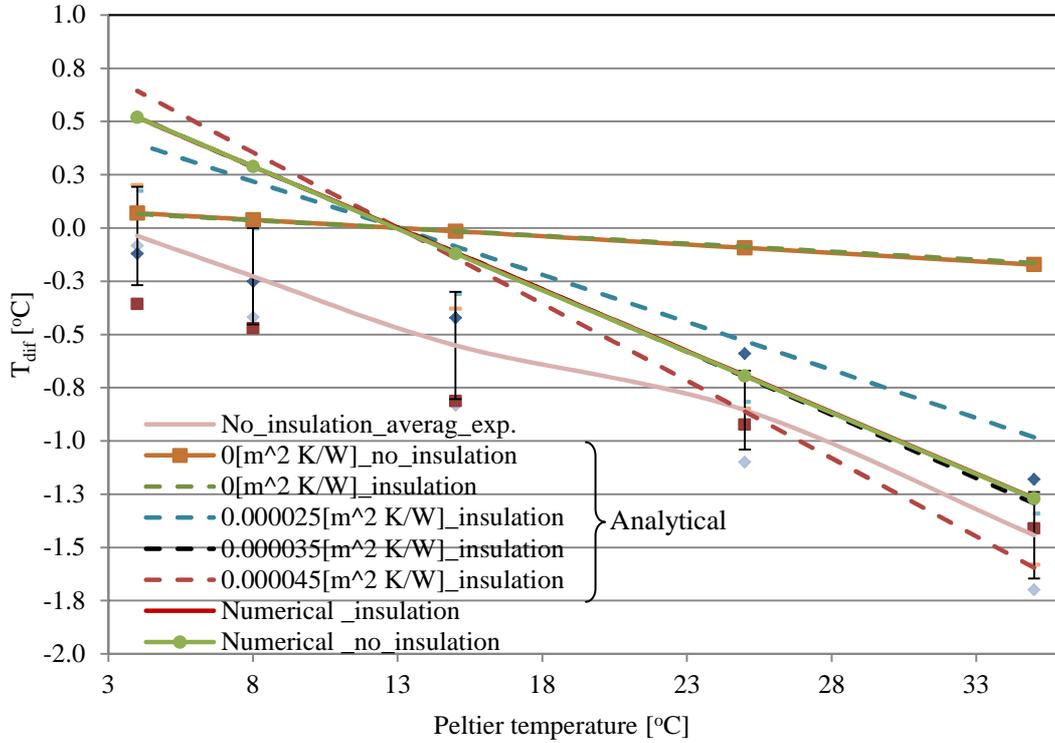
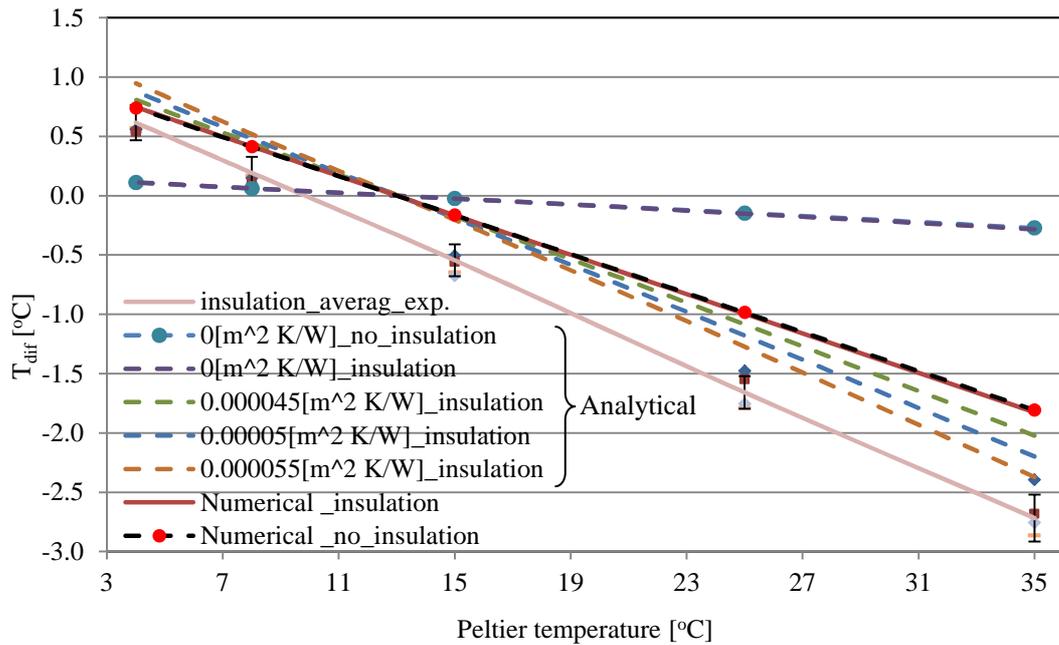


Figure 9 Electrical resistance analogy of thermocouple thermal resistance.



(a) 80µm



(b) 200µm

Figure 10 Comparison between experimental, analytical and numerical results for different values of thermal contact resistance between probe of and peltier surface for thermocouple sizes: (a) 80μm and (b) 200μm. 20mm stripped insulation of the experimental results is chosen for comparison.

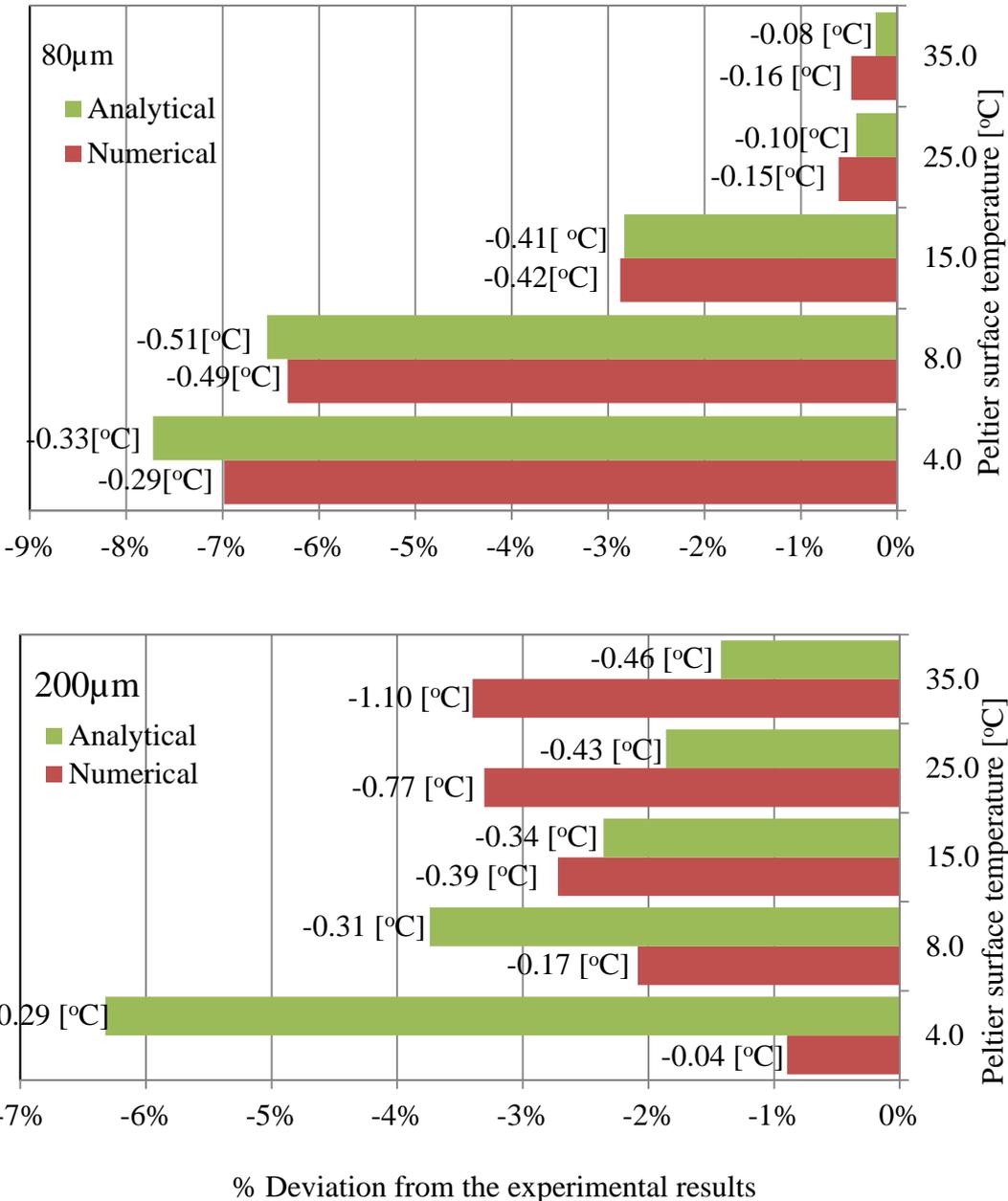


Figure 11 Comparison between the experimental, analytical and numerical results of the values of  $R_c$  0.00035 [m<sup>2</sup>K/W] for the size 80μm and 0.000055[m<sup>2</sup>K/W] for the size 200μm. Bars lengths and labels represent percentage deviation and temperature difference from experimental results respectively. 20mm stripped insulation is chosen for comparison.

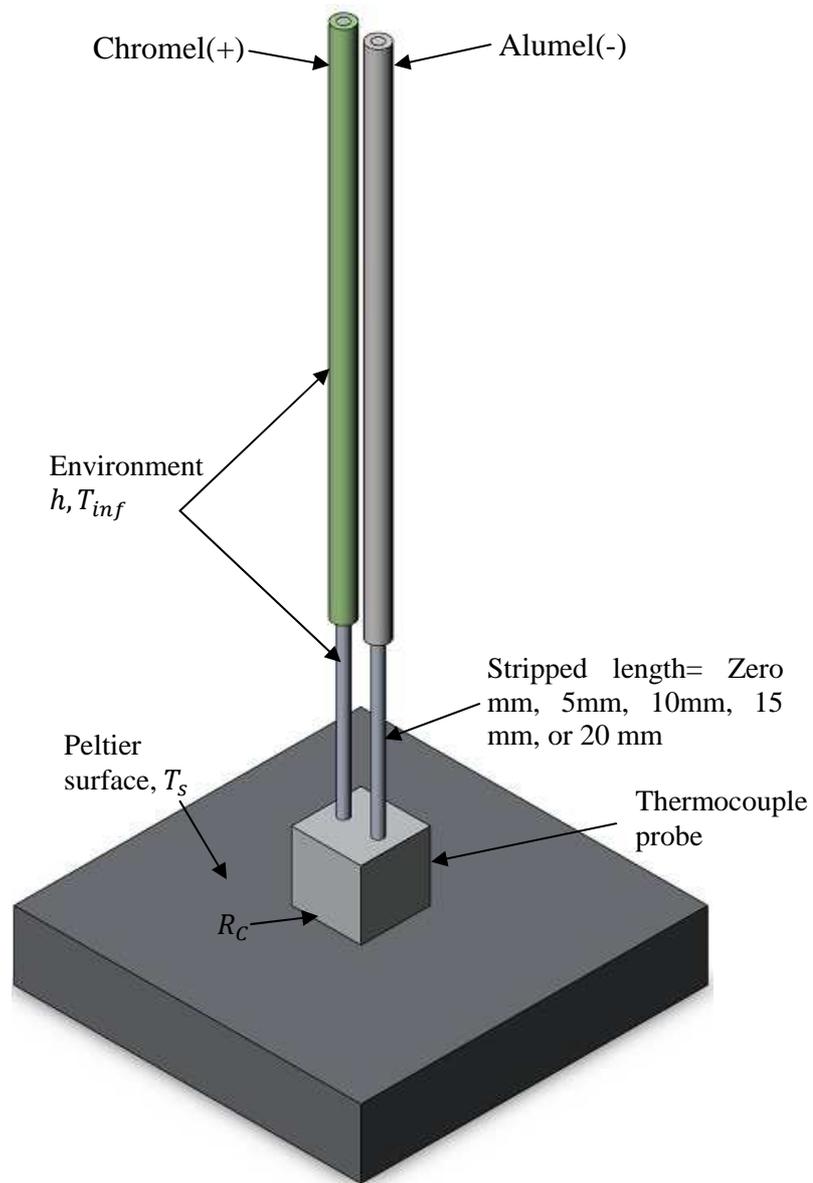


Figure 12 Demonstration graph of thermocouple three-dimensional model for the Numerical analysis.

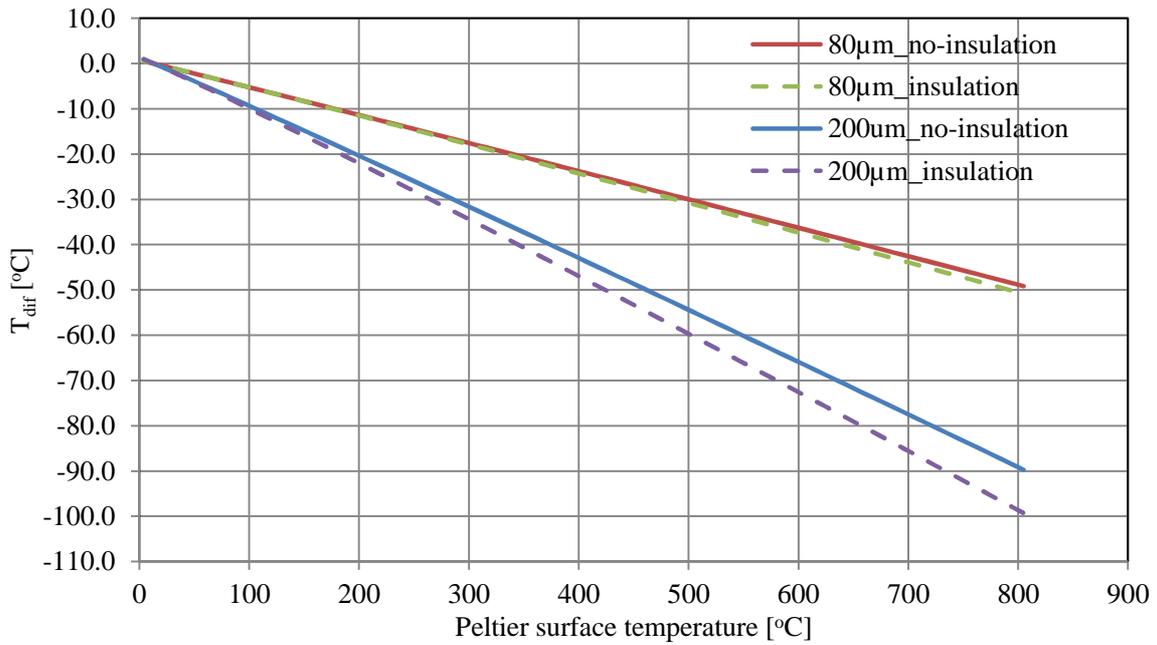


Figure 13 Analytical results for the thermocouple error measurement for peltier surface temperature up to 800[°C] beyond the experimental working range.

Table 1 Thermocouple type K material properties.

Thermocouple wire components	Thermocouple wires properties		Thermocouple insulation properties	
	Chromel[17]	Alumel[17]	80µm [PFA][29]* part no. Z2-K-2 X 5	200µm [PTFE][30]* part no. ZO-PFA-K-1 X 5
Thermal conductivity [W/(m K)]	19.2 ( $k_{w1}$ or $k_{w2}$ )	29.77( $k_{w1}$ or $k_{w2}$ )	0.3 ( $k_{ins}$ )	0.25 ( $k_{ins}$ )
Density [kg/m <sup>3</sup> ]	8730	8600	7900	2130-2230
Heat capacity [J/(kg K)]	447.7	523.34	500	1000

\*manufacturer lab facility

Table 2 Equivalent diameters for three sizes of thermocouple.

Bare wire diameter [µm]*	80	200
Wire diameter with insulation [µm]*, $D_{ins}$	250	500

$t_{ins}[\mu\text{m}]$	85	150
$D_{weq}[\mu\text{m}]$ Eq.(1)	113.14	283
$t_{eqins}[\mu\text{m}]$ Eq.(2)	198.33	285
$D_{eqins}=D_{weq}+2t_{eqins}[\mu\text{m}]$	311.5	633 $\mu\text{m}$
$h$ Eq.(10) [ $\text{W}/\text{m}^2 \text{K}$ ]	124	53.3
$r_{cr}$ Eq.(12)[ $\mu\text{m}$ ]	2419.4	5464.5

\* measured by accurate micrometer.

Table 3 Air properties at atmospheric pressure and 20°C [24].

$\nu_a[\text{m}^2/\text{s}]$	$\alpha_a[\text{m}^2/\text{s}]$	$k_a[\text{W}/\text{m K}]$	$P_{ra}$	$\beta_a [1/\text{K}]$	$\mu_a [\text{kg}/\text{m.s}]$	$C_{pa}[\text{kJ}/\text{kg.K}]$
$15 \times 10^{-6}$	$20.8 \times 10^{-6}$	0.025	0.72	$3.403 \times 10^{-3}$	$18.1 \times 10^{-6}$	1.006

### Nomenclature

Symbol	Description
$A_{weq}$	Equivalent single wire model sectional area [ $\text{m}^2$ ]
$A_p$	Thermocouple probe cross-sectional area [ $\text{m}^2$ ]
$C_{pa}$	Air specific heat capacity [ $\text{J}/\text{kg K}$ ]
$D$	Thin cylinder diameter [m]
$D_{weq}$	Thermocouple bare wire equivalent diameter[m]
$D_w$	Thermocouple metal wire diameter. [m]
$D_{ins}$	Thermocouple metal wire diameter with insulation [m]
$D_{eqins}$	Equivalent thermocouple wire insulation diameter
$D_p$	Thermocouple probe diameter [m]
$g$	Gravitational acceleration [ $\text{m}/\text{s}^2$ ]
$H$	Thin cylinder height [m]
$h$	Heat transfer coefficient of free convection [ $\text{W}/\text{m}^2 \text{K}$ ]
$h_j$	Thermal joint conductivity[ $\text{W}/\text{m}^2 \text{K}$ ]
$h_p$	Heat transfer coefficient of free convection around the probe [ $\text{W}/\text{m}^2 \text{K}$ ]
$k_{weq}$	Equivalent thermal conductivity of thermocouple wire [ $\text{W}/\text{m K}$ ]
$k_{w1} \& k_{w2}$	Individual thermal conductivity of wires 1&2[ $\text{W}/\text{m K}$ ]
$k_a$	Air thermal conductivity[ $\text{W}/\text{m K}$ ]
$k_{ins}$	Thermocouple insulation thermal conductivity [ $\text{W}/\text{m K}$ ]
$k_p$	Probe thermal conductivity[ $\text{W}/\text{m K}$ ]
$p_p$	Probe perimeter [m]
$q_p$	Heat flux to the probe [W]

$q_w$	Heat flux to the wire [W]
$r_{cr}$	Critical radius [m]
$R_m$	Constriction thermal resistance [ $m^2$ K/W]
$R_c$	Contact thermal resistance resistance [ $m^2$ K/W]
$R_p$	Probe thermal resistance [ $m^2$ K/W]
$R_w$	Wires thermal resistance [ $m^2$ K/W]
$t_{eqins}$	Thermocouple equivalent insulation thickness [m]
$T$	Temperature [ $^{\circ}$ C]
$T_{inf}$	Environmental temperature [ $^{\circ}$ C]
$T_{dif}$	$T_{measured} - T_{peltier}$ [ $^{\circ}$ C]
$T_s$	Undisturbed surface temperature [ $^{\circ}$ C]
$T_m$	Modified surface temperature [ $^{\circ}$ C]
$T_b$	Bottom temperature of the probe [ $^{\circ}$ C]
$T_L$	Top temperature of the probe [ $^{\circ}$ C]
$T_p$	Temperature variable of the probe [ $^{\circ}$ C]
$T_w$	Temperature variable of the wire [ $^{\circ}$ C]

*Greek symbols*

$\theta_p$	$T_p - T_{inf}$
$\theta_L$	$T_L - T_{inf}$
$\theta_b$	$T_b - T_{inf}$
$\theta_w$	$T_w - T_{inf}$
$\nu_a$	Air kinematic viscosity [ $m^2/s$ ]
$\alpha_a$	Air thermal diffusivity [ $m^2/s$ ]
$\beta_a$	Air thermal expansion coefficient [ $1/K$ ]
$\mu_a$	Air dynamic viscosity [ $kg/m.s$ ]
$\alpha$	$A_w/A_p$