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LIBERAL EGALITARIANISM AND THE HARM PRINCIPLE*

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We analyse the implications of classical liberal and libertarian approaches for distributive justice in the context of social welfare orderings. We study an axiom capturing a liberal non-interfering view of society, the Weak Harm Principle, whose roots can be traced back to John Stuart Mill. We show that liberal views of individual autonomy and freedom can provide consistent foundations for welfare judgements. In particular, a liberal non-interfering approach can help to adjudicate some fundamental distributive issues relative to intergenerational justice. However, a strong relation is established between liberal views of individual autonomy and non-interference, and egalitarian principles in the Rawlsian tradition.

What are the implications of classical liberal and libertarian approaches for distributive justice? Can liberal views of individual autonomy and freedom provide consistent foundations for social welfare judgements? In particular, can a liberal non-interfering approach help to adjudicate some fundamental distributive issues relative to intergenerational justice? What is the relation between classical liberal political philosophy and the egalitarian tradition stemming from John Rawls's seminal book *A Theory of Justice* (Rawls, 1971)? This article addresses these questions and, in so doing, it contributes to three different strands of the literature.

In recent work, Mariotti and Veneziani (2013, 2014) have explored a new notion of respect for individual autonomy in social judgements, suited for social welfare orderings (henceforth, SWOs)[1] whose philosophical roots can be traced back to John Stuart Mill's essay *On Liberty*, and his view of a sphere of individual freedom:

> there is a sphere of action in which society . . . has, if any, only an indirect interest; comprehending all that portion of a person's life and conduct *which affects only himself*. . . This, then, is the appropriate region of human liberty [italics added].
> (Mill, 2003 [1859], p. 82)

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[1] A social welfare relation is a reflexive and transitive relation to evaluate alternative allocations of welfare to individuals in society. A social welfare ordering is a complete social welfare relation. For a formal definition, see Section 1 below.

The principle of non-interference (or non-interference, in short) embodies the idea that 'an individual has the right to prevent society from acting against him in all circumstances of change in his welfare, *provided* that the welfare of no other individual is affected' (Mariotti and Veneziani, 2013, p. 1690).

Formally, non-interference can be illustrated as follows: in a society with two individuals, consider two allocations, or profiles, $u = (u_1, u_2)$ and $v = (v_1, v_2)$, describing the welfare levels of the two agents in two alternative scenarios. Suppose that, for whatever reason, u is strictly socially preferred to v. Suppose then that agent 1 either suffers a welfare loss, or enjoys a welfare increase in both profiles, while agent 2's welfare is unchanged, giving rise to two new profiles $u' = (u'_1, u_2)$ and $v' = (v'_1, v_2)$, with either $u'_1 > u_1$ and $v'_1 > v_1$; or $u'_1 < u_1$ and $v'_1 < v_1$. non-interference says that, if agent 1's welfare is strictly higher at u' than at v', then society should not reverse the strict preference between u and v to a strict preference for v' over u'. An agent 'can veto society from a strict preference switch after a positive or negative change that affects only [her] and nobody else' (Mariotti and Veneziani, 2013, p. 1690).

The veto power accorded to individuals is weak because a switch to indifference is admitted and because non-interference is silent in a number of cases (e.g. if agent 1's welfare changes in opposite directions, with $(u'_1 - u_1)(v'_1 - v_1) < 0$, or if her welfare is higher at v' than at u'). There are numerous non-dictatorial, and even anonymous SWOs that satisfy non-interference. Yet, Mariotti and Veneziani (2013) have proved that, in societies with a finite number of agents, dictatorial SWOs are the only ones compatible with non-interference among those satisfying weak Pareto.[2] Lombardi and Veneziani (2012) and Alcantud (2013*b*) have extended this result to societies with a countably infinite number of agents.

This impossibility proves the limitations of liberal approaches to Paretian social judgements: there cannot be any 'protected sphere' for individuals even if nobody else is affected. As Mariotti and Veneziani (2013, p.1691) put it, 'Of the appeals of the individuals to be left alone because "nobody but me has been affected", at least some will necessarily have to be overruled'. The first contribution of this article to the literature on liberal approaches is to analyse a specific, ethically relevant weakening of non-interference and provide a series of positive results, both in the finite and in the infinite context.

To be precise, we limit the bite of non-interference by giving individuals a veto power only in situations in which they suffer a decrease in welfare. Arguably, this captures the most intuitive aspect of a liberal ethics of non-interference, as it protects individuals in situations where they suffer a damage, while nobody else is affected: a switch in society's strict preferences against an individual after she has incurred a welfare loss would represent a punishment for her.

Formally, in the two-agent example above, we restrict non-interference to hold in situations where $u'_1 < u_1$ and $v'_1 < v_1$. We call this axiom the Weak Harm Principle – for it represents a strict weakening of the Harm Principle first introduced by Mariotti

[2] The anonymity and weak Pareto axioms are formally defined in Section 1 below.

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and Veneziani (2009) – and show that a limited liberal ethics of non-interference can lead to consistent social judgements.[3]

The implications of liberal principles of non-interference (in conjunction with standard axioms in social choice), however, turn out to be fairly surprising. For there is a strong formal and conceptual relation between liberal views, as incorporated in the Weak Harm Principle, and egalitarian social welfare relations (henceforth, SWRs). The analysis of this relation is the second main contribution of the article.

Formally, we provide a number of fresh characterisations of widely used Rawlsian SWRs. Standard characterisations of the difference principle, or of its lexicographic extension, are based either on informational invariance and separability properties (d'Aspremont, 1985; d'Aspremont and Gevers, 2002) or on axioms with a marked egalitarian content such as the classic Hammond equity axiom (Hammond, 1976 1979).[4]

We prove that both the Rawlsian difference principle and its lexicographic extension can be characterised by the Weak Harm Principle, together with standard efficiency, fairness and – where appropriate – continuity properties. The adoption of SWRs with a strong egalitarian bias can thus be justified by a liberal principle of non-interference which is logically distinct from informational invariance and separability axioms, but has no egalitarian content. Indeed the Weak Harm Principle has a marked individualistic flavour (in the sense of Hammond, 1996).

This relation between liberal approaches and egalitarian SWRs has been originally established by Mariotti and Veneziani (2009), who have characterised the leximin SWO in finite societies based on the Harm Principle. We extend and generalise their insight in various directions.

First of all, as noted above, we focus on a strict weakening of the Harm Principle. This is important both formally and conceptually. Formally, it has been argued that the characterisation in Mariotti and Veneziani (2009) is less surprising than it seems, because under anonymity, the Harm Principle implies Hammond equity (Alcantud, 2013*b*, Proposition 4). This conclusion does not hold with the Weak Harm Principle: even under anonymity, the Weak Harm Principle and Hammond equity are logically independent and the original insight of Mariotti and Veneziani (2009) is therefore strengthened. Conceptually, by ruling out only a strict preference switch in social judgements, the Weak Harm Principle captures liberal and libertarian views more clearly than the Harm Principle, for it emphasises the negative prescription at the core of Mill's analysis of non-interference and assigns a significantly weaker veto power to individuals.

Further, based on the Weak Harm Principle, we also provide new characterisations of Rawls's difference principle. Compared to the leximin, the maximin SWR may be deemed undesirable because it defines rather large indifference classes. Yet, in a number of settings, its relatively simpler structure is a significant advantage, which

[3] Mariotti and Veneziani (2012*a*) analyse different restrictions of non-interference and characterise Nash-type orderings. For a related analysis of utilitarianism, see Mariotti and Veneziani (2012*b*).

[4] See also Tungodden (1999, 2000) and Bosmans and Ooghe (2013). Similar axioms are used also in the infinite context; see, for example, Lauwers (1997); Asheim and Tungodden (2004); Asheim *et al.* (2007); Bossert *et al.* (2007); Alcantud (2013*a*); Asheim and Zuber (2013).

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allows one to capture the core egalitarian intuitions in a technically parsimonious way. Moreover, unlike the leximin, the maximin satisfies continuity and therefore egalitarian judgements based on the difference principle are more robust to small measurement mistakes, e.g. in empirical analysis. This probably explains the wide use of the maximin in modern theories of equality of opportunity (Roemer, 1998, 2002; Gotoh and Yoshihara, 2003), in experimental approaches to distributive justice (Konow, 2002; Bolton and Ockenfels, 2006), in the analysis of the ethics of exhaustible resources and global warming (Solow, 1974; Cairns and Long, 2006; Llavador *et al.*, 2011; Roemer, 2011) and, in the context of intergenerational justice, (Silvestre, 2002; Llavador *et al.*, 2010).[5] In the analysis of intergenerational justice and environmental economics, the maximin principle is often taken to embody the very notion of sustainability (Llavador *et al.*, 2015).

Indeed, and this is the third main contribution of the article, we analyse liberal and libertarian approaches to intergenerational justice. On the one hand, the intergenerational context provides a natural framework for the application of liberal principles of non-interference. For there certainly are many economic decisions whose effects do not extend over time and leave the welfare of other generations unchanged. Moreover, liberal principles of non-interference capture some widespread ethical intuitions in intergenerational justice (Wolf, 1995). In the seminal Brundtland (1987, p. 43) report, for example, sustainable development is defined precisely as 'development that meets the needs of the present without compromising the ability of future generations to meet their needs'.

On the other hand, the application of liberal principles to intergenerational justice raises complex theoretical and technical issues. Lombardi and Veneziani (2012) and Alcantud (2013*b*) have shown that there exists no fair and Paretian SWR that satisfies a fully non-interfering view in societies with a countably infinite number of agents. More generally, the analysis of distributive justice among an infinite number of generations is problematic for all of the main approaches, and impossibility results often emerge (Lauwers, 1997; Basu and Mitra, 2003; Fleurbaey and Michel, 2003; Zame, 2007; Hara *et al.*, 2008; Crespo *et al.*, 2009). Several recent contributions have provided characterisation results by dropping either completeness (Asheim and Tungodden, 2004; Asheim *et al.*, 2007; Basu and Mitra, 2007; Bossert *et al.*, 2007) or transitivity (Sakai, 2010).[6] But the definition of suitable anonymous and Paretian criteria is still an open question in the infinite context (for a thorough discussion, see Asheim, 2010).

Our main contribution to this literature is a novel analysis of liberal egalitarianism in economies with a countably infinite number of agents. To be specific, we provide a new characterisation of one of the main extensions of the leximin criterion in infinitely-lived societies, namely the leximin overtaking proposed by Asheim and Tungodden (2004). As in the finite-horizon case, we show that the Weak Harm Principle can be used to provide a simple and intuitive characterisation, without appealing to any

[5] Maximin preferences are prominent also outside normative economics – for example, in decision theory and experimental economics. See, *inter alia*, the classic papers by Maskin (1979); Barberà and Jackson (1988); Gilboa and Schmeidler (1989); and, more recently, Sarin and Vahid (2001); de Castro *et al.* (2011).

[6] Asheim and Zuber (2013) have recently proposed a complete and transitive extension of the leximin which overcomes the impossibility by requiring only sensitivity to the interests of generations whose consumption has finite rank.

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informational invariance or separability property or to axioms with an egalitarian content. Indeed, although we focus on a specific extension of the leximin that is prominent in the literature on evaluating infinite utility streams, our arguments can be modified to obtain new characterisations for all of the main approaches.

We also extend the analysis of Rawls's difference principle to the intergenerational context. As already noted, if the leximin is adopted, social judgements are sensitive to tiny changes in welfare profiles and measurement errors. In the intergenerational context, an additional issue concerns the significant incompleteness of leximin SWRs which may hamper social evaluation in a number of ethically relevant scenarios (see the discussion in Asheim *et al.*, 2010). Therefore we provide a novel characterisation of the maximin SWO (more precisely, the infimum rule, Lauwers, 1997) in societies with a countably infinite number of agents: based on the Weak Harm Principle, we identify a complete egalitarian criterion that allows for robust social evaluation of intergenerational distributive conflicts.

Our result differs from other characterisations in the literature in two key respects. Conceptually, the characterisation is again obtained by focusing on standard efficiency, fairness and continuity properties together with a liberal principle of non-interference: neither egalitarian axioms, nor informational invariance or separability properties are necessary. Formally, unlike in Lauwers's (1997) seminal paper, the proof of the characterisation result in the infinite context echoes very closely that in finite societies: both the axiomatic framework and the method of proof – and thus the underlying ethical intuitions – are essentially invariant.

In the light of our results, we can provide some tentative answers to the questions posed in the opening paragraph. Liberal and libertarian approaches emphasising individual autonomy and freedom are logically consistent and provide useful guidance in social judgements (including in the analysis of intergenerational justice), provided the notion of non-interference is suitably restricted. Perhaps counterintuitively, however, a liberal non-interfering approach emphasising individual protection in circumstances of welfare losses leads straight to welfare egalitarianism. Based on the Weak Harm Principle, it is possible to provide a unified axiomatic framework to analyse a set of SWRs originating from Rawls's difference principle in a welfaristic framework. Thus, our analysis sheds new light on the normative foundations of standard egalitarian principles and provides a rigorous justification for the label 'liberal egalitarianism' usually associated with Rawls's approach.

The rest of the article is structured as follows. Section 1 lays out the basic framework. Section 2 introduces our main liberal axiom and characterises the leximin SWO in economies with a finite number of agents. Section 3 analyses the implications of liberal views for robust (continuous) SWOs and derives a characterisation of the difference principle. Sections 4 and 5 extend the analysis to the intergenerational context. Section 6 concludes.

1. The Framework

Let $\mathcal{N}$ denote the (non-empty) set of individuals in society and let $\mathcal{T}$ be the cardinality of $\mathcal{N}$. In this article, we analyse societies with both a finite and an infinite number of agents. In the former case, $\mathcal{T}$ is a natural number T and the set of agents is denoted by

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N. In the latter case, $\mathcal{T} = \infty$ and the set of agents corresponds to the set of strictly positive natural numbers $\mathbb{N}$. In the rest of this Section, we provide the basic definitions and axioms that are common to both the finite and the infinite case.

Each agent i in $\mathcal{N}$ can reach a certain level of welfare, which is normalised so that the set of all welfare levels that agent i can achieve is the closed interval $X \equiv [0,1]$. A list of welfare levels achieved by each agent, a welfare profile, is a point in $X^{\mathcal{T}} \equiv [0,1]^{\mathcal{T}}$, where for any profile $u \in X^{\mathcal{T}}$, and for every agent i in $\mathcal{N}$, u_i denotes i's welfare at u. Thus, for example, X^{∞} denotes the set of countably infinite welfare profiles for agents in $\mathbb{N}$.[7]

We are interested in the social judgements over alternative allocations of welfare among individuals in society. Let $\succcurlyeq$ be a (binary) relation on $X^{\mathcal{T}}$. For any $u, v \in X^{\mathcal{T}}$, $u \succcurlyeq v$ stands for $(u, v) \in \succcurlyeq$ and $u \not\succcurlyeq v$ for $(u, v) \notin \succcurlyeq$; $\succcurlyeq$ stands for 'at least as good as'. The asymmetric part $\succ$ of $\succcurlyeq$ is defined by $u \succ v$ if and only if $u \succcurlyeq v$ and $v \not\succcurlyeq u$, and the symmetric part $\sim$ of $\succcurlyeq$ is defined by $u \sim v$ if and only if $u \succcurlyeq v$ and $v \succcurlyeq u$. They stand respectively for 'strictly better than' and 'indifferent to'. A relation $\succcurlyeq$ is said to be: reflexive if, for any $u \in X^{\mathcal{T}}$, $u \succcurlyeq u$; and transitive if, for any $u, v, w \in X^{\mathcal{T}}$, $u \succcurlyeq v \succcurlyeq w$ implies $u \succcurlyeq w$. A relation $\succcurlyeq$ on $X^{\mathcal{T}}$ is a SWR if it is reflexive and transitive.

In this article, we study some desirable properties of SWRs, which incorporate notions of efficiency, fairness and liberal views of non-interference. In this Section, we present some basic axioms that are used in the rest of the article.

A property of SWRs that is *a priori* desirable is that they be able to rank all possible alternatives. Formally:[8]

COMPLETENESS. *For all $u, v \in X^{\mathcal{T}}$, if $u \neq v$, then $u \succcurlyeq v$ or $v \succcurlyeq u$.*

A relation $\succcurlyeq$ on $X^{\mathcal{T}}$ is a SWO if it is a complete SWR.

For any $x \in X$, let ${}_{con}x = (x, x, \ldots)$ denote the profile in which all agents reach the same level of welfare equal to x. The standard way of capturing efficiency properties is by means of the Pareto axioms.[9]

STRONG PARETO. *For all $u, v \in X^{\mathcal{T}}$, if $u > v$, then $u \succ v$.*

WEAK PARETO. *For all $u, v \in X^{\mathcal{T}}$, and all $\epsilon \in X$ with $\epsilon > 0$, if $u \geq v + {}_{con}\epsilon$, then $u \succ v$.*

Strong Pareto states that if all agents are at least as well off in u as in v, and some of them are strictly better off, then u should be socially strictly preferred to v. Weak Pareto is weaker in that it requires all agents to be (discernibly) strictly better off in u as in v.

[7] The focus on the space of bounded profiles is standard in the literature (Lauwers, 1997; Basu and Mitra, 2003, 2007; Zame, 2007; Hara *et al.*, 2008; Asheim, 2010; Asheim and Banerjee, 2010). It is worth noting in passing that, from a theoretical viewpoint, the $\mathcal{T}$-dimensional unit box can be interpreted as the set of all conceivable distributions of opportunities, where the latter are conceived of as chances in life, or probabilities of success as in Mariotti and Veneziani (2012*a*, *b*).

[8] Note that if $u = v$, then $u \succcurlyeq v$ is guaranteed by reflexivity.

[9] The notation for vector inequalities is as follows: for any $u, v \in X^{\mathcal{T}}$, $u \geq v$ if and only if $u_i \geq v_i$, for each $i \in \mathcal{N}$; $u > v$ if and only if $u \geq v$ and $u \neq v$; and $u \gg v$ if and only if $u_i > v_i$, for each $i \in \mathcal{N}$.

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Observe that if $\mathcal{T} = T$, then weak Pareto is equivalent to the standard weak Pareto axiom.

A basic requirement of fairness is embodied in the next axiom, which states that social judgements ought to be neutral with respect to agents' identities (Basu and Mitra, 2003).

ANONYMITY. *For all* $u, v \in X^{\mathcal{T}}$*, if there exist two agents* $i, j \in \mathcal{N}$ *such that* $u_i = v_j$ *and* $u_j = v_i$*, and, for all other agents* $k \in \mathcal{N} \setminus \{i, j\}$, $u_k = v_k$*, then* $u \sim v$.

Because every permutation of a finite number of elements in a set is the composition of a finite number of permutations of two elements in that set, by transitivity, anonymity implies that for every SWR, any two profiles u, v that are identical up to a finite permutation of their elements must be indifferent.

2. The Weak Harm Principle

We study the implications of liberal views of non-interference in fair and Paretian social welfare judgements. In this Section, we define and discuss the main liberal principle and then present a novel characterisation of the leximin SWO in the finite context.

The key features of liberal views in social choice are captured by the Weak Harm Principle, according to which agents have a right to prevent society from turning against them in all situations in which they suffer a welfare loss, provided no other agent is affected. Formally:

WEAK HARM PRINCIPLE. *For all* $u, v \in X^T$*, if* $u \succ v$ *and if* $u', v' \in X^T$ *are such that*

$$u'_i < u_i, v'_i < v_i, \text{for some agent } i \in N \text{ and}$$
$$u'_j = u_j, v'_j = v_j, \text{for each agent } j \neq i,$$

then $v' \not\succ u'$ if $u'_i > v'_i$.

In other words, consider two profiles u and v such that, for whatever reason, u is strictly socially preferred to v. Then suppose that agent i suffers a welfare loss in both profiles, while all other agents' welfare is unchanged, giving rise to two new profiles u' and v'. The Weak Harm Principle says that, if agent i strictly prefers u' to v' (his welfare is higher at u' than at v') then society should not reverse the strict preference between u and v to a strict preference for v' over u'.

The Weak Harm Principle captures a liberal view of non-interference whenever individual choices have no effect on others. The decrease in agent i's welfare may be due to negligence or bad luck, but in any case the principle states that society should not strictly prefer v' over u': having already suffered a welfare loss in both profiles, an adverse switch in society's strict preferences against agent i would represent an unjustified punishment for him.

The Weak Harm Principle assigns a veto power to individuals in situations in which they suffer harm and no other agent is affected. This veto power is weak in that it only applies to certain welfare configurations (individual preferences after the welfare loss

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must coincide with society's initial preferences) and, crucially, the individual cannot force society's preferences to coincide with his own.

It is important to stress that the principle incorporates some key liberal intuitions, and so it may conflict with different views on distributive justice. For there may be many non-liberal reasons for society to switch from $u \succ v$ to $v' \succ u'$. For example, it may be the case that the sum (the product) of individual utilities is higher at u than at v but the opposite is true when the primed alternatives are considered, then, in a classical utilitarian (Nash/prioritarian) approach, one would have $u \succ v$, but $v' \succ u'$.

In this case, the Weak Harm Principle may seem objectionable as it requires ignoring all information concerning the size of the changes in welfare. The key point here is that the axiom is not meant to capture utilitarian, Nash/prioritarian, or indeed any other distributive intuitions: it aims to incorporate some liberal views of autonomy and protection from interference, for which issues of interpersonal comparability of welfare changes are at best irrelevant. The axiom has an individualistic and non-aggregative structure (focusing on changes in the situation of a single agent when everyone else is indifferent) precisely in order to capture this important intuition of liberal and libertarian approaches.

The Weak Harm Principle is weaker than the Principle of Non-Interference formulated by Mariotti and Veneziani (2013) since it only focuses on welfare losses incurred by agents. It also represents a strict weakening of the Harm Principle proposed by Mariotti and Veneziani (2009) because, unlike the latter, it does not require that society's preferences over u' and v' be identical with agent i's, but only that society should not reverse the strict preference between u and v to a strict preference for v' over u' (possibly except when i's welfare is higher at v'). This weakening is important for both conceptual and formal reasons.

Conceptually, the Weak Harm Principle aims to capture – in a welfaristic framework – a negative freedom that is central in classical liberal and libertarian approaches, namely, freedom from interference from society, when no other individual is affected. The name of the axiom itself is meant to echo John Stuart Mill's famous formulation in his essay *On Liberty*:[10]

> As soon as any part of a person's conduct *affects prejudicially the interests of others*, society has a jurisdiction over it, and the question whether the general welfare will or will not be promoted by interfering with it, becomes open to discussion. But there is no room for entertaining any such question when a person's conduct *affects the interests of no persons besides himself* [italics added].
>
> (Mill, 2003 [1859], p. 139)

In this sense, by only requiring that agent i should not be punished in the SWR by changing social preferences against her, the liberal content of the axiom is much clearer and the Weak Harm Principle strongly emphasises the negative prescription of Mill's principle.

Formally, our weakening of the Harm Principle has relevant implications. Mariotti and Veneziani (2009, Theorem 1, p. 126) prove that, jointly with strong Pareto,

[10] For a comprehensive philosophical discussion, see Mariotti and Veneziani (2014).

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anonymity, and completeness, the Harm Principle characterises the leximin SWO, according to which that society is best which lexicographically maximises the welfare of its worst-off members. For any profile $u \in X^T$, let $\bar{u} = (\bar{u}_1, \ldots, \bar{u}_T)$ be a permutation of u such that the components are ranked in ascending order with $\bar{u}_1 \leq \bar{u}_2 \leq \ldots \leq \bar{u}_T$.

DEFINITION 1. *The leximin SWO $\succcurlyeq_T^{LM}$ on X^T is defined as follows. For all $u, v \in X^T$,*

(*i*) $u \sim_T^{LM} v \Leftrightarrow \bar{u}_i = \bar{v}_i$ *for all* $i \in N$;
(*ii*) $u \succ_T^{LM} v \Leftrightarrow$ *there is some* $i \in N$ *such that* $\bar{u}_i > \bar{v}_i$ *and* $\bar{u}_j = \bar{v}_j$ *for each* $j < i$.

The leximin SWO is usually considered to have a strong egalitarian bias and so a characterisation based on a liberal principle with no explicit egalitarian content is surprising. To clarify this point, note that the classic characterisation by Hammond (1976) states that a SWR is the leximin SWO if and only if it satisfies strong Pareto, anonymity, completeness and the following axiom:

HAMMOND EQUITY. *For all $u, v \in X^T$, if $u_i < v_i < v_j < u_j$ for two agents $i,j \in N$, and $u_k = v_k$ for all other agents $k \in N \backslash \{i,j\}$, then $v \succcurlyeq u$.*

Unlike the Harm Principle, Hammond equity expresses a clear concern for equality, for it asserts that among two welfare profiles which are not Pareto-ranked and differ only in two components, society should prefer the more egalitarian one.

Hammond equity and the Harm Principle are conceptually distinct and logically independent. Yet, it has been argued that the characterisation of the leximin SWO in Mariotti and Veneziani (2009) is formally not surprising because, under anonymity and completeness, the Harm Principle implies Hammond equity (Alcantud, 2013*b*, Proposition 4).[11] This objection does not hold if one considers the Weak Harm Principle. To see this, consider the following example.

EXAMPLE 1 *(Sufficientarianism). Suppose that welfare units can be normalised so that a welfare level equal to $1/2$ represents a decent living standard. Then one can define a SWR $\succcurlyeq^s$ on X^T according to which that society is best in which the highest number of people reach a decent living standard. Formally, for all $u \in X^T$, let $P(u) = \{i \in N : u_i \geq 1/2\}$ and let $|P(u)|$ denote the cardinality of $P(u)$. Then, for all $u, v \in X^T$:*

$$u \succcurlyeq^s v \Leftrightarrow |P(u)| \geq |P(v)|.$$

It is immediate to see that $\succcurlyeq^s$ is a SWO and it satisfies anonymity and the Weak Harm Principle, but violates both Hammond equity and the Harm Principle.[12]

The absence of any conceptual and formal relations between the Weak Harm Principle and Hammond equity, even under anonymity, established in Example 1 is not a mere technical artefact. The Suppes-Sen grading principle, for instance, satisfies anonymity and the Weak Harm Principle and violates Hammond equity but one may

[11] The argument is originally due to François Maniquet in unpublished correspondence.
[12] Consider, for example, the profiles $u = (1,0)$ and $v = (1/3, 1/4)$. By definition $u \succ^s v$, which violates Hammond equity. Therefore, since $\succcurlyeq^s$ is anonymous and complete, it also violates the Harm Principle.

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object that this is due to its incompleteness. In contrast, the SWR in Example 1 is complete and it embodies a prominent approach to distributive justice in political philosophy and social choice (Frankfurt, 1987; Roemer, 2004). Thus, even under anonymity and completeness, liberal principles of non-interference incorporate substantially different normative intuitions than standard equity axioms. Example 1 also highlights the theoretical relevance of our weakening of the Harm Principle, for the Weak Harm Principle is consistent with a wider class of SWOs, including some – such as the sufficientarian – which embody widely shared views on distributive justice.

Given this, it is remarkable that the characterisation result provided in Mariotti and Veneziani (2009) can be strengthened.[13]

PROPOSITION 1. *A SWR $\succcurlyeq$ on X^T is the leximin SWO $\succcurlyeq_T^{LM}$ if and only if it satisfies anonymity, strong Pareto, completeness and the Weak Harm Principle.*

In the light of our discussion of the Weak Harm Principle and Example 1, it is worth stressing some key theoretical implications of Proposition 1. First, it is possible to eschew impossibility results by weakening the Principle of Non-Interference proposed by Mariotti and Veneziani (2014) while capturing some core liberal intuitions. For by Proposition 1 there exist anonymous and strongly Paretian SWOs consistent with liberal non-interfering views, as expressed in the Weak Harm Principle.

Second, by Proposition 1 Hammond equity and the Weak Harm Principle are equivalent in the presence of anonymity, completeness, and strong Pareto, even though they are logically independent. However, it can be proved that if $T = 2$, then under strong Pareto and completeness, Hammond equity implies the Weak Harm Principle, but the converse is never true (Mariotti and Veneziani, 2014). Together with Example 1, this implies that Proposition 1 is far from trivial. For even under completeness and either anonymity or strong Pareto, the Weak Harm Principle is not stronger than Hammond equity and it is actually strictly weaker, at least in some cases.

Third, Proposition 1 puts the normative foundations of leximin in a rather different light. For, unlike in standard results, the egalitarian SWO is characterised without appealing to any axioms with a clear egalitarian content.[14] Actually, strong Pareto, completeness and the Weak Harm Principle are compatible with some of the least egalitarian SWOs, namely the lexicographic dictatorships, which proves that the Weak Harm Principle imposes no significant egalitarian restriction. As a result, Proposition 1 highlights the normative strength of Anonymity in determining the egalitarian outcome, an important insight which is not obvious in standard characterisations based on Hammond equity.[15]

[13] The properties in Proposition 1 are clearly independent. The proof of Proposition 1 is a generalisation of the proof of Theorem 1 in Mariotti and Veneziani (2009) and is available from the authors upon request.

[14] Nor to any invariance or separability axioms.

[15] To be sure, from a purely formal viewpoint, it is the interaction of the four axioms which drives the egalitarian outcome in Proposition 1. As the discussion of Example 1 above shows, the removal of either strong Pareto or completeness allows for SWRs that are not necessarily egalitarian. Similarly, several non-egalitarian or even inegalitarian SWRs are compatible with completeness, anonymity and strong Pareto, such as the utilitarian or the leximax SWOs. Conceptually, however, in our framework, anonymity is the axiom which most clearly embodies a notion of justice as fairness.

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In order to illustrate the role of the different axioms in ruling out inegalitarian SWRs, and the basic intuition behind the proof of Proposition 1, consider a society with only two agents. For any $x \in X$, a minimal egalitarian principle requires that $(x-\epsilon, x+\epsilon) \not\succ (x, x)$ for any positive real number ϵ such that $(x-\epsilon, x+\epsilon) \in X^2$. For the sake of concreteness and without loss of generality, consider $x = 1/2$ and $\epsilon = 1/4$. We show that for any SWR that satisfies our four axioms, indeed $(1/4, 3/4)$ cannot be strictly preferred to $(1/2, 1/2)$. Suppose, to the contrary, that $(1/4, 3/4) \succ (1/2, 1/2)$. Then by the Weak Harm Principle, together with completeness, $(1/4, u_2) \succcurlyeq (1/2, 1/4)$ for any $1/4 < u_2 < 3/4$. But then we can use strong Pareto, together with transitivity, to break the indifferences and obtain $(1/4, 1/2) \succ (1/2, 1/4)$, which yields a contradiction by anonymity. The next Sections significantly extend and generalise these intuitions

3. Liberal Egalitarianism Reconsidered

One common objection to the leximin SWO is its sensitivity to small changes in welfare profiles, and so to measurement errors and minor variations in policies. Albeit possibly secondary in theoretical analyses, these issues are relevant in empirical applications and policy debates. As Chichilnisky (1982, p. 346) aptly noted,

> Continuity is a natural assumption that is made throughout the body of economic theory, and it is certainly desirable as it permits approximation of social preferences on the basis of a sample of individual preferences, and makes mistakes in identifying preferences less crucial. These are relevant considerations in a world of imperfect information.

In this Section, we study the implications of liberal non-interfering approaches for social evaluations that are robust to small changes in welfare profiles. A standard way of capturing this property is by an interprofile condition requiring the SWR to vary continuously with changes in welfare profiles.

CONTINUITY. *For all $u \in X^T$, $\{v \in X^T | v \succcurlyeq u\}$ and $\{v \in X^T | u \succcurlyeq v\}$ are closed sets.*

By Proposition 1, if continuity is imposed in addition to the Weak Harm Principle, completeness, strong Pareto and anonymity, an impossibility result immediately obtains. Indeed, it is possible to show that a conflict exists even if anonymity is dropped. To see this, consider the following example.

EXAMPLE 2 *(Impossibility). Consider a society with only two agents, without loss of generality. Suppose that a SWR $\succcurlyeq$ on X^2 satisfies the Weak Harm Principle, completeness, strong Pareto and continuity. By strong Pareto, $(1, 1/2) \succ (3/4, 1/2)$. By continuity and completeness, there exists a sufficiently small real number $\epsilon > 0$ such that $(1, 1/2) \succ (3/4, (1/2) + \epsilon)$. For all natural numbers $k > 0$, let the sequence $(u_1^k)_{k \in \mathbb{N}}$ be defined by $u_1^k = 1/2 + 1/2^{k+1}$. Because the sequence lies entirely in the open interval $(1/2, 1)$, by the Weak Harm Principle and completeness, $(u_1^k, 1/2) \succcurlyeq (1/2, (1/2) + \epsilon)$ for all k. Therefore since the sequence converges*

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monotonically to $1/2$, *continuity implies that* $(1/2, 1/2) \succcurlyeq (1/2, (1/2) + \epsilon)$, *in violation of strong Pareto.*

Therefore, given our interest in liberal principles of non-interference, we weaken our efficiency requirement to focus on weak Pareto and show that the combination of the five axioms characterises Rawls's difference principle, according to which that society is best which maximises the welfare of the worst off individual.[16]

DEFINITION 2. *The maximin* SWO $\succcurlyeq^M$ *on* $X^{\mathcal{T}}$ *is defined as follows. For all* $u, v \in X^{\mathcal{T}}$,

$$u \succcurlyeq^M v \Leftrightarrow \inf_{i \in \mathcal{N}} u_i \geq \inf_{i \in \mathcal{N}} v_i.$$

Theorem 1 states that the standard requirements of fairness, efficiency, completeness and continuity, together with our liberal axiom characterise the maximin SWO.[17]

THEOREM 1. *A* SWR $\succcurlyeq$ *on* X^T *is the maximin* SWO $\succcurlyeq^M$ *if and only if it satisfies anonymity, weak Pareto, completeness, continuity and the Weak Harm Principle.*

Proof. Since the proof of the $(\Rightarrow)$ part of the statement can be easily verified, we shall only prove its $(\Leftarrow)$ part.

$(\Leftarrow)$ Let $\succcurlyeq$ on X^T be a SWR satisfying the specified set of axioms. We show that $\succcurlyeq$ is the maximin SWO. We prove that, for all $u, v \in X^T$,

$$u \succ^M v \Leftrightarrow u \succ v \tag{1}$$

and

$$u \sim^M v \Leftrightarrow u \sim v. \tag{2}$$

Note that as $\succcurlyeq$ is transitive and satisfies anonymity, in what follows we can focus either on u and v, or on the corresponding ranked profiles $\bar{u}$ and $\bar{v}$, without loss of generality. Further, by construction, $\inf_{i \in N} \bar{u}_i = \bar{u}_1$ and $\inf_{i \in N} \bar{v}_i = \bar{v}_1$.

First, we show that the implication $(\Rightarrow)$ of (1) is satisfied. Take any $u, v \in X^T$. Suppose that $u \succ^M v \Leftrightarrow \bar{u}_1 > \bar{v}_1$. We proceed by contradiction, first proving that $v \succ u$ is impossible and then ruling out $v \sim u$.

Suppose that $v \succ u$, or equivalently, $\bar{v} \succ \bar{u}$. As weak Pareto holds, $\bar{v}_j \geq \bar{u}_j$ for some $j \in N$, otherwise a contradiction immediately obtains. We proceed according to the following steps.

Step 1. Let

$$k = \inf\{l \in N | \bar{v}_l \geq \bar{u}_l\}.$$

By anonymity and the transitivity of $\succcurlyeq$, let $v_i = \bar{v}_k$ and let $u_i = \bar{u}_1$. Then, consider two real numbers $d_1, d_2 > 0$, and two profiles u^*, v' – together with the corresponding ranked profiles $\bar{u}^*$, $\bar{v}'$ – formed from $\bar{u}, \bar{v}$ as follows: $\bar{u}_1$ is lowered to $\bar{u}_1 - d_1 > \bar{v}_1$; $\bar{v}_k$ is

[16] Note that Definition 2 holds both in the finite and in the infinite context.
[17] The properties in Theorem 1 are independent (details are available from the authors upon request).

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lowered to $\bar{u}_k > \bar{v}_k - d_2 > \bar{u}_1 - d_1$; and all other entries of $\bar{u}$ and $\bar{v}$ are unchanged. By construction $u^*, v' \in X^T$ and $\bar{u}^*_j > \bar{v}'_j$ for all $j \leq k$, whereas by the Weak Harm Principle, completeness, anonymity and transitivity we have $\bar{v}' \succcurlyeq \bar{u}^*$.

Step 2. Let ϵ be a real number such that

$$0 < \epsilon < \inf\{\bar{u}^*_j - \bar{v}'_j | j \leq k\}$$

and define $\bar{u}' = \bar{u}^* -_{con} \epsilon$. By construction, $\bar{u}' \in X^T$ and $\bar{u}^* \gg \bar{u}'$. Weak Pareto implies $\bar{u}^* \succ \bar{u}'$. As $\bar{v}' \succcurlyeq \bar{u}^*$, by step 1, the transitivity of $\succcurlyeq$ implies $\bar{v}' \succ \bar{u}'$.

If $\bar{u}'_j > \bar{v}'_j$ for all $j \in N$, weak Pareto implies $\bar{u}' \succ \bar{v}'$, a contradiction. Otherwise, let $\bar{v}'_l \geq \bar{u}'_l$ for some $l > k$. Then, let

$$k' = \inf\{l \in N | \bar{v}'_l \geq \bar{u}'_l\}.$$

The above steps 1 and 2 can be applied to $\bar{u}'$, $\bar{v}'$ to derive profiles $\bar{u}''$, $\bar{v}'' \in X^T$ such that $\bar{u}''_j > \bar{v}''_j$ for all $j \leq k'$, whereas $\bar{v}'' \succ \bar{u}''$. By weak Pareto, a contradiction is obtained whenever $\bar{u}''_j > \bar{v}''_j$ for all $j \in N$. Otherwise, let $\bar{v}''_l \geq \bar{u}''_l$ for some $l > k'$. And so on. After a finite number s of iterations, two profiles $\bar{u}^s, \bar{v}^s \in X^T$ can be derived such that $\bar{v}^s \succ \bar{u}^s$, by steps 1 and 2, but $\bar{u}^s \succ \bar{v}^s$, by weak Pareto, a contradiction.

Therefore, by completeness, it must be $\bar{u} \succcurlyeq \bar{v}$ whenever $\bar{u} \succ^M \bar{v}$. We have to rule out the possibility that $\bar{u} \sim \bar{v}$. We proceed by contradiction. Suppose that $\bar{u} \sim \bar{v}$. Since $\bar{v}_1 < \bar{u}_1$, there exists $\epsilon \in X$ with $\epsilon > 0$ such that $\bar{u}^\epsilon = \bar{u} -_{con} \epsilon$, $\bar{u}^\epsilon \in X^T$, and $\bar{v}_1 < \bar{u}^\epsilon_1$ so that $\bar{u}^\epsilon \succ^M \bar{v}$. However, by weak Pareto and the transitivity of $\succcurlyeq$ it follows that $\bar{v} \succ \bar{u}^\epsilon$. Apply the above reasoning to $\bar{v}$ and $\bar{u}^\epsilon$ to obtain the desired contradiction.

Now, we show that the implication ($\Rightarrow$) of (2) is met as well. Suppose $\bar{u}_1 = \bar{v}_1$. If $\bar{u}_1 = 1$, the result follows by reflexivity. Hence suppose $\bar{u}_1 < 1$. Let $N(u) = \{i \in N : u_i = \bar{u}_1\}$ and let u^K be such that $u^K_i = u_i$, all $i \notin N(u)$, and $u^K_i = u_i + K^{-1}$, all $i \in N(u)$, where K is any natural number such that $u_i + K^{-1} < 1$, all $i \in N(u)$. By construction, $u^K \in X^T$ and $\bar{u}^k_1 > \bar{v}_1$ for all $k \geq K$. Since $\lim_{k\to\infty} u^k = u$ and $u^k \in \{w \in X^T | w \succcurlyeq v\}$ for all $k \geq K$, continuity implies $u \succcurlyeq v$. A symmetric argument proves that $v \succcurlyeq u$, and so $u \sim v$. This completes the proof of Theorem 1.

Theorem 1 has two main implications in the context of our analysis. First, it shows that anonymous and (weakly) Paretian liberal SWOs exist that are also continuous. This is particularly interesting given that the consistency between weak Pareto, continuity properties, and liberal principles in the spirit of Sen's celebrated *minimal liberalism* axiom has been recently called into question by Kaplow and Shavell (2001).

Second, Theorem 1 provides a novel characterisation of the difference principle that generalises the key insight of Section 2. Standard characterisations focus either on informational invariance and separability properties (d'Aspremont and Gevers, 2002; Segal and Sobel, 2002), or on axioms incorporating a clear inequality aversion such as Hammond equity (Bosmans and Ooghe, 2013) or the Pigou-Dalton principle (Fleurbaey and Tungodden, 2010). Theorem 1 characterises an egalitarian SWO by using an axiom – the Weak Harm Principle – that, unlike informational invariance properties, has a clear ethical foundation but it has no egalitarian content as it only incorporates a liberal, non-interfering view of society.

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4. A Liberal Principle of Intergenerational Justice

In the previous Sections, we study the implications of liberal principles of non-interference in societies with a finite number of agents. We now extend our analysis to societies with a countably infinite number of agents, or generations. A liberal non-interfering approach seems particularly appropriate in the analysis of intergenerational distributive issues: although the welfare of a generation is often affected by decisions taken by their predecessors, there certainly are many economic decisions whose effects do not extend over time and leave the welfare of other generations unchanged. In this Section (and the next), we explore the implications of fair and Paretian liberal approaches to intergenerational justice.[18]

As a first step, we introduce some additional notation that is relevant only in the infinite context. For any generation T and any welfare profile $u \in X^{\infty}$, ${}_1u_T = (u_1, \ldots, u_T)$ denotes the T-head of u and ${}_{T+1}u = (u_{T+1}, u_{T+2}, \ldots)$ denotes its T-tail, so that u is the combined profile $({}_1u_T, {}_{T+1}u)$. Further, let $\succcurlyeq$ and $\succcurlyeq'$ be SWRS defined on X^{∞}, we say that $\succcurlyeq'$ is an extension of $\succcurlyeq$ if for any $u, v \in X^{\infty}$, $u \succcurlyeq v$ implies $u \succcurlyeq' v$, and $u \succ v$ implies $u \succ' v$.

The extension of the main liberal principle to the analysis of intergenerational justice is rather straightforward and needs no further comment, except possibly noting that in this context, the Weak Harm Principle is weakened to hold only for pairs of welfare profiles whose tails can be Pareto-ranked.

WEAK HARM PRINCIPLE*. *For all* $u, v \in X^{\infty}$ *with* $v \equiv ({}_1v_T, ({}_{T+1}u + {}_{con}\epsilon))$ *for some generation* $T \geq 1$ *and some* $\epsilon \geq 0$, *if* $u \succ v$ *and if* $u', v' \in X^{\infty}$ *are such that*

$$u'_i < u_i, v'_i < v_i, \text{ for some generation } i \leq T, \text{ and}$$
$$u'_j = u_j, v'_j = v_j, \text{ for each generation } j \neq i,$$

then $v' \not\succ u'$ *if* $u'_i > v'_i$.

As already noted, economies with an infinite number of agents raise several formal and conceptual issues and different definitions of the main criteria (including utilitarianism, egalitarianism, the Nash SWR, and so on) can be provided in order to compare (countably) infinite welfare profiles. Here, we derive a novel characterisation of one of the main approaches in the literature, namely the leximin overtaking recently formalised by Asheim and Tungodden (2004), in the tradition of Atsumi (1965) and von Weizsäcker (1965). Yet, as argued at the end of the Section, our key results are robust and the Weak Harm Principle can be used to provide normative foundations to *all* of the main extensions of the leximin SWR to the infinite context.

[18] As noted by an anonymous referee, from a formal viewpoint, our analysis applies to any societies with a countably infinite number of agents and not exclusively to the intergenerational context. Yet, conceptually, we believe that our framework is particularly suited to analyse intergenerational justice, especially given that some of the axioms rely on a natural ordering of the agents.

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With the help of Definition 1, the leximin overtaking SWR can be defined as follows.

DEFINITION 3 (ASHEIM AND TUNGODDEN, 2004, DEFINITION 2, P. 224). *The leximin overtaking SWR $\succsim^{LM^*}$ on X^∞ is defined as follows. For all $u, v \in X^\infty$,*

(*i*) $u \sim^{LM^*} v \Leftrightarrow$ *there is a generation* $\tilde{T} \geq 1$ *such that* ${}_1u_T \sim_T^{LM} {}_1v_T$ *for all* $T \geq \tilde{T}$;
(*ii*) $u \succ^{LM^*} v \Leftrightarrow$ *there is a generation* $\tilde{T} \geq 1$ *such that* ${}_1u_T \succ_T^{LM} {}_1v_T$ *for all* $T \geq \tilde{T}$.

According to Definition 3, an infinite welfare profile u is strictly preferred to another profile v if and only if there is a finite period $\tilde{T}$ such that, for every period T after $\tilde{T}$, the welfare levels of the first T generations at u strictly leximin dominate those of the first T generations at v. Similarly, u is indifferent to v if and only if there is a period $\tilde{T}$ such that, for every period T after $\tilde{T}$, the T-head of u is leximin indifferent to the T-head of v.

In order to characterise the leximin overtaking SWR, we need to weaken completeness and require that the SWR be (at least) able to compare profiles with the same tail.

MINIMAL COMPLETENESS. *For all $u, v \in X^\infty$ with $u = ({}_1u_T, {}_{T+1}v)$ for some generation $T \geq 1$, if $u \neq v$, then $u \succsim v$ or $v \succsim u$.*

Finally, in the analysis of intergenerational justice, we follow the literature and consider a mainly technical requirement to deal with infinite-dimensional profiles (Asheim and Tungodden, 2004; Basu and Mitra, 2007; Asheim, 2010; Asheim and Banerjee, 2010).

WEAK PREFERENCE CONTINUITY. *For all $u, v \in X^\infty$, if there is generation $\tilde{T} \geq 1$ such that $({}_1u_T, {}_{T+1}v) \succ v$ for each successive generation $T \geq \tilde{T}$, then $u \succ v$.*

Axioms such as weak preference continuity (and the analogous preference continuity analysed in Section 5 below) establish 'a link to the standard finite setting of distributive justice, by transforming the comparison of any two infinite utility paths to an infinite number of comparisons of utility paths each containing a finite number of generations' (Asheim and Tungodden, 2004, p. 223).

Theorem 2 proves that anonymity, strong Pareto, the Weak Harm Principle*, minimal completeness and weak preference continuity characterise the leximin overtaking.[19]

THEOREM 2. *A SWR $\succsim$ on X^∞ is an extension of the leximin overtaking SWR $\succsim^{LM^*}$ if and only if $\succsim$ satisfies anonymity, strong Pareto, minimal completeness, the Weak Harm Principle* and weak preference continuity.*

Proof. ($\Rightarrow$) Suppose that the SWR $\succsim$ on X^∞ is an extension of the leximin overtaking SWR $\succsim^{LM^*}$. It is easy to see that $\succsim$ meets anonymity and strong Pareto. By observing that $\succsim^{LM^*}$ is complete for comparisons between profiles with the same tail it is also easy to see that $\succsim$ satisfies minimal completeness and weak preference continuity.

[19] The properties in Theorem 2 are independent (details are available from the authors upon request).

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We show that $\succcurlyeq$ meets the Weak Harm Principle*. Take any profiles $u, v, u', v' \in X^\infty$ satisfying its premises. We prove that $u' \succ v'$, whenever $u'_i > v'_i$. Because $\succcurlyeq^{LM^*}$ is complete for comparisons between profiles whose tails differ by a nonnegative constant, $u \succ^{LM^*} v$. Then take any generation $T' \geq \max\{i, \tilde{T}\}$ where $\tilde{T}$ corresponds to part (*ii*) of Definition 3. If $u'_i > v'_i$, then Theorem 1 in Mariotti and Veneziani (2009, p. 126) implies that ${}_1u'_{T'} \succ^{LM}_{T'} {}_1v'_{T'}$. Because the choice of $T' \geq \max\{i, \tilde{T}\}$ was arbitrary, $u' \succ v'$, as sought.

($\Leftarrow$) Suppose that the SWR $\succcurlyeq$ on X^∞ satisfies the above set of axioms. We show that the SWR $\succcurlyeq$ is an extension of the leximin overtaking SWR $\succcurlyeq^{LM^*}$. Take any $u, v \in X^\infty$.

That $u \sim^{LM^*} v$ implies $u \sim v$ follows as in Asheim and Tungodden (2004), by Anonymity and the transitivity of $\succcurlyeq$. Therefore we only show that $u \succ^{LM^*} v$ implies $u \succ v$.

Let $u \succ^{LM^*} v$. Take any $T \geq \tilde{T}$ that corresponds to part (*ii*) of Definition 3 and consider the combined profile $w \equiv ({}_1u_T, {}_{T+1}v) \in X^\infty$. Note that $w \succ^{LM^*} v$. We show that $w \succ v$.

For any $u \in X^\infty$, and all $T \geq 1$, let ${}_1\bar{u}_T = (\bar{u}_1, \ldots, \bar{u}_T)$ be a permutation of the T-head of u such that the components are ranked in ascending order. By anonymity and transitivity, we can focus on the combined profiles $\tilde{w} \equiv ({}_1\bar{u}_T, {}_{T+1}v)$ and $\tilde{v} \equiv ({}_1\bar{v}_T, {}_{T+1}v)$. Note that since $w \succ^{LM^*} v$, by Definition 3 there exists $t \in \{1, 2, \ldots, T\}$ such that $\bar{u}_t > \bar{v}_t$ and $\bar{u}_j = \bar{v}_j$ for each $j < t$.

By minimal completeness, suppose – by way of contradiction – that $\tilde{v} \succcurlyeq \tilde{w}$. We distinguish two cases.

Case 1. $\tilde{v} \succ \tilde{w}$.

As strong Pareto holds it must be the case that $\tilde{v}_l > \tilde{w}_l$ for some l, where $t < l \leq T$. Let:

$$k = \inf\{t < l \leq T | \tilde{v}_l > \tilde{w}_l\}.$$

By anonymity, let $v_i = \tilde{v}_k$ and let $w_i = \tilde{w}_{k-g}$, for some $1 \leq g < k$, where $\tilde{w}_{k-g} > \tilde{v}_{k-g}$. Then, let two real numbers $d_1, d_2 > 0$, and consider profiles $w', v' \in X^\infty$ formed from $\tilde{w}$, $\tilde{v}$ as follows: $\tilde{w}_{k-g}$ is lowered to $\tilde{w}_{k-g} - d_1$ such that $\tilde{w}_{k-g} - d_1 > \tilde{v}_{k-g}$; $\tilde{v}_k$ is lowered to $\tilde{v}_k - d_2$ such that $\tilde{w}_k > \tilde{v}_k - d_2 > \tilde{w}_{k-g} - d_1$; and all other entries of $\tilde{w}$ and $\tilde{v}$ are unchanged. By anonymity and transitivity, consider the combined profiles $\tilde{w}' \equiv ({}_1\bar{w}'_T, {}_{T+1}v)$ and $\tilde{v}' \equiv ({}_1\bar{v}'_T, {}_{T+1}v)$. By construction $\tilde{w}'_j \geq \tilde{v}'_j$ for all $j \leq k$, with $\tilde{w}'_{k-g} > \tilde{v}'_{k-g}$, whereas the Weak Harm Principle*, combined with minimal completeness, anonymity, and transitivity implies $\tilde{v}' \succcurlyeq \tilde{w}'$. Furthermore, by strong Pareto, it is possible to choose $d_1, d_2 > 0$, such that $\tilde{v}' \succ \tilde{w}'$, without loss of generality. Consider two cases:

Case 1.

(*a*) suppose that $\tilde{v}_k > \tilde{w}_k$, but $\tilde{w}_l \geq \tilde{v}_l$ for all $l > k$. It follows that $\tilde{w}' > \tilde{v}'$, and so strong Pareto implies that $\tilde{w}' \succ \tilde{v}'$, a contradiction and

(*b*) suppose that $\tilde{v}_l > \tilde{w}_l$ for some l, $k < l \leq T$. Note that by construction $\tilde{v}'_l = \tilde{v}_l$ and $\tilde{w}'_l = \tilde{w}_l$ for all $l > k$. Then, let:

$$k' = \inf\{k < l \leq T | \tilde{v}'_l > \tilde{w}'_l\}.$$

The above argument can be applied to $\tilde{w}'$, $\tilde{v}'$ to derive combined profiles $\tilde{w}'', \tilde{v}'' \in X^\infty$ such that $\tilde{w}''_j \geq \tilde{v}''_j$ for all $j \leq k'$, with at least one strict inequality, whereas the Weak

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Harm Principle*, combined with minimal completeness, anonymity, strong Pareto, and transitivity implies $\tilde{v}'' \succ \tilde{w}''$. And so on. After a finite number of iterations s, two combined profiles $\tilde{w}^s, \tilde{v}^s \in X^\infty$ can be derived such that, by the Weak Harm Principle*, combined with minimal completeness, anonymity, strong Pareto, and transitivity we have that $\tilde{v}^s \succ \tilde{w}^s$, but strong Pareto implies $\tilde{w}^s \succ \tilde{v}^s$, yielding a contradiction.

Case 2. $\tilde{v} \sim \tilde{w}$.

Since, by our supposition, $\tilde{v}_t < \bar{u}_t \equiv \tilde{w}_t$, there exists a real number $\epsilon > 0$ such that $\tilde{v}_t < \tilde{w}_t - \epsilon < \tilde{w}_t$. Let $\tilde{w}^\epsilon \in X^\infty$ be a profile such that $\tilde{w}^\epsilon_t = \tilde{w}_t - \epsilon$ and $\tilde{w}^\epsilon_j = \tilde{w}_j$ for all $j \neq t$. It follows that $\tilde{w}^\epsilon \succ^{LM^*} \tilde{v}$ but $\tilde{v} \succ \tilde{w}^\epsilon$ by Strong Pareto and the transitivity of $\succcurlyeq$. Hence, the argument of Case 1 above can be applied to $\tilde{v}$ and $\tilde{w}^\epsilon$, yielding the desired contradiction.

Therefore minimal completeness implies $\tilde{w} \succ \tilde{v}$. Then anonymity, combined with the transitivity of $\succcurlyeq$, implies $({}_1u_T, {}_{T+1}v) \succ v$. Since the choice of $T \geq \tilde{T}$ was arbitrary, weak preference continuity implies $u \succ v$, as desired. This completes the proof of Theorem 2.

Theorem 2 shows that, if the principle of non-interference analysed by Lombardi and Veneziani (2012) and Alcantud (2013*b*) is suitably restricted to hold only for welfare losses, then intergenerational distributive conflicts can be adjudicated by means of liberal, fair and Paretian social criteria. Indeed, Theorem 2 provides a novel characterisation of one of the main extensions of the leximin to economies with an infinite number of agents, based on the Weak Harm Principle*, thus confirming the link between a liberal and libertarian concern for individual autonomy, and egalitarian criteria, in the intergenerational context also.[20]

These conclusions are robust and can be extended to alternative definitions of the leximin.[21] For example, if weak preference continuity is replaced with a stronger continuity requirement, a stronger version of the leximin overtaking (the *S-Leximin*, see Asheim and Tungodden, 2004, Definition 1, p. 224) can easily be derived. Perhaps more interestingly, Bossert *et al.* (2007) have dropped continuity properties and have characterised a larger class of extensions of the leximin criterion satisfying strong Pareto, anonymity, and an infinite version of Hammond equity.[22] Lombardi and Veneziani (2009) have shown that it is possible to provide a characterisation of the leximin relation defined by Bossert *et al.* (2007) based on strong Pareto, anonymity and the Weak Harm Principle. Further, the Weak Harm Principle can be

[20] It is worth noting in passing that Theorem 2 can be further strengthened by requiring the Weak Harm Principle* to hold only for profiles with the same tail, namely $\epsilon = 0$.

[21] The proofs of the following claims are available from the authors upon request.

[22] Formally, the relationship between the characterisation of the leximin by Bossert *et al.* (2007) and that by Asheim and Tungodden (2004) is analogous to the relationship between the characterisation of the utilitarian SWR by Basu and Mitra (2007) and the characterisations of the more restrictive utilitarian SWR induced by the overtaking criterion (see the discussion in Bossert *et al.* (2007, p. 580)).

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used – instead of various versions of Hammond equity axiom – to characterise the leximin criterion proposed by Sakai (2010), which drops transitivity but retains completeness; and the time-invariant leximin overtaking proposed by Asheim *et al.* (2010).[23]

5. The Intergenerational Difference Principle

In Section 3, we argued that a potential shortcoming of the leximin criterion is its sensitivity to infinitesimal changes in welfare profiles and explored the implications of liberal principles together with a continuity requirement that incorporates a concern for robustness in social judgements. In the context of intergenerational distributive justice, a further problem of the various extensions of the leximin criterion is their incompleteness, which makes them unable to produce social judgements in a large class of pairwise comparisons of welfare profiles.

In this Section, we complete our study of liberal principles of non-interference by analysing the implications of the Weak Harm Principle* for intergenerational justice when social welfare criteria are required to be continuous and to be able to adjudicate all distributive conflicts. This is by no means a trivial question, for it is well known that continuity is a problematic requirement for SWOs in economies with an infinite number of agents and impossibility results often emerge.[24]

The main axioms incorporating completeness, fairness, efficiency, and liberal non-interference are the same as in previous Sections. Further, we follow the standard practice in the literature (Lauwers, 1997) and define continuity based on the sup metric.

Sup Continuity. *For all $u \in X^\infty$, if there is a sequence of profiles $\{v^k\}_{k=1}^\infty$ in X^∞ such that $\lim_{k\to\infty} v^k = v \in X^\infty$ with respect to the sup metric d_∞, and $v^k \succcurlyeq u$ ($u \succcurlyeq v^k$) for all $k \in \mathbb{N}$, then $u \not\succ v$ ($v \not\succ u$).*

Observe that in general Sup continuity is weaker than the standard continuity axiom but it is equivalent to the latter if the SWR is complete as in Theorem 3 below.[25]

As in the previous Section, we also impose a technical requirement to deal with comparisons of infinite-dimensional profiles.

Preference Continuity. *For all $u, v \in X^\infty$, if there is a generation $\tilde{T} \geq 1$ such that $({}_1u_T,\ {}_{T+1}v) \succcurlyeq v$ for each successive generation $T \geq \tilde{T}$, then $u \succcurlyeq v$.*

[23] As compared to the standard overtaking criterion, the time invariant version does not rely on a natural ordering of generations. Thus, it is possible to drop weak preference continuity and replace it with a similar consistency axiom that does not entail a preference for earlier generations.

[24] See the classic paper by Diamond (1965). For more recent contributions see Hara *et al.* (2008) and the literature cited therein.

[25] It is also weaker than the continuity axiom recently proposed by Asheim *et al.* (2012, p. 271), although the two properties are equivalent for complete SWRs.

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Our next result extends the key insights on liberal egalitarianism to the intergenerational context: anonymity, weak Pareto, completeness, sup continuity, the Weak Harm Principle* and preference continuity characterise the maximin SWO $\succcurlyeq^M$ on X^∞ (see Definition 2).[26]

THEOREM 3. *A SWR $\succcurlyeq$ on X^∞ is the maximin SWO $\succcurlyeq^M$ if and only if it satisfies anonymity, weak Pareto, completeness, sup continuity, the Weak Harm Principle* and preference continuity.*

Proof. Since the proof of the ($\Rightarrow$) part of the statement can be easily verified, we shall only prove its ($\Leftarrow$) part.

($\Leftarrow$) Let $\succcurlyeq$ on X^∞ be a SWR satisfying the specified set of axioms. To show that $\succcurlyeq$ is the maximin SWO, it suffices to prove that for all $u, v \in X^\infty$,

$$\inf_{i\in\mathbb{N}} u_i > \inf_{i\in\mathbb{N}} v_i \Rightarrow u \succ v \tag{3}$$

and

$$\inf_{i\in\mathbb{N}} u_i = \inf_{i\in\mathbb{N}} v_i \Rightarrow u \sim v. \tag{4}$$

Consider (3). Take any $u, v \in X^\infty$ such that $\inf_{i\in\mathbb{N}} u_i > \inf_{i\in\mathbb{N}} v_i$. In order to prove that $u \succ v$, we first demonstrate that ${}_{con}\hat{x} \succcurlyeq v$ holds, where:

$$\hat{x} = \frac{\inf_{i\in\mathbb{N}} u_i + \inf_{i\in\mathbb{N}} v_i}{2}.$$

To this end, we consider two cases, according to whether $\sup_{i\in\mathbb{N}} v_i < 1$ or $\sup_{i\in\mathbb{N}} v_i = 1$.

Case 1. $\sup_{i\in\mathbb{N}} v_i < 1$.

As a first step, we prove the following claim.

CLAIM 1. *There is a generation $T \geq 1$ such that for all $t \geq T$ and for any real number $\epsilon > 0$ such that the combined profile $({}_1\hat{x}_t, {}_{t+1}v + {}_{con}\epsilon)$ is in X^∞, we have $({}_1\hat{x}_t, {}_{t+1}v + {}_{con}\epsilon) \succcurlyeq v$.*

Proof of Claim 1. Suppose not. Since $\succcurlyeq$ on X^∞ satisfies completeness, it follows that for each generation $T \geq 1$ there exist a generation $t \geq T$ and a real number $\epsilon > 0$ such that $({}_1\hat{x}_t, {}_{t+1}v + {}_{con}\epsilon) \in X^\infty$ and $v \succ ({}_1\hat{x}_t, {}_{t+1}v + {}_{con}\epsilon)$.

Since $\hat{x} > \inf_{i\in\mathbb{N}} v_i$, it follows that there exists a generation $T^* \geq 1$ such that $\hat{x} > v_{T^*} \geq \inf\{v_1, \ldots, v_{T^*}\}$. By the contradicting hypothesis, and since $\succcurlyeq$ satisfies completeness, there exists a generation $t^* \geq T^*$ and a real number $\epsilon > 0$ such that

[26] The properties in Theorem 3 are independent (details are available from the authors upon request). It is worth noting in passing that the characterisation of the maximin SWO can be obtained without the full force of completeness, by adopting an axiom similar to minimal completeness. We thank Geir Asheim for this suggestion.

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$({}_1\hat{x}_{t^*},\ {}_{t^*+1}v +_{con}\epsilon) \in X^\infty$ and $v \succ ({}_1\hat{x}_{t^*},\ {}_{t^*+1}v +_{con}\epsilon)$. For the sake of notational simplicity, let $x \equiv ({}_1\hat{x}_{t^*},\ {}_{t^*+1}v +_{con}\epsilon)$. Observe that $\hat{x} > \inf\{v_1, \ldots, v_{T^*}\} \geq \inf\{v_1, \ldots, v_{t^*}\}$.

For any $u \in X^\infty$, and all $T \geq 1$, recall that ${}_1\bar{u}_T = (\bar{u}_1, \ldots, \bar{u}_T)$ is a permutation of the T-head of u such that the components are ranked in ascending order. Let $\tilde{v} \equiv ({}_1\bar{v}_{t^*},\ {}_{t^*+1}v)$. By anonymity and transitivity, $\tilde{v} \succ x$. Suppose that ${}_1x_{t^*} \gg {}_1\bar{v}_{t^*}$. Then, there exists a real number a such that $0 < a < \inf\{\inf\{x_t - \tilde{v}_t | t \leq t^*\}, \epsilon/2\}$ and $x_t \geq \tilde{v}_t + a$ for all $t \in \mathbb{N}$. But then weak Pareto implies $x \succ \tilde{v}$ yielding a contradiction.

Therefore, suppose that for some $1 < t \leq t^*$ we have that $\tilde{v}_t \geq x_t = \hat{x}$. We proceed according to the following steps.

Step 1. Let:

$$q = \inf\{1 < t \leq t^* | \tilde{v}_t \geq \hat{x}\}.$$

Then, consider two real numbers $d_1, d_2 > 0$, and two profiles $x^1, v' \in X^\infty$ formed from x, $\tilde{v}$ as follows: x_q is lowered to $x_q^1 = x_q - d_1 = \hat{x} - d_1 > \tilde{v}_1 = \inf\{v_1, \ldots, v_{t^*}\}$; $\tilde{v}_q$ is lowered to $v_q' = \tilde{v}_q - d_2$ where $\hat{x} > \tilde{v}_q - d_2 > \hat{x} - d_1$; and all other entries of x and $\tilde{v}$ are unchanged. Let $\tilde{x}^1 \equiv ({}_1\bar{x}^1_{t^*}, {}_{t^*+1}x)$, $\tilde{v}' \equiv ({}_1\bar{v}'_{t^*}, {}_{t^*+1}v)$. By construction, $\tilde{x}_t^1 > \tilde{v}_t'$ for all $1 \leq t \leq q$, whereas by the Weak Harm Principle*, completeness, anonymity, and transitivity we have $\tilde{v}' \succsim \tilde{x}^1$.

Step 2. Let:

$$0 < k < \inf\left\{\inf\{\tilde{x}_t^1 - \tilde{v}_t' | t \leq q\}, \inf\{1 - \tilde{v}_t' | q < t \leq t^*\}, \frac{\epsilon}{2t^*}\right\} < \epsilon \tag{5}$$

and define $\tilde{v}^1 = \tilde{v}' +_{con} k$. By construction, $\tilde{v}^1 \in X^\infty$ and $\tilde{v}_i^1 \geq \tilde{v}_i' + k$ for all $i \in \mathbb{N}$, and so weak Pareto implies $\tilde{v}^1 \succ \tilde{v}'$. Since $\tilde{v}' \succsim \tilde{x}^1$, then transitivity implies that $\tilde{v}^1 \succ \tilde{x}^1$.

Suppose that ${}_1\tilde{x}^1_{t^*} \gg {}_1\tilde{y}^1_{t^*}$. Then, since $\inf_{i\in\mathbb{N}} \tilde{x}_i^1 > \inf_{i\in\mathbb{N}} \tilde{v}_i^1$ and ${}_{t^*+1}\tilde{x}^1 \gg {}_{t^*+1}\tilde{v}^1$, there exists a real number $a \in (0, \inf\{\inf\{\tilde{x}_t^1 - \tilde{v}_t^1 | t \leq t^*\}, k/2t^*\})$ such that $\tilde{x}_i^1 \geq \tilde{v}_i^1 + a$ for all $i \in \mathbb{N}$. Weak Pareto implies $\tilde{x}^1 \succ \tilde{v}^1$ yielding a contradiction. Otherwise, let $\tilde{v}_t^1 \geq \tilde{x}_t^1$ for some t, with $q < t \leq t^*$. Let:

$$q' = \inf\{q < t \leq t^* | \tilde{v}_t^1 \geq \tilde{x}_t^1\}.$$

Noting that by (5), $\epsilon - k = \epsilon' > 0$ so that ${}_{t^*+1}\tilde{x}^1 - {}_{t^*+1}\tilde{v}^1 = {}_{con}\epsilon' \gg {}_{con}0$, the above steps 1 and 2 can be applied to $\tilde{x}^1$, $\tilde{v}^1$ to derive profiles $\tilde{x}^2, \tilde{v}^2 \in X^\infty$ such that $\tilde{x}_t^2 > \tilde{v}_t^2$ for all $1 \leq t \leq q'$, whereas $\tilde{v}^2 \succ \tilde{x}^2$. By weak Pareto, a contradiction can be obtained if ${}_1\bar{x}^2_{t^*} \gg {}_1\bar{v}^2_{t^*}$. Otherwise, let $\tilde{x}_t^2 \leq \tilde{v}_t^2$ for some $q' < t \leq t^*$. And so on. After a finite number $s < t^*$ of iterations, two profiles $\tilde{x}^s, \tilde{v}^s \in X^\infty$ can be derived such that $\tilde{v}^s \succ \tilde{x}^s$, by steps 1 and 2, but ${}_1\bar{x}^s_{t^*} \gg {}_1\bar{v}^s_{t^*}$, and so $\tilde{x}^s \succ \tilde{v}^s$ can be obtained by applying weak Pareto, a contradiction. This completes the proof of Claim 1.

We can now show that ${}_{con}\hat{x} \succsim v$. To this end, choose a natural number H such that $v +_{con} h^{-1} \in X^\infty$ for all natural numbers $h \geq H$: the existence of H is guaranteed by the assumption $\sup_{i\in\mathbb{N}} v_i < 1$. By Claim 1, there exists $T \geq 1$ such that $({}_1\hat{x}_t, {}_{t+1}v +_{con} h^{-1}) \succsim v$ for all $t \geq T$ and all $h \geq H$. Fix any generation $t \geq T$. Then, since $\lim_{h\to\infty}({}_1\hat{x}_t,\ {}_{t+1}v +_{con} h^{-1}) = ({}_1\hat{x}_t,\ {}_{t+1}v) \in X^\infty$ and $({}_1\hat{x}_t,\ {}_{t+1}v +_{con} h^{-1}) \succsim v$ for any

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$h \geq H$, sup continuity and completeness imply that $({}_1\hat{x}_t,\ {}_{t+1}v) \succsim v$. Because the choice of generation $t \geq T$ was arbitrary, it follows that $({}_1\hat{x}_t,\ {}_{t+1}v) \succsim v$ for all $t \geq T$, and so preference continuity implies that ${}_{con}\hat{x} \succsim v$, as sought.

Case 2. $\sup_{i\in\mathbb{N}} v_i = 1$.

As $\inf_{i\in\mathbb{N}} u_i > \inf_{i\in\mathbb{N}} v_i$, choose a natural number K large enough such that the set $\mathbb{N}(K)$ defined below is non-empty:

$$\mathbb{N}(K) \equiv \left\{ i \in \mathbb{N} | 1 - \frac{1}{K} < v_i \leq 1, v_l < v_i - \frac{1}{K} \text{ for some generation } l \right\}.$$

Consider profile v^K formed from v as follows: $v_i^K = v_i - (1/K)$, for all $i \in \mathbb{N}(K)$, and $v_i^K = v_i$ for all $i \notin \mathbb{N}(K)$. By construction, $v^K \in X^\infty$, $\sup_{i\in\mathbb{N}} v_i^K \leq 1 - (1/K)$ and $\inf_{i\in\mathbb{N}} u_i > \inf_{i\in\mathbb{N}} v_i^K = \inf_{i\in\mathbb{N}} v_i$. By the same argument as in Case 1, it follows that ${}_{con}\hat{x} \succsim v^K$. Then, for any natural number $k \geq K$, consider the sequence $v_i^k = v_i - (1/k)$, for all $i \in \mathbb{N}(K)$, and $v_i^k = v_i$ for all $i \notin \mathbb{N}(K)$. Since the above arguments hold for any natural number $k \geq K$, then ${}_{con}\hat{x} \succsim v^k$ for all $k \geq K$. Further, $\lim_{k\to\infty}(v^k) = v$ and so ${}_{con}\hat{x} \succsim v$ follows from completeness and sup continuity.

We have established that ${}_{con}\hat{x} \succsim v$. In order to complete the proof of (3), we note that by construction, $u \gg {}_{con}\hat{x}$ and $\inf_{i\in\mathbb{N}} u_i > \hat{x}$, and so weak Pareto implies that $u \succ {}_{con}\hat{x}$. By transitivity we conclude that $u \succ v$, as sought.

Next, we show that (4) holds as well. Suppose that $\inf_{i\in\mathbb{N}} u_i = \inf_{i\in\mathbb{N}} v_i$. If $\inf_{i\in\mathbb{N}} u_i = 1$, then the result follows by reflexivity. Hence suppose $\inf_{i\in\mathbb{N}} u_i < 1$. Choose a natural number K sufficiently large such that the set

$$\mathbb{N}(u, K) \equiv \left\{ l \in \mathbb{N} | 1 - \frac{1}{K} > u_l \geq \inf_{i\in\mathbb{N}} u_i \right\}$$

is non-empty. Then, for all natural numbers $k \geq K$, consider u^k formed from u as follows: $u_i^k = u_i + (1/k)$, all $i \in \mathbb{N}(u, K)$, and $u_i^k = u_i$, all $i \notin \mathbb{N}(u, K)$. By construction, $u^k \in X^\infty$ and $\inf_{i\in\mathbb{N}} u_i^k > \inf_{i\in\mathbb{N}} v_i$, and so $u^k \succ v$ by (3) for all $k \geq K$. Therefore since $\lim_{k\to\infty} u^k = u$, completeness and sup continuity imply $u \succsim v$. A similar argument proves $v \succsim u$, and thus we obtain $u \sim v$. This completes the proof of Theorem 3.

Theorem 3 establishes an interesting possibility result for liberal approaches in economies with an infinite number of agents. For it proves that there exist fair, Paretian and continuous SWOs that respect a liberal principle of non-interference. Indeed, the maximin SWO satisfies even the stronger version of the Weak Harm Principle (analogous to that presented in Section 2) extended to hold for any countably infinite welfare profiles.

Further, Theorem 3 provides a novel, and interesting characterisation of the maximin SWO in the intergenerational context. Lauwers (1997) characterises the maximin SWO in the infinite context by focusing on weak Pareto, anonymity,[27]

[27] Actually, the characterisation by Lauwers (1997) relies on a strong anonymity axiom that considers all permutations of the welfare profiles.

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continuity, repetition approximation and either a strong version of Hammond equity,[28] or ordinal level comparability. Theorem 3 provides a completely different liberal foundation to the maximin SWO, because the Weak Harm Principle* is logically and theoretically distinct both from axioms with an egalitarian content, such as Hammond equity, and from informational invariance conditions.

6. Conclusions

A number of recent contributions have raised serious doubts on the possibility of a fair and efficient liberal approach to distributive justice that incorporates a fully non-interfering view. This article shows that possibility results do emerge, in societies with both a finite and an infinite number of agents, provided the bite of non-interference is limited in an ethically relevant way. Anonymous and Paretian criteria exist which incorporate a notion of protection of individuals (or generations) from unjustified interference, in situations in which they suffer a welfare loss, provided no other agent (or generation) is affected.

A weaker version of a liberal axiom – the Harm Principle – recently proposed by Mariotti and Veneziani (2009), together with standard properties, allows us to derive a set of new characterisations of the maximin and of its lexicographic refinement, including in the intergenerational context. This is surprising, because the Weak Harm Principle is meant to capture a liberal and libertarian requirement of non-interference and it incorporates no obvious egalitarian content. Thus, our results shed new light on the ethical foundations of the egalitarian approaches stemming from Rawls's difference principle, and provide new meaning to the label of liberal egalitarianism usually attached to Rawls's theory.

From the viewpoint of liberal approaches emphasising a notion of individual autonomy, or freedom, however, our results have a rather counterintuitive implication. For they prove that, in various contexts, liberal non-interfering principles lead straight to welfare egalitarianism.

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[28] Formally, for any $u, v \in X^{\infty}$ such that $u_i \geq v_i \geq v_j \geq u_j$ for some $i, j \in \mathbb{N}$ and $u_k = v_k$ for all $k \in \mathbb{N} \backslash \{i, j\}$, $v \succcurlyeq u$ (Lauwers, 1997, p. 46).

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