On tapered warp-free laminates with single-ply terminations.

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Abstract: This article reviews the current state of the art in the design of traditional uni-directional fibre laminate construction; beyond the ubiquitous balanced and symmetric design. A ply termination algorithm is then employed to develop permissible tapered designs, with single-ply terminations and ply contiguity constraints, which are free from undesirable changes in mechanical coupling characteristics. More importantly however, is the fact that all tapered designs have immunity to thermal warping distortion; which include all combinations of anti-symmetric (or cross-symmetric), non-symmetric and symmetric angle- and cross-ply sub-sequence symmetries. Tapered designs are presented for laminates with fully uncoupled properties, and those possessing extension-shearing and/or bending-twisting coupling. Such designs represent typical fuselage skin thicknesses, i.e., with between (n =) 12 and 16 plies, but due consideration is also given to new fuselage design concepts with grid-stiffeners and/or geodesic stiffener arrangements, for which thinner designs (n e 8) are of interest.

Keywords: A. Laminate Taper; B. Extension-Shearing Coupling; C. Bending-Twisting Coupling; D. Laminate Design Heuristics.

1. Introduction

Tapered composite laminates have been studied extensively [1,2] in the context of delamination initiation and propagation in the region of ply terminations. However, little consideration has been given to the extent of the laminate design space and the extent to which arbitrary plies may be dropped without introducing undesirable changes in mechanical coupling characteristics or thermal warping distortion during the curing process or in-service operation.

Symmetric stacking sequences are ubiquitous in modern composite laminate design practice, for the simple reason that their use guarantees the laminate remains flat, or warp free, after high temperature curing. Non-symmetric laminates are commonly associated with, or often (incorrectly) used to describe

[**3**], configurations that warp extensively after high temperature curing, and for which the deformed shape is also difficult to predict reliably [4]; requiring non-linear analysis techniques.

With very few exceptions, tapered designs in aircraft construction are currently certified only for balanced and symmetric laminates [**5**], despite the severe design constraint that 1 angle-ply termination requires a further 3 angle-ply terminations: 2 terminations to maintain balanced angle-plies, and a further 2 to maintain symmetry. Balanced stacking sequences guarantee that Extension-Shearing coupling is eliminated by using matching pairs of angle-ply layers [**6**]. Symmetric laminates guarantee that coupling between in-plane and out-of-plane behaviour is eliminated, along with thermal warping distortions that would otherwise be expected. However, tapering of symmetric laminates with fewer than the requisite 4 angle-ply terminations is believed to be common in industrial practice [7], despite the fact that the effects on the structural integrity are not well understood; these effects are simply minimised by ensuring that terminations are made at the laminate mid-plane.

An obvious, but somewhat controversial solution is to adopt unbalanced and non-symmetric stacking sequence configurations to exploit a larger, but generally unknown design space. Such laminate architectures give rise to *Extension-Shearing* and *Bending-Twisting* coupling, respectively, where extension-shearing coupling gives rise to *Bending-Twisting* coupling deformation in aircraft wing-box structures when top and bottom skins have identical bias fibre alignment, but this effect can be eliminated with opposing bias fibre alignment. *Bending-Twisting* coupling, at the laminate level, results in weaker compression buckling strength compared to the equivalent fully uncoupled laminate (with matching stiffness properties), although there is evidence that this continues to be ignored [7], leading to unsafe designs. *Bending-Twisting* coupling offers potential improvement in shear buckling strength with respect to the equivalent fully uncoupled laminate, but only when the resulting principal compressive stress direction and the principal bending stiffness direction are in the same sense [6].

Recent research on laminate design has demonstrated that fully uncoupled laminates [8], or those with *Extension-Shearing* [6] and/or *Bending-Twisting* coupling [9,10] all have immunity to thermal warping distortion and, collectively, have a design space containing all possible combinations of anti-symmetric (or cross-symmetric), non-symmetric and symmetric angle- and cross-ply sub-sequence symmetries.

The results presented in this article are based on the four laminate classes, illustrated in Fig. 1 under free thermal contraction. All are immune to thermal warping distortions by virtue of the fact that their coupling stiffness properties are null (**B** = 0); as would be expected from symmetric laminate configurations. Laminate classes with non-zero coupling stiffness (**B** $\neq$ 0), but with warp free or hygro-thermally curvature-stable (HTCS) properties, have been shown to require 4 ply terminations with standard ply orientations [11], but can be reduced to 2 ply terminations with non-standard ply orientations [12].

The first two classes contain balanced angle-ply layers, leading to uncoupled extensional stiffness properties. The *Simple* laminate in the first column is uncoupled in bending, whilst the laminate class in the second column possesses *Bending-Twisting* coupling. The final two laminate classes possess unbalanced angle-ply layers, leading to *Extension-Shearing* coupling properties. The laminate class in the third column is uncoupled in bending, whereas the laminate class in the fourth column has both *Extension-Shearing* and *Bending-Twisting* coupling, as would arise from unbalanced and symmetric laminates.

It should be emphasized that unbalanced laminates, otherwise referred to as *Extension-Shearing* coupled laminates, remain warp free for all solutions presented here, irrespective of the number of layers in the laminate. This is in marked contrast to other recent studies [13], where approximate solutions have been derived, which converge towards the thermo-mechanically curvature-stable, or warp-free condition only when the number of layers in the laminate becomes very large.

Similarly, symmetry has previously been shown [8,10] to be a limiting design rule, which serves only to mask the potential design space containing *Simple* and *Bending-Twisting* coupled laminates, where symmetric stacking sequences are the exception rather than the rule. Indeed, improvements in damage tolerance have been demonstrated, in the context of delamination buckling after impact [14], for fully uncoupled laminates using anti-symmetric designs; symmetric designs were found to perform no better than non-symmetric designs.

The remainder of this article is arranged as follows. Section 2 provides a summary of the derivation of definitive listings for the warp free laminate classes given in Fig. 1. Section 3 provides information on the design space for each class, including the dominant forms of sub-sequence symmetries. A ply termination algorithm is described in Section 4, which is then applied to the definitive listings of laminates from each of the four laminate classes to develop tapered laminate designs and compatible stacking sequences for single-ply terminations; for computation expedience, the definitive listings are prefiltered against ply contiguity constraints, i.e., the maximum number of adjacent plies with the same orientation. Tapered laminate examples are presented, illustrating the limited range of symmetric solutions in comparison to other sub-sequence symmetries. Finally, lamination parameters are introduced to allow the available design space to be visually interrogated, before conclusions are drawn in Section 6.

2 Derivation of stacking sequences

The common feature relating the *Simple*, *Extension-Shearing* and/or *Bending-Twisting* coupled laminate classes is that all are decoupled, i.e. $B_{ij} = 0$; hence in-plane and out-of-plane behaviour are independent and can therefore be treated separately. The constitutive relations simplify as follows:

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ & A_{22} & A_{26} \\ \text{Sym.} & & A_{66} \end{bmatrix} \begin{Bmatrix} \mu_x \\ \mu_y \\ \gamma_{xy} \end{Bmatrix}$$

$$\begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ & D_{22} & D_{26} \\ \text{Sym.} & & D_{66} \end{bmatrix} \begin{Bmatrix} {}^{o}{}_{x} \\ {}^{o}{}_{y} \\ {}^{o}{}_{xy} \end{Bmatrix} \qquad (1)$$

where the elements of the stiffness matrices are derived from the well know relationships:

$$A_{ij} = \Sigma Q'_{ij}(z_k - z_{k-1})$$

$$B_{ij} = \Sigma Q'_{ij}(z_k^{2} - z_{k-1}^{2})/2 = 0 \qquad (2)$$

$$D_{ij} = \Sigma Q'_{ij}(z_k^{3} - z_{k-1}^{3})/3$$

in which: the summations extend over all n plies; Q'_{ij} are the transformed reduced stiffnesses (i, j = 1, 2, 6) and; z_k represents the distance from the laminate mid-plane of the k^{th} ply.

In the derivation of the database of stacking sequences, which assumes (but is not restricted to) combinations of standard fibre angle orientations, i.e. 0, 90 and/or ±θ° (= ±45°), the general rule of symmetry is relaxed. Neither cross plies nor angle plies are constrained to be symmetric about the laminate mid-plane. The derivation involved the added restrictions that *each layer in the laminate*: *has identical orthotropic material properties; has identical thickness, t, and; differs only by its orientation.* For compatibility with the previously published data, similar symbols have been adopted for defining all stacking sequences, i.e., ○, ●, + and – are used in place of standard ply angle orientations 0, 90, +45 and –45°, respectively.

Stacking sequences are characterized by their sub-sequence symmetries using a double prefix notation, the first character of which relates to the form of symmetry produced by the location of the angle plies with respect to the laminate mid-plane, and the second character to the cross plies; referred to as the angle-ply sub-sequence and cross-ply sub-sequence, respectively. The double prefix contains combinations of the following characters: *A* to indicate Anti-symmetric form; *N* for Non-symmetric; and *S* for Symmetric. Additionally, for cross-ply sub-sequences only, *C* is used to indicate Cross-symmetric form. Sub-sequence should not be confused with sub-laminate. Plies in a sub-sequence are not necessarily adjacent, whereas a sub-laminate is a contiguous ply block.

To avoid the trivial solution of a stacking sequences with cross plies only, all sequences have an angle-ply (+) on one outer surface of the laminate. As a result, the other outer surface may be an angle-ply of equal (+) or opposite (–) orientation or a cross ply (○), which may be either 0 or 90°.

2.1 Non-dimensional parameters

Non-dimensional parameters allow the extensional and bending stiffness properties to be readily calculated for any fibre/matrix system and angle-ply orientation and provide a compact data set alongside each laminate stacking sequence derived.

The development of non-dimensional parameters is demonstrated, by way of an example for a 15-ply non-symmetric laminate stacking sequence $[+/-/+/\bigcirc/+/\bigcirc/+_3/-/+/-/+/\bigcirc/+]_T$, in Table 1. The first two columns of Table 1 provide the ply number and orientation, respectively, whilst subsequent columns illustrate the summations, for each ply orientation, of $(z_k - z_{k-1})$, $(z_k^2 - z_{k-1}^2)$ and $(z_k^3 - z_{k-1}^3)$, relating to the **A**, **B** and **D** matrices, respectively. Here, the distance from the laminate mid-plane, z, is expressed in terms of ply thickness t; assumed to be unit value.

The non-dimensional parameters arising from the tabular summations are as follows. For the extension stiffness matrix [**A**]: $n_\bigcirc$, the number of (0° or 90°) cross plies (= ${}_A\Sigma_\bigcirc$) = 3, n_-, the number of negative angle plies (= ${}_A\Sigma_-$) = 3, n_+, the number of positive angle plies (= ${}_A\Sigma_+$) = 9. The coupling stiffness matrix [**B**] summations confirm that $B_{ij} = 0$ for this laminate. For the bending stiffness matrix [**D**]: $\zeta_\bigcirc$ the bending stiffness parameter for (0° or 90°) cross plies (= $4 \times {}_D\Sigma_\bigcirc = 4 \times 169.25$) = 675, ζ_-, the bending stiffness parameter for negative angle plies (= $4 \times {}_D\Sigma_- = 4 \times 169.25$) = 675, ζ_+, the bending stiffness parameter for positive angle plies (= $4 \times {}_D\Sigma_+ = 4 \times 506.25$) = 2025, where $n^3 = 15^3 = \zeta = \zeta_\bigcirc + \zeta_- + \zeta_+ =$ 3375.

This *Extension-Shearing* and *Bending-Twisting* coupled ($\mathbf{A}_F\mathbf{B}_0\mathbf{D}_F$) laminate satisfies the following non-dimensional parameter criteria:

n_+ ` n_-

ζ_+ ` ζ_- (3)

whilst $n_+ = n_-$ and/or $\zeta_+ = \zeta_-$ are the conditions giving rise to the *Bending-Twisting* coupled and *Simple* or *Extension-Shearing* coupled laminate classes in Fig. 1, respectively.

These non-dimensional parameters, together with the transformed reduced stiffness, Q'_{ij}, for each ply orientation of constant ply thickness, *t*, facilitate simple calculation of the elements of the extensional, coupling and bending stiffness matrices from:

$$A_{ij} = \{n_+Q'_{ij+} + n_-Q'_{ij-} + n_\circ Q'_{ij\circ} + n_\bullet Q'_{ij\bullet}\}t$$

$$B_{ij} = 0 \tag{4}$$

$$D_{ij} = \{\zeta_+Q'_{ij+} + \zeta_-Q'_{ij-} + \zeta_\circ Q'_{ij\circ} + \zeta_\bullet Q'_{ij\bullet}\}t^3/12$$

A factor of 4, applied to the bending stiffness parameters, accounts for the recasting of the third of Eq. (**2**) as Eq. (**4**).

The transformed reduced stiffnesses, Q'_{ij}, are defined by:

$$Q'_{11} = Q_{11}\cos^4\theta + 2(Q_{12} + 2Q_{66})\cos^2\theta\sin^2\theta + Q_{22}\sin^4\theta$$

$$Q'_{12} = Q'_{21} = (Q_{11} + Q_{22} - 4Q_{66})\cos^2\theta\sin^2\theta + Q_{12}(\cos^4\theta + \sin^4\theta)$$

$$Q'_{16} = Q'_{61} = \{(Q_{11} - Q_{12} - 2Q_{66})\cos^2\theta + (Q_{12} - Q_{22} + 2Q_{66})\sin^2\theta\}\cos\theta\sin\theta$$

$$Q'_{22} = Q_{11}\sin^4\theta + 2(Q_{12} + 2Q_{66})\cos^2\theta\sin^2\theta + Q_{22}\cos^4\theta$$

$$Q'_{26} = Q'_{62} = \{(Q_{11} - Q_{12} - 2Q_{66})\sin^2\theta + (Q_{12} - Q_{22} + 2Q_{66})\cos^2\theta\}\cos\theta\sin\theta$$

$$Q'_{66} = (Q_{11} + Q_{22} - 2Q_{12} - 2Q_{66})\cos^2\theta\sin^2\theta + Q_{66}(\cos^4\theta + \sin^4\theta) \tag{5}$$

and the reduced stiffness terms, Q_{ij}, are calculated from the orthotropic material properties:

$$Q_{11} = E_1/(1 - \nu_{12}\nu_{21})$$

$$Q_{12} = \nu_{12}E_2/(1 - \nu_{12}\nu_{21})$$

$$Q_{22} = E_2/(1 - \nu_{12}\nu_{21})$$

$$Q_{66} = G_{12} \tag{6}$$

Note that orthotropic material properties imply that $Q_{16} = Q_{26} = 0$.

3. Stacking sequences and sub-sequence symmetries

The total (Σ) numbers of *Simple*, *Extension-Shearing* (*E-S*) coupled, *Bending-Twisting* (*B-T*) coupled and *Extension-Shearing* and *Bending-Twisting* (*E-S*;*B-T*) coupled laminate stacking sequences for each ply number grouping, *n*, with up to (*n* =) 16 plies, are given in Tables 2 – 5, together with the number in each sub-symmetric group. These are expressed as a percentage of the total design space (Σ) for each ply number grouping to reveal the dominant form of sub-sequence symmetry. Example stacking sequences, representing the minimum number of plies, *n*, within each sub-symmetric group, are given in the footnotes of each table. Note that in order to avoid the trivial solution of cross-ply laminates, the first (outer) ply in all the stacking sequences derived is an angle ply (+).

Table 2 reveals that fully uncoupled or *Simple* laminates are dominated by anti-symmetric (*AS*) stacking sequences, yet anti-symmetric laminates are commonly assumed to possess *Extension-Twisting* coupling. Similarly, Table 3 reveals that symmetric laminates (*SS*) with *Bending-Twisting* coupling dominate the design space, but only up to 12 plies; thereafter, non-symmetric laminates are dominant. Symmetric laminates are commonly and often incorrectly assumed to be *Simple* laminates [7], for which classical closed form buckling solutions lead to strength predictions on the unsafe side.

Table 4 demonstrates that *Extension-Shearing* coupled laminates exist only for even pair groupings over the range investigated; despite having unbalanced angle plies. Hence, this laminate class is precluded from the single ply terminations considered in this article. Finally, Table 5 reveals that *Extension-Shearing* and *Bending-Twisting* coupled laminates follow a similar pattern to the *Bending-Twisting* coupled laminates of Table 3, where symmetry dominates in laminates with up to 12 plies; non-symmetric sub-sequence symmetries dominate thereafter.

4. Laminate Tapering Algorithm

Tapered laminate are certified for symmetric laminate construction, the majority of which possess *Bending-Twisting* coupling, but such designs have a severe design constraint, i.e., a single angle-ply termination requires a further three angle-ply terminations to maintain balanced and symmetric construction. This section investigates the extent to which this restriction can be overcome by considering the definitive list of laminates presented in the previous section. Note, however, the results of Table 4 reveal that neither single-ply nor multiple-ply taper is possible for *Extension-Shearing* coupled laminates within the range of ply number groupings previously investigated [6]. Other forms of sub-sequence symmetries are known to exist in higher ply number groupings, hence tapered solutions may yet be found in this laminate class, but *Extension-Shearing* coupled laminates are not considered further in this study.

Tapered laminate designs have been developed in a two stage process. The first stage can be described as a top down process, in which each ply number grouping n is algorithmically filtered through ply number grouping n–1, representing a single ply termination. A second stage, which can be described as a bottom up process, begins with compatible stacking sequences representing the minimum ply number grouping of interest. These sequences are then algorithmically filtered through higher ply number groupings, in turn, but now only sequences compatible with the minimum ply number grouping are retained.

Design heuristics are first applied to the data presented in Tables 2 – 5, but only in the context of a ply contiguity constraint, which serves to reduce the design space by eliminating large numbers of consecutive plies with the same orientation; this so called 'ply blocking' is known to increase the potential for delamination initiation [5]. Only the most severe constraints of up to 2-ply contiguity are considered herein. The maximum fibre angle difference between contiguous plies is also often restricted to 45°, which implies that standard angle plies, +45° and -45° (or 135°), must be separated by a 0° (or 90°) ply. Whilst consideration is given to this design rule, it is not applied as a design constraint, since the stacking sequences presented in this article are given in symbolic form, i.e., +, –, ○ and ●, which may represent shallow angles (+22.5°, -22.5°, 0°) as well as standard angles (+45°, -45°, 0° and 90°), respectively.

Applying contiguity constraints to the definitive listings of Tables 2, 3 and 5, leads to the reduced design space given in Tables 6 – 8, respectively. Note that contiguity constraints are also important in the context of computational expedience; they were not applied to a preliminary study, reported elsewhere [15], where it became clear that high performance computing facilities would be required to complete such a study.

The termination scheme involves: single ply terminations, applied in turn to individual plies of every stacking sequence with n plies; comparison with all stacking sequences with n-1 plies and; recording exact matches. The results of the tapering process are presented in Table 9, for *Simple* laminates, noting that the first (or upper surface) ply and the last (or lower surface) ply are assumed to be continuous throughout the tapering process; this represents a practical design constraint to prevent surface ply delamination. Column (1) contains the ply number grouping, n, ranging from 8 – 16 plies. Column (2) presents the number of stacking sequences with ply contiguity ≤ 2, from summation of the *Simple* laminate results of Table 6. Column (3) gives the results of the single ply termination scheme, which represent the total number of stacking sequences with *n* plies, for which a matching stacking sequence is identified with *n*-1 plies. Matching sequences with *n*+1 ply laminates are presented in parentheses. Hence the first number in column (3) indicates that for laminates with 16 plies (*n* = 16), there are 84 of the 260 stacking sequences, which match 88 of the 246 stacking sequences with 15 plies (*n*-1). The first number in parentheses in column (3) indicates that 162 stacking sequences with 16 plies (*n* = 16) match stacking sequences with 17 plies (*n*+1), not shown.

Repeated sequences are removed from the data presented in column (3), i.e., where multiple matches arise from different ply terminations within a single stacking sequence. The final ply number grouping (*n* = 9) to which this termination scheme is applied, leaves a single match with (*n*–1 =) 8 plies; no terminations are possible for *n* = 8 to *n* = 7. This forms a starting point for the second stage of the tapering algorithm.

The second stage of the tapering algorithm can be described as a bottom up process, and begins with compatible stacking sequences representing the minimum ply number grouping of interest. These sequences are then algorithmically filtered through higher ply number groupings, in turn, but now only sequences compatible with the minimum ply number grouping are retained.

Once again, the single ply termination scheme involves the removal of individual plies, in turn, for each stacking sequence with *n*+1 plies, but now using the column (3) results in parentheses, and a comparison made against each stacking sequence with *n* plies, and matches recorded. This procedure ensures that all solutions reported here will be compatible with higher ply number groupings beyond those given, i.e., *n* > 16.

Hence, beginning with the single 8-ply solution given at the bottom of column (3), which corresponds to the anti-symmetric stacking sequence: $[+/–/–/+/–/+/+/–]_T$, and applying the termination scheme to the solutions in parentheses for (*n*+1 =) 9 plies, two matches are found: $[+/–/–/+/\bigcirc/–/+/+/–]_T$ and $[+/–/–/+/\bullet/–/+/+/–]_T$. These two results are reported in column (4), together with an indication of which plies are terminated (○ or ●/+ or –). There are no single angle-ply terminations (+ or –) due to the balanced nature of this laminate class, i.e. $n_+ = n_-$, which explains why the second value within parentheses is zero throughout column (4); signifying that only ○ or ● cross plies may be terminated. The two matching 9-ply solutions are retained for comparison with the results of the termination scheme applied now to the four (n+1 =) 10-ply stacking sequences reported in parentheses in column (3). Only two of these four sequences match: $[+/-/-/+/\bigcirc/\bigcirc/-/+/+/-]_T$ and $[+/-/-/+/\bullet/\bullet/-/+/+/-]_T$. The four solutions reported in column (4) result from the fact that the termination of the 1st or 2nd cross ply is possible from either of the two stacking sequences will lead to one of the two 9-ply solutions. This procedure continues to the highest ply number grouping of interest.

The tapering of laminates between 12 and 16 plies, representing typical fuselage thicknesses, requires only a different starting point in the second stage of the algorithm; the procedure is otherwise identical to that described above. The 14 laminates, given in parentheses in column (3), must be considered in the bottom up process. Matches are sought within the 24 laminates with (n+1 =) 13 plies, given in parentheses in column (3). In this case, all stacking sequences from the minimum ply number grouping of interest (n = 12) are compatible with the higher ply groupings.

The sub-sequence symmetries for this reduced data set are given in Table 10(a), for single- and 2-ply contiguity, for comparison with those of the complete design space given in Table 2. These results demonstrate why designs with single-ply contiguity alone cannot be achieved. Table 10(b) reveals that all designs within this tapered group (n = 12 – 16) have anti-symmetric angle-ply and symmetric cross-ply sub-sequences.

Note that parentheses are used for the 8-ply solution of Table 10(a) since this result arises from single ply terminations applied to 9-ply laminates. Similarly, in Table 10(b) the 12-ply solutions arise from single ply terminations applied to 13-ply laminates, with ply contiguity ≤ 2, which explains the difference between the 12-ply results reported in Table 10(a).

Table 11 provides comparable results for *Bending-Twisting* coupled laminates. As in *Simple* laminates, the balanced nature of this class of laminate precludes the possibility of single angle-ply terminations. However, unlike the *Simple* laminates, not all of the (203) 12-ply sequences from column (3) are compatible with the (492) 13 ply sequences in the bottom up phase of the termination scheme given in column (5); with 12-ply laminates as the minimum ply number grouping of interest.

Table 12 reveals that the resulting tapered designs now have several sub-sequence symmetries, although symmetric designs do dominate. This can be understood from Table 12(a) where symmetric laminates are the only form for ply number groupings below (n =) 11 plies and continue to dominate the design space up to (n =) 15 plies. Other forms of sub-sequence symmetry appear only in higher ply number groupings. Note that single ply terminations first appear in (*n* =) 12 ply laminates with *SC* subsequence symmetry, but are lost in 14 and 15 ply laminates, yet solutions are found in the tapered group (*n* = 12 – 16) of Table 12(b), when allowing ply contiguity to vary (≤ 2) between ply number groupings.

Tapered results for *Extension-Shearing* and *Bending-Twisting* coupled laminates are given in Table 13; noting that stacking sequences with a single ply orientation only, i.e.: sequences of the form $[+/\ldots./+]_T$, have been omitted. Once again, columns (4) and (5) list the total number of solutions, together with an indication of which ply orientations are terminated (○ or ●/+/–). Unlike the previous laminate classes, these now represent the number of cross-plies (○ or ●), and separately, the positive and negative angle-plies (+/–), due to the unbalanced nature (n_+ ` n_-) of this laminate class; where the relative number of angle-ply terminations differ because of the fixed outer (+) ply.

Results from an analysis of the sub-sequence symmetries for *Extension-Shearing* and *Bending-Twisting* coupled laminates are given Table 14. Whilst non-symmetric laminates clearly dominate the single ply termination results above 12 plies in Table 14(a) and despite the many other angle- and cross-ply sub-sequence symmetries present, Table 14(b) reveals that designs within this tapered group (*n* = 12 – 16) are again dominated by symmetric laminates. This demonstrates that the minimum ply number of interest has a marked effect on both the number of sub-symmetries and the number of solutions in given tapered group.

5. Tapered Laminate Designs

For practical laminate design, tapering must be possible without introducing unwanted coupling behaviour. This section therefore presents a number of tapered laminate designs with single-ply terminations, highlighting the potential for achieving single-ply contiguity, enforcing a maximum ply angle difference in contiguous plies, and exploiting unconventional laminates and unusual forms of stacking sequence symmetries in order to increase the design space, hence design flexibility. Standard ply orientations +45, -45, 0 and 90° are represented here by symbols +, –, ○ and ●, respectively. Each stacking sequence is followed by the subscript T, denoting that the total stacking sequence is given. The double subscript that then follows in parentheses denotes the form of symmetry for the angle-ply subsequence and cross-ply subsequence symmetries, respectively, with *A* for anti-symmetric, *N* for non-symmetric, *S* for symmetric and for cross-ply subsequences only, *C* for cross-symmetric.

*5.1 **Simple laminate designs:***

All tapered laminate designs, with single ply terminations between 12 and 16 plies, contain anti-symmetric angle-ply and symmetric cross-ply sub-sequences. The first of two examples, illustrated in Fig. 2(a), both of which have anti-symmetric angle plies and symmetric cross plies (*AS*), demonstrates that single-ply contiguity cannot be achieved throughout the tapered design since cross plies must be terminated as alternating consecutive pairs; terminations are also constrained to the laminate mid-plane and form a symmetric central ply block, which serves to maintain the fully uncoupled orthotropic nature of each stacking sequence. The second example demonstrates the difficulty in achieving a maximum ply angle difference of 45° between contiguous plies, within the two-ply contiguity constraint, due to the necessary introduction of 0 and 90° cross plies in the symmetric central ply block.

The examples of Fig. 2(a) highlight a simple single ply termination pattern that can be developed for higher ply number groupings, i.e., by the introduction of consecutive pairs of cross plies at the laminate mid-plane to maintain symmetry within the central ply block.

It would be incorrect to assume that only anti-symmetric laminates are permissible in *Simple* tapered laminate designs. Tapering between other ply number groupings reveals additional sub-sequence symmetries, e.g. the example in Fig. 2(b), with single ply terminations between 16 and 20 plies, contains non-symmetric angle- and cross-ply sub-sequences. Here, the two-ply contiguity constraint is violated within the central ply block of the 18-ply laminate, but more significantly, this example reveals that the central ply block can also become non-symmetric.

*5.2 **Bending-Twisting coupled laminate designs:***

Single ply terminations in ***B-T*** coupled laminates also involve only cross plies, but now with a broader range of sub-sequence symmetries than *Simple* tapered laminates between 12 and 16 plies. These examples demonstrate the same symmetric central ply block configurations, irrespective of the overall subsequence symmetry. Here also, single ply contiguity, together with single ply terminations, cannot be achieved throughout the tapered design: cross plies may be terminated only at the laminate mid-plane to preserve the *Bending-Twisting* coupled nature of each stacking sequence, hence they are terminated as alternating consecutive pairs, which again has the effect of changing sub-sequence symmetries in some tapered sequences, e.g. from cross-symmetric cross plies to non-symmetric cross-plies, and/or violating the single ply contiguity constraint. Here, symbols are used to denote cross-plies (○ or ●), and positive and negative angle-plies (+, −).

The examples of Fig. 2(c) highlight the same simple single ply termination pattern seen in Fig. 2(a), which can be developed for higher ply number groupings, i.e., by the introduction of consecutive pairs of cross plies at the laminate mid-plane to maintain symmetry within the central ply block.

5.3 ***Extension-Shearing and Bending-Twisting coupled laminate designs:***

Single ply terminations in *E-S*;*B-T* coupled laminates incorporate cross plies and angle plies and result in the broadest range of sub-sequence symmetries. These examples also demonstrate single- and two-ply contiguity combinations throughout the tapered design with single ply terminations, but now they may contain combinations of cross plies and/or angle plies, whilst maintaining the *Extension-Shearing* and *Bending-Twisting* coupling nature of each stacking sequence. As in all of the tapered laminates between 12 and 16 plies presented here, ply terminations are again found within a symmetric ply block at the laminate mid-plane, which has the effect changing sub-sequence symmetries in some tapered sequences, e.g. from cross-symmetric cross plies to non-symmetric cross-plies, and/or violating the single ply contiguity constraint.

All the examples of Fig. 2(d) highlight a simple single ply termination pattern that can be developed for higher ply number groupings, i.e., by the introduction of consecutive pairs of cross plies or angle plies at the laminate mid-plane to maintain symmetry within the central ply block.

Whilst many strange forms of sub-sequence symmetry may be found in these tapered laminates, the most significant feature for *E-S*;*B-T* coupled laminates arises from the possibility to change the blend ratio ($n_+/n_\pm$) of the angle-ply sub-sequence, hence the degree of *Extension-Shearing* coupling in the laminate. Applications of this design concept have previously been demonstrated as a passive control mechanism in bend-twist coupled wing box structures [16], but can now be seen to extend to geodesic fuselage designs, involving angled or helical stiffener arrangements [17], in order to counteract the tendency for bending-twisting coupling behaviour due to angled stiffeners at +ϕ on the inner surface of the fuselage skin and -ϕ on the outer surface.

5.4 ***Lamination parameters for design space interrogation.***

Lamination parameters, originally conceived by Tsai and Hahn [**18**] offer an alternative set of non-dimensional expressions when ply angles are a design constraint. They were first applied to optimum design by Miki [19] and presented in graphical form by Fukunaga and Vanderplaats [20]. Optimized lamination parameters may be matched against a corresponding set of stacking sequences. Graphical representations help with this design process, since arguably the greatest challenge to the composite laminate designer, is the inverse problem of generating practical laminate configurations, which satisfy the optimized lamination parameters.

Elements of the extensional (**A**) and bending (**D**) stiffness matrices are each related to lamination parameters, ξ_i, and laminate invariants, U_i, respectively, by:

$$A_{11} = \{U_1 + \xi_1 U_2 + \xi_2 U_3\} \times H$$

$$A_{12} = A_{21} = \{-\xi_2 U_3 + U_4\} \times H$$

$$A_{16} = A_{61} = \{\xi_3 U_2/2 + \xi_4 U_3\} \times H$$

$$A_{22} = \{U_1 - \xi_1 U_2 + \xi_2 U_3\} \times H$$

$$A_{26} = A_{62} = \{\xi_3 U_2/2 - \xi_4 U_3\} \times H$$

$$A_{66} = \{-\xi_2 U_3 + U_5\} \times H \tag{7}$$

and

$$D_{11} = \{U_1 + \xi_9 U_2 + \xi_{10} U_3\} \times H^3/12$$

$$D_{12} = D_{21} = \{U_4 - \xi_{10} U_3\} \times H^3/12$$

$$D_{16} = D_{61} = \{\xi_{11} U_2/2 + \xi_{12} U_3\} \times H^3/12$$

$$D_{22} = \{U_1 - \xi_9 U_2 + \xi_{10} U_3\} \times H^3/12$$

$$D_{26} = D_{62} = \{\xi_{11} U_2/2 - \xi_{12} U_3\} \times H^3/12$$

$$D_{66} = \{-\xi_{10} U_3 + U_5\} \times H^3/12 \tag{8}$$

where the laminate thickness H (= number of plies, n, × constant ply thickness, t) and the laminate invariants, U_i, are calculated from the reduced stiffness terms, Q_{ij}, of Eq. (6):

$$U_1 = \{3Q_{11} + 3Q_{22} + 2Q_{12} + 4Q_{66}\}/8$$

$$U_2 = \{Q_{11} - Q_{22}\}/2$$

$$U_3 = \{Q_{11} + Q_{22} - 2Q_{12} - 4Q_{66}\}/8$$

$$U_4 = \{Q_{11} + Q_{22} + 6Q_{12} - 4Q_{66}\}/8$$

$$U_5 = \{Q_{11} + Q_{22} - 2Q_{12} + 4Q_{66}\}/8 \tag{9}$$

These ply orientation dependent lamination parameters are also related to the non-dimensional parameters developed in Table 1 by the following expressions:

$\xi_1 = \{n_+\cos(2\theta_+) + n_-\cos(2\theta_-) + n_\circ\cos(2\theta_\circ) + n_\bullet\cos(2\theta_\bullet)\}/n$

$\xi_2 = \{n_+\cos(4\theta_+) + n_-\cos(4\theta_-) + n_\circ\cos(4\theta_\circ) + n_\bullet\cos(4\theta_\bullet)\}/n$

$\xi_3 = \{n_+\sin(2\theta_+) + n_-\sin(2\theta_-) + n_\circ\sin(2\theta_\circ) + n_\bullet\sin(2\theta_\bullet)\}/n$

$\xi_4 = \{n_+\sin(4\theta_+) + n_-\sin(4\theta_-) + n_\circ\sin(4\theta_\circ) + n_\bullet\sin(4\theta_\bullet)\}/n$ (10)

$\xi_9 = \{\zeta_+\cos(2\theta_+) + \zeta_-\cos(2\theta_-) + \zeta_\circ\cos(2\theta_\circ) + \zeta_\bullet\cos(2\theta_\bullet)\}/n^3$

$\xi_{10} = \{\zeta_+\cos(4\theta_+) + \zeta_-\cos(4\theta_-) + \zeta_\circ\cos(4\theta_\circ) + \zeta_\bullet\cos(4\theta_\bullet)\}/n^3$

$\xi_{11} = \{\zeta_+\sin(2\theta_+) + \zeta_-\sin(2\theta_-) + \zeta_\circ\sin(2\theta_\circ) + \zeta_\bullet\sin(2\theta_\bullet)\}/n^3$

$\xi_{12} = \{\zeta_+\sin(4\theta_+) + \zeta_-\sin(4\theta_-) + \zeta_\circ\sin(4\theta_\circ) + \zeta_\bullet\sin(4\theta_\bullet)\}/n^3$ (11)

Noting that for standard ply orientations 0, 90 and ±45°, assumed here, $\xi_4 = \xi_{12} = 0$. Lamination parameters permit an interrogation of the extent of resulting design space for tapered laminates, since individual laminate stacking sequences can now be represented by a single point in either a 2- or 3-dimensional space, representing either the extensional stiffness properties or the bending stiffness properties. Each point, within the resulting point cloud, defines a co-ordinate set, from which the stiffness properties may be readily determined using Eq. (7) or Eq. (8).

Interrogation of the resulting lamination design spaces reveals clearly discernible strings of points. One such string is highlighted in Fig. 3, for *Bending-Twisting* coupled laminates, which are represented by a 2-dimensional space for the extensional stiffness and a 3-dimensional space for the bending stiffness. The highlighted string represents a series of 9 compatible stacking sequences forming, collectively, the tapered design of Fig. **4**(a), with symmetric angle-plies as well as symmetric cross-plies (*SS*), and where single ply terminations occur in consecutive cross-ply pairs.

Single ply termination of consecutive pairs of 0 or 90° plies results in a zig-zig pattern in the lamination parameter points representing extensional stiffness, highlighted in Fig. 3(d), where each linear segment is aligned with the top right or top left corner of the design space. These two corners represent, respectively, the co-ordinate positions of laminates with 0 or 90° plies only, hence terminating 90° (or ●) plies from the 16- and 15-ply stacking sequences results in a migration of the lamination parameter co-ordinate away from the top left corner of the design space. The migration then leads away from the top right of the design space as 0° (or ○) plies are terminated from the 14- and 13-ply laminates. Lamination parameter points with $\xi_1 = 0$ indicate equal numbers of 0° (or ○) and 90° (or ●) plies, whilst a lamination parameter point with coordinate $(\xi_1,\xi_2) = (0,-1)$ in Fig. 3(d) represents a laminate with angle plies, ±45° (or ±), only. The relative proportion of each ply orientation within a given laminate therefore determines the coordinate of the lamination parameter point within the design space. Note that the final 8-ply stacking sequence in this tapered design has equal numbers of each ply orientation: 0, 90 and ±45° (or ○, ●, and ±); and is therefore extensionally isotropic; this corresponds to lamination parameters $\xi_1 = \xi_2 = 0$ in Fig. 3(d).

The close proximity of points in each string of bending stiffness lamination parameters, highlighted in Fig. 3(a) – (c), is explained by the fact that single ply terminations at the laminate mid-plane have only a small effect on the laminate bending stiffness properties, giving rise to the well-defined strings of points. The curved nature of the strings of points arise from the fact that the bending stiffness parameters, to which the lamination parameters are directly linked through Eq. (11), have a non-linear dependency with respect to a change in the number of plies, *n*, see Table 1; this is in contrast to the linear dependency of the extensional stiffness parameters.

Migration of lamination parameter points away from $\xi_{11} = 0$ indicates increasing bending-twisting coupling, i.e. $D_{16}^* = D_{26}^*$ $(= 12D_{16}/H^3)$, as the laminate thickness is reduced from 16 to 8 plies. Such designs are known to have a reduced compression buckling strength compared to those with $D_{16} = D_{26} = 0$, but increases in buckling strength can result when shear loading becomes dominant [**6**].

Lamination parameter design spaces for the complete set of tapered solutions reported here are provided in the electronic annex to this article.

It should be noted that the bias of the point cloud towards the front of the lamination parameter design space, i.e. $\xi_{11} < 0$, as seen from the side elevation of Fig. 3(c), is explained by the fact that the first, or outer ply in all the laminates derived, is a positive angle ply. This constraint was applied to avoid trivial solutions containing cross-plies only. Had the first ply been a negative angle ply, the bias would be towards the back of the design space, i.e. $\xi_{11} > 0$.

*5.5 **Bi-angle non-crimp fabric hybrid laminates***

The continuing requirement for more efficient manufacturing of composite structures has resulted in the recent development of bi-angle non-crimp fabrics (NCF). Indeed, a recent study on repeating bi-angle

$[\underline{\theta/0}]_{rT}$ NCF designs [13], has now led to commercially available forms, including shallow angle designs (θ = 22.5°), with the potential to reduce wet lay-up times by half; in comparison to traditional UD tape. Note that these unbalanced, non-symmetric, $\underline{E\text{-}S}$;$\underline{B\text{-}T}$ coupled laminate designs also possess coupling between extension and bending (B ` 0) and therefore undesirable thermo-mechanical behaviour; which dissipates only as the number of repeats, *r*, becomes very large.

New design strategies must be considered here, since traditional design rules are no longer generally applicable. Additionally, single-ply terminations can be achieved only through hybrid designs, i.e., using a combination of bi-angle $[\underline{\theta/0}]$ NCF and UD plies, as illustrated in Fig. **4**(b) for a (n =) 16 ply laminate, tapered to (n =) 8 plies.

As seen in the UD laminate sequences of Fig. 2(d), this tapered $\underline{E\text{-}S}$;$\underline{B\text{-}T}$ coupled laminate with single ply terminations highlights a simple pattern that can be developed for higher ply number groupings, i.e., by the introduction of consecutive pairs of cross plies or angle plies at the laminate mid-plane to maintain symmetry within the central ply block; but this comes at the expense of necessarily increasing the ply contiguity constraint to 3 in the (n =) 10 ply laminate.

Note that a 24-ply fully isotropic (π/4) laminate can also be constructed from $\underline{0/45}$ and $\underline{0/\text{-}45}$ bi-angle NCF: $[\underline{\text{-}45/90}/\underline{0/45}/\underline{0/45}/\underline{90/45}/\underline{\text{-}45/0}/\underline{\text{-}45/90}/\underline{\text{-}45/90}/\underline{45/90}/\underline{0/\text{-}45}/\underline{0/45}/\underline{0/45}/\underline{\text{-}45/90}]_{rT}$, by are either flipping/reversing ($\underline{\text{-}45/0}$), rotating ($\underline{90/45}$) or both ($\underline{45/90}$ and $\underline{\text{-}45/90}$). This provides an important benchmark for assessing the relative stiffness, strength or damage tolerance of competing NCF designs.

Single ply terminations applied to NCF materials are equivalent to two adjacent ply terminations in traditional UD materials [21] and therefore introduce a constraint that reduces the design space substantially in contrast to a general two ply termination scheme [22], particularly in 'thin laminate' designs. However, this can be overcome by adopting 'thin-ply' or 'spread-tow' technology, which will allow an exponential increase in tailoring opportunities by bringing design flexibilities, found only in traditionally thick laminate construction, into the thin laminate domain.

Thin-ply or *spread tow* material technology offers the prospect of an 8-ply NCF stacking sequence $[\underline{\theta/0/0/\theta/0/\theta/\theta/0}]_{rT}$ with the same thickness as a single ply of traditional UD material. This design also follows the repeating bi-angle $[\underline{\theta/0}]_{rT}$ philosophy proposed by Tsai [13], but with immunity to thermal warping distortions irrespective of the number of 8-ply NCF layers. This design also possesses quasi-homogeneous anisotropic properties with matching $\mathbf{A}^*$ (= A_{ij}/H) = $\mathbf{D}^*$ (= $12D_{ij}/H^3$), where the laminate thickness H is derived from the product of the number of plies, n, and constant ply thickness, t. The quasi-homogeneous relationship remains throughout the tapered laminate.

6. Conclusions

This article has investigated the extent of design space for tapered laminates when the ubiquitous balanced and/or symmetric design rules are relaxed. Consideration of design heuristics, involving single ply terminations and ply contiguity (≤ 2), representing the most severe design constraints for tapered laminates, have significantly influenced the form of the designs, i.e., single ply terminations are possible only at the laminate mid-plane for consistent mechanically (de-)coupled characteristics. Nevertheless, this feature may be desirable for mitigation of load path fluctuations and in-plane loading eccentricities, which generally result from ply termination. Several key observations have been demonstrated through the results presented:

- Tapered laminate solutions arise in non-symmetric, as well as symmetric laminates, with consistent mechanical coupling properties and immunity to thermal warping throughout for *Simple* laminate configurations as well as those with *Extension-Shearing* and/or *Bending-Twisting* coupling.
- Single ply terminations are permissible in *Simple* or *Bending-Twisting* coupled laminates, but only for cross plies within a mid-plane ply block; which can be non-symmetric.
- Single angle-ply terminations are permissible only in laminate configurations possessing *Extension-Shearing* and *Bending-Twisting* coupling.
- A simple design rule has been identified in which consistent mechanical *Extension-Shearing* and *Bending-Twisting* coupling is preserved by modifying a mid-plane symmetric ply block, within an otherwise non-symmetric laminate configuration, which is applicable to both single angle-ply and/or cross-ply terminations.
- Lamination parameter point clouds reveal clear strings of points representing the tapered designs found. These graphical representations of the design space offer insight into the potential for tailoring of stiffness properties.

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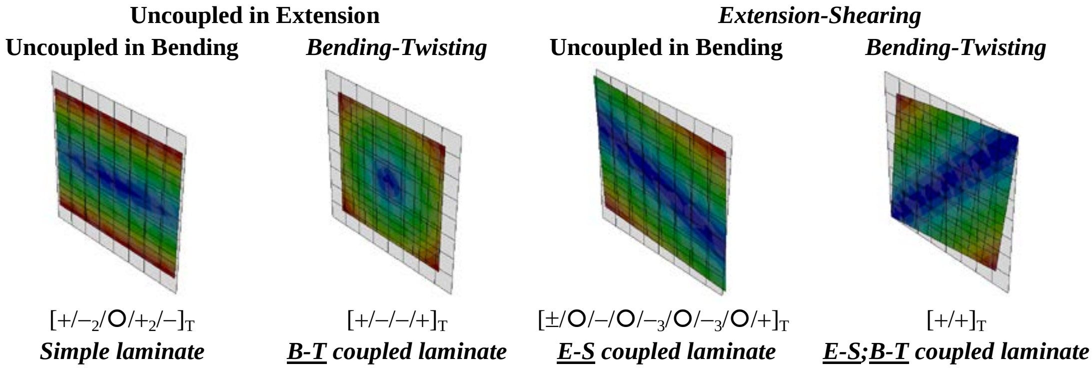

Figure 1 – In-plane thermal contraction responses (exaggerated) resulting from a typical high temperature curing process. All examples shown are square, initially flat, composite laminates. The stacking sequences provided, in symbolic form, are representative of the minimum ply number grouping for each laminate class, with standard ply orientations ±45, 0 and 90° in place of symbols +, –, ○ and ●, respectively.

[+/–/○/●/–/+/●/○/○/●/–/+/●/○/+/–]$_{\mathrm{T}(AS)}$
[+/–/○/●/–/+/●/○/●/–/+/●/○/+/–]$_{\mathrm{T}(AS)}$
[+/–/○/●/–/+/●/●/–/+/●/○/+/–]$_{\mathrm{T}(AS)}$
[+/–/○/●/–/+/●/–/+/●/○/+/–]$_{\mathrm{T}(AS)}$
[+/–/○/●/–/+/–/+/●/○/+/–]$_{\mathrm{T}(AS)}$

[+/○/–/–/○/+/○/●/●/○/–/○/+/+/○/–]$_{\mathrm{T}(AS)}$
[+/○/–/–/○/+/○/●/○/–/○/+/+/○/–]$_{\mathrm{T}(AS)}$
[+/○/–/–/○/+/○/○/–/○/+/+/○/–]$_{\mathrm{T}(AS)}$
[+/○/–/–/○/+/○/–/○/+/+/○/–]$_{\mathrm{T}(AS)}$
[+/○/–/–/○/+/–/○/+/+/○/–]$_{\mathrm{T}(AS)}$

(a)

[+/–/●/○/○/●/–/+/○/●/●/○/○/–/●/+/+/●/–/○]$_{\mathrm{T}(NN)}$
[+/–/●/○/○/●/–/+/○/●/○/○/–/●/+/+/●/–/○]$_{\mathrm{T}(NN)}$
[+/–/●/○/○/●/–/+/○/○/○/–/●/+/+/●/–/○]$_{\mathrm{T}(NN)}$
[+/–/●/○/○/●/–/+/○/○/–/●/+/+/●/–/○]$_{\mathrm{T}(NN)}$
[+/–/●/○/○/●/–/+/○/–/●/+/+/●/–/○]$_{\mathrm{T}(NN)}$

(b)

[+/–/○/+/○/–/●/○/○/●/○/–/+/–/○/+]$_{\mathrm{T}(NN)}$
[+/–/○/+/○/–/●/○/●/○/–/+/–/○/+]$_{\mathrm{T}(NN)}$
[+/–/○/+/○/–/●/●/○/–/+/–/○/+]$_{\mathrm{T}(NN)}$
[+/–/○/+/○/–/●/○/–/+/–/○/+]$_{\mathrm{T}(NN)}$
[+/–/○/+/○/–/○/–/+/–/○/+]$_{\mathrm{T}(NN)}$

[+/–/–/+/+/–/○/●/●/○/+/–/–/+/–/+]$_{\mathrm{T}(NS)}$
[+/–/–/+/+/–/○/●/○/+/–/–/+/–/+]$_{\mathrm{T}(NS)}$
[+/–/–/+/+/–/○/○/+/–/–/+/–/+]$_{\mathrm{T}(NS)}$
[+/–/–/+/+/–/○/+/–/–/+/–/+]$_{\mathrm{T}(NS)}$
[+/–/–/+/+/–/+/–/–/+/–/+]$_{\mathrm{T}(NS)}$

[+/○/●/●/○/–/○/●/●/○/–/●/○/○/●/+]$_{\mathrm{T}(SN)}$
[+/○/●/●/○/–/○/●/○/–/●/○/○/●/+]$_{\mathrm{T}(SN)}$
[+/○/●/●/○/–/○/○/–/●/○/○/●/+]$_{\mathrm{T}(SN)}$
[+/○/●/●/○/–/○/–/●/○/○/●/+]$_{\mathrm{T}(SC)}$
[+/○/●/●/○/–/–/●/○/○/●/+]$_{\mathrm{T}(SC)}$

[+/○/–/●/–/+/●/○/○/●/+/–/●/–/○/+]$_{\mathrm{T}(SS)}$
[+/○/–/●/–/+/●/○/●/+/–/●/–/○/+]$_{\mathrm{T}(SS)}$
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[+/○/–/●/–/+/●/+/–/●/–/○/+]$_{\mathrm{T}(SS)}$
[+/○/–/●/–/+/+/–/●/–/○/+]$_{\mathrm{T}(SS)}$

(c)

[+/○/●/–/○/+/●/+/+/●/○/+/–/●/+/○]$_{\mathrm{T}(NN)}$
[+/○/●/–/○/+/●/+/●/○/+/–/●/+/○]$_{\mathrm{T}(NN)}$
[+/○/●/–/○/+/●/●/○/+/–/●/+/○]$_{\mathrm{T}(NN)}$
[+/○/●/–/○/+/●/○/+/–/●/+/○]$_{\mathrm{T}(NN)}$
[+/○/●/–/○/+/○/+/–/●/+/○]$_{\mathrm{T}(NN)}$

[+/–/+/●/+/–/●/+/+/●/+/–/●/–/+/+]$_{\mathrm{T}(NS)}$
[+/–/+/●/+/–/●/+/●/+/–/●/–/+/+]$_{\mathrm{T}(NS)}$
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[+/–/+/●/+/–/+/–/●/–/+/+]$_{\mathrm{T}(NS)}$

[+/○/●/+/●/○/+/–/–/+/●/○/+/○/●/+]$_{\mathrm{T}(SC)}$
[+/○/●/+/●/○/+/–/+/●/○/+/○/●/+]$_{\mathrm{T}(SC)}$
[+/○/●/+/●/○/+/+/●/○/+/○/●/+]$_{\mathrm{T}(SC)}$
[+/○/●/+/●/○/+/●/○/+/○/●/+]$_{\mathrm{T}(SC)}$
[+/○/●/+/●/○/●/○/+/○/●/+]$_{\mathrm{T}(SC)}$

[+/●/○/+/○/●/+/●/●/+/○/●/+/●/○/+]$_{\mathrm{T}(SN)}$
[+/●/○/+/○/●/+/●/+/○/●/+/●/○/+]$_{\mathrm{T}(SC)}$
[+/●/○/+/○/●/+/+/○/●/+/●/○/+]$_{\mathrm{T}(SC)}$
[+/●/○/+/○/●/+/○/●/+/●/○/+]$_{\mathrm{T}(SC)}$
[+/●/○/+/○/●/○/●/+/●/○/+]$_{\mathrm{T}(SC)}$

[+/○/○/●/●/○/+/–/–/+/●/○/○/●/○/+]$_{\mathrm{T}(SN)}$
[+/○/○/●/●/○/+/–/+/●/○/○/●/○/+]$_{\mathrm{T}(SN)}$
[+/○/○/●/●/○/+/+/●/○/○/●/○/+]$_{\mathrm{T}(SN)}$
[+/○/○/●/●/○/+/●/○/○/●/○/+]$_{\mathrm{T}(SN)}$
[+/○/○/●/●/○/●/○/○/●/○/+]$_{\mathrm{T}(SN)}$

[+/+/○/●/●/+/●/+/+/●/+/●/●/○/+/+]$_{\mathrm{T}(SS)}$
[+/+/○/●/●/+/●/+/●/+/●/●/○/+/+]$_{\mathrm{T}(SS)}$
[+/+/○/●/●/+/●/●/+/●/●/○/+/+]$_{\mathrm{T}(SS)}$
[+/+/○/●/●/+/●/+/●/●/○/+/+]$_{\mathrm{T}(SS)}$
[+/+/○/●/●/+/+/●/●/○/+/+]$_{\mathrm{T}(SS)}$

(d)

Figure 2 – Tapered: (a) Simple laminate designs with anti-symmetric stacking sequences; (b) Simple laminate with non-symmetric stacking sequences; (c) laminate designs for *B-T* coupled laminates with different sub-sequence symmetries and; (d) laminate designs for *E-S*;*B-T* coupled laminates with different (including mixed) sub-sequence symmetries.

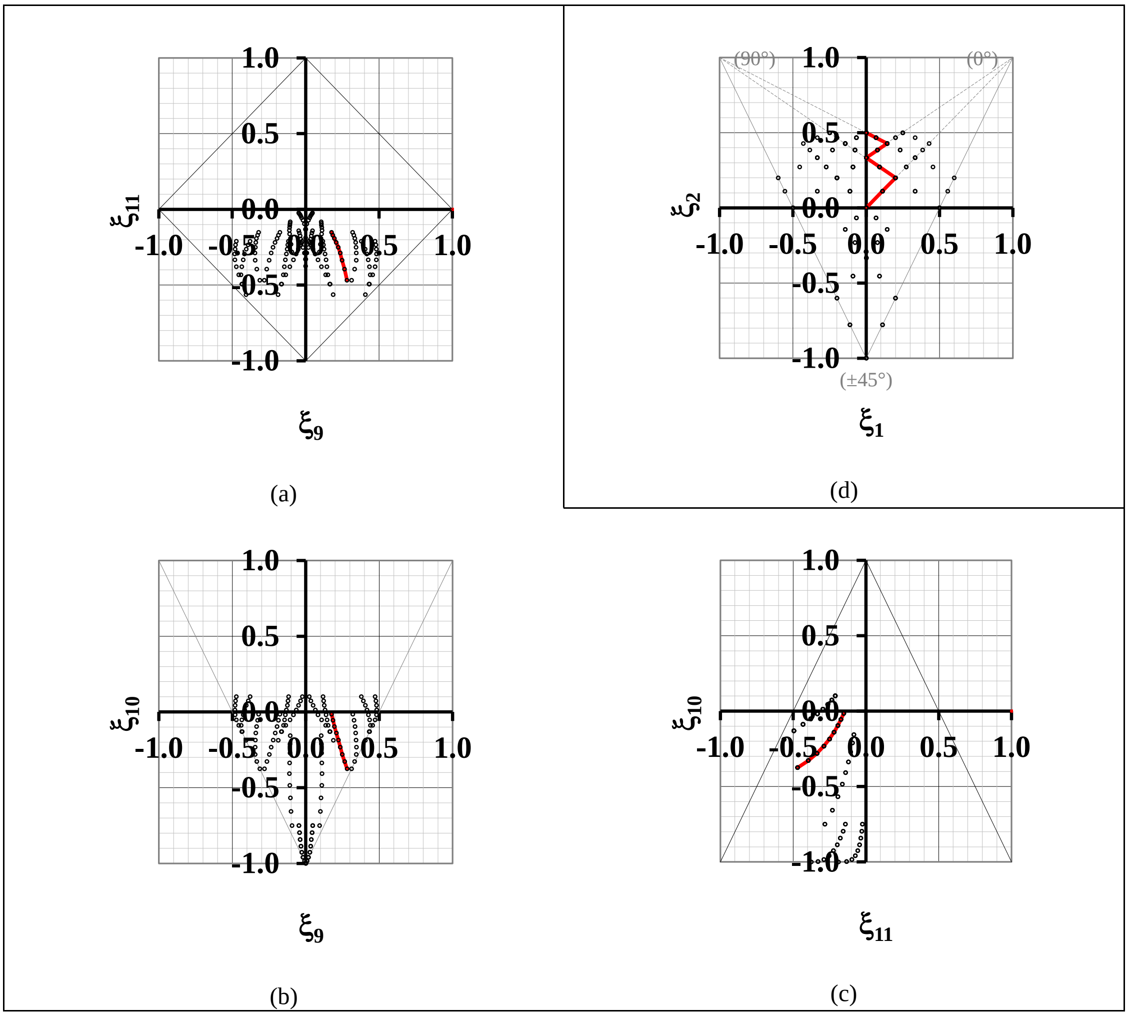

Figure 3 – Lamination parameter design space for tapered *Bending-Twisting* coupled laminates with 8-16 plies, representing the 3-dimensional space for bending stiffness: (a) plan; (b) front elevation and; (c) side elevation. (d) The 2-dimensional space for extensional stiffness.

[+/○/–/●/○/●/○/●/●/○/●/○/●/–/○/+]$_{T(SS)}$
[+/○/–/●/○/●/○/●/○/●/○/●/–/○/+]$_{T(SS)}$
[+/○/–/●/○/●/○/○/●/○/●/–/○/+]$_{T(SS)}$
[+/○/–/●/○/●/○/●/○/●/–/○/+]$_{T(SS)}$
[+/○/–/●/○/●/●/○/●/–/○/+]$_{T(SS)}$
[+/○/–/●/○/●/○/●/–/○/+]$_{T(SS)}$
[+/○/–/●/○/○/●/–/○/+]$_{T(SS)}$
[+/○/–/●/○/●/–/○/+]$_{T(SS)}$
[+/○/–/●/●/–/○/+]$_{T(SS)}$

(a)

$[\underline{\theta/0}/\underline{0/\theta}/\theta/0/-\theta/0/0/-\theta/0/\theta/\underline{0/\theta}/\underline{\theta/0}]_T$
$[\underline{\theta/0}/\underline{0/\theta}/\theta/0/-\theta/0/-\theta/0/\theta/\underline{0/\theta}/\underline{\theta/0}]_T$
$[\underline{\theta/0}/\underline{0/\theta}/\theta/0/-\theta/-\theta/0/\theta/\underline{0/\theta}/\underline{\theta/0}]_T$
$[\underline{\theta/0}/\underline{0/\theta}/\theta/0/-\theta/0/\theta/\underline{0/\theta}/\underline{\theta/0}]_T$
$[\underline{\theta/0}/\underline{0/\theta}/\theta/0/0/\theta/\underline{0/\theta}/\underline{\theta/0}]_T$
$[\underline{\theta/0}/\underline{0/\theta}/\theta/0/\theta/\underline{0/\theta}/\underline{\theta/0}]_T$
$[\underline{\theta/0}/\underline{0/\theta}/\theta/\theta/\underline{0/\theta}/\underline{\theta/0}]_T$
$[\underline{\theta/0}/\underline{0/\theta}/\theta/\underline{0/\theta}/\underline{\theta/0}]_T$
$[\underline{\theta/0}/\underline{0/\theta}/\underline{0/\theta}/\underline{\theta/0}]_T$

(b)

Figure 4 – Tapered laminate design: (a) highlighted in Fig. 3 and; (b) using a combination of bi-angle [$\underline{\theta/0}$] NCF and UD plies.

Table 1 – Development of non-dimensional parameters for a *Extension-Shearing* and *Bending-Twisting* coupled laminates.

		[A]				[B]				[D]			
Ply	θ	$(z_k - z_{k-1})$	${}_A\Sigma_\cap$ <u>3</u>	${}_A\Sigma_-$ <u>3</u>	${}_A\Sigma_+$ <u>9</u>	$(z_k^2 - z_{k-1}^2)$	${}_B\Sigma_\cap$ <u>0</u>	${}_B\Sigma_-$ <u>0</u>	${}_B\Sigma_+$ <u>0</u>	$(z_k^3 - z_{k-1}^3)$	${}_D\Sigma_\cap$ <u>169.25</u>	${}_D\Sigma_-$ <u>169.25</u>	${}_D\Sigma_+$ <u>506.25</u>
1	+	1			1	-14			-14	147.25			147.25
2	–	1		1		-12		-12		108.25		108.25	
3	+	1			1	-10			-10	75.25			75.25
4	○	1	1			-8	-8			48.25	48.25		
5	+	1			1	-6			-6	27.25			27.25
6	○	1	1			-4	-4			12.25	12.25		
7	+	1			1	-2			-2	3.25			3.25
8	+	1			1	0			0	0.25			0.25
9	+	1			1	2			2	3.25			3.25
10	–	1		1		4		4		12.25		12.25	
11	+	1			1	6			6	27.25			27.25
12	–	1		1		8		8		48.25		48.25	
13	+	1			1	10			10	75.25			75.25
14	○	1	1			12	12			108.25	108.25		
15	+	1			1	14			14	147.25			147.25

Table 2 – Number (%) of *Simple* laminate stacking sequences for each ply number grouping, *n*, arranged by sub-sequence symmetry using a double prefix notation: 1st prefix for Angle-ply sub-sequence; 2nd prefix for Cross-ply sub-sequence. Example stacking sequences, representing the minimum number of plies within each symmetry grouping, are given in the footnote.

n	8	9	10	11	12	13	14	15	16
AC								6.7	3.3
AN								6.7	
AS	100	100	100	100	84.0	100	86.4	80.0	74.4
NN								5.6	11.9
NS									0.6
SC									0.6
SN									
SS					16.0		13.6	1.1	9.2
Σ	1	6	6	24	25	84	88	360	360

A – Anti-symmetric; *C* – Cross-symmetric; *N* – Non-symmetric; *S* – Symmetric

AC:
[+/○/●/–/–/●/○/●/●/○/+/+/○/●/–]$_T$

AN:
[+/○/●/–/–/●/●/●/○/●/+/+/○/●/–]$_T$

AS:
[+/–/–/●/+/+/–]$_T$

NN:
[+/–/–/○/+/○/–/+/+/+/–/–/–/○/+]$_T$

NS:
[+/–/–/–/+/+/+/+/–/–/+/–/–/+/–/+]$_T$

SC:
[+/–/○/–/●/+/●/●/○/○/+/○/–/●/–/+]$_T$

SN:
[+/–/●/○/–/●/●/+/●/+/○/○/–/●/●/–/+]$_T$

SS:
[+/–/–/●/+/●/●/+/●/–/–/+]$_T$

Table 3 – Number (%) of <u>*B-T*</u> coupled laminate stacking sequences for each ply number grouping, *n*, arranged by sub-sequence symmetry using a double prefix notation: 1st prefix for Angle-ply sub-sequence; 2nd prefix for Cross-ply sub-sequence. Example stacking sequences, representing the minimum number of plies within each symmetry grouping, are given in the footnote.

n	8	9	10	11	12	13	14	15	16
NC									
NN		16.7		35.8	20.0	52.1	32.0	68.0	54.0
NS				7.5	6.2	10.8	11.2	10.5	11.8
SC				3.8	2.8	0.9		0.9	1.1
SN						4.8	4.8	4.3	4.0
SS	100	83.3	100	52.8	71.0	31.4	52.0	16.3	29.1
Σ	15	36	56	212	290	1,336	1,500	9,666	10,210

C – Cross-symmetric; *N* – Non-symmetric; *S* – Symmetric

NC:
[+/–/+/○/–/●/●/+/●/–/○/○/–/●/–/+/+]$_{\mathrm{T}}$

NN:
[+/●/–/●/–/+/–/+/●]$_{\mathrm{T}}$

NS:
[+/–/–/–/+/●/+/+/+/–/–]$_{\mathrm{T}}$

SC:
[+/●/○/–/○/●/●/–/●/○/+]$_{\mathrm{T}}$

SN:
[+/●/●/○/○/–/●/–/●/●/●/○/+]$_{\mathrm{T}}$

SS:
[+/–/–/+]$_{\mathrm{T}}$

Table 4 – Number (%) of <u>*E-S*</u> coupled laminate stacking sequences for each ply number grouping, *n*, arranged by sub-sequence symmetry using a double prefix notation: 1st prefix for Angle-ply sub-sequence; 2nd prefix for Cross-ply sub-sequence. Example stacking sequences, representing the minimum number of plies within each symmetry grouping, are given in the footnote.

n	8	9	10	11	12	13	14	15	16
$_{+}NN_{+}$							100		100
$_{+}NN_{-}$									
$_{+}NN_{\circ}$									
Σ	0	0	0	0	0	0	4	0	8

N – Non-symmetric

$_{+}NN_{+}$:
[+/–/○/–/○/–$_3$/○/–$_3$/○/+]$_{\mathrm{T}}$

$_{+}NN_{-}$:
[+/–$_2$/+/○/+/○/+/○/+$_5$/–/+/○/–]$_{\mathrm{T}}$

$_{+}NN_{\circ}$:
[+/○/–$_2$/+/–/●/○/–$_3$/○/–/●/–$_2$/+/–/+/○]$_{\mathrm{T}}$

Table 5 – Number (%) of *E-S*;*B-T* coupled laminate stacking sequences for each ply number grouping, *n*, arranged by sub-sequence symmetry using a double prefix notation: 1st prefix for Angle-ply sub-sequence; 2nd prefix for Cross-ply sub-sequence. Example stacking sequences, representing the minimum number of plies within each symmetry grouping, are given in the footnote. Symmetric laminates of the form $[+/\ldots/+]_T$ have been disregarded in all of the results presented in this article.

n	8	9	10	11	12	13	14	15	16
AC								0.03	
AN								0.03	
AS		1.9		1.3		0.7		0.3	
NC								0.1	
NN	4.0	20.6	13.4	37.3	23.5	53.1	40.1	67.9	56.2
NS		5.0	3.3	9.3	6.2	10.1	8.5	9.5	9.4
SC					0.7	1.4	1.1	0.7	0.4
SN		2.5		1.8	1.3	3.2	2.1	3.7	3.5
SS	96.0	70.1	83.3	50.3	68.3	31.5	48.2	17.7	30.5
Σ	50	321	241	1,843	1,191	11,651	6,847	83,573	43,830

A – Anti-symmetric; *C* – Cross-symmetric; *N* – Non-symmetric; *S* – Symmetric

AC:
$[+/○/●/–/–/●/○/–/●/○/+/+/○/●/–]_T$
AN:
$[+/○/●/–/–/●/●/–/○/●/+/+/○/●/–]_T$
AS:
$[+/–/–/–/+/+/–]_T$
NC:
$[+/●/+/○/+/○/+/+/+/●/+/●/+/○/+]_T$
NN:
$[+/●/●/●/+/+/●]_T$
NS:
$[+/–/–/–/●/+/–/+/–]_T$
SC:
$[+/●/○/○/●/○/●/●/○/+]_T$
SN:
$[+/●/○/○/●/●/●/○/+]_T$
SS:
$[+/–/+]_T$

Table 6 – Number of *Simple* laminates from (*n* =) 16 plies down to (*n* =) 8 plies subject to contiguity constraints.

	Ply Contiguity			
n	1	2	>2	Σ
16	30	230	115	375
15	56	190	130	376
14	10	64	14	88
13	20	34	30	84
12	2	20	3	25
11	8	10	6	24
10	2	2	2	6
9	4	2	-	6
8	-	1	-	1

Table 7 – Number of *Bending-Twisting* coupled laminates from (n =) 16 plies down to (n =) 8 plies, subject to contiguity constraints.

	Ply Contiguity			
n	1	2	>2	Σ
16	210	5,717	4,283	10,210
15	602	5,452	3,612	9.666
14	40	940	520	1,500
13	156	722	458	1,336
12	6	197	87	290
11	40	108	64	212
10	-	42	14	56
9	14	14	8	36
8	-	12	3	15

Table 8 – Number of *Extension-Shearing and Bending-Twisting* coupled laminates from (n =) 16 plies down to (n =) 8 plies, subject to contiguity constraints.

	Ply Contiguity			
n	1	2	>2	Σ
16	414	19,949	23,467	43,830
15	3,413	39,622	40,538	83,573
14	88	3,463	3,296	6,847
13	925	5,382	5,344	11,651
12	10	665	516	1,191
11	243	845	755	1,843
10	4	145	92	241
9	75	122	124	321
8	-	35	15	50

Table 9 – Results from the single ply termination algorithm applied to *Simple* laminates.

(1)	(2)	(3)	(4)	(5)
16	260	84 (162)	4 (2/0)	44 (22/0)
15	246	68 (88)	2 (1/0)	22 (11/0)
14	74	24 (46)	4 (2/0)	44 (22/0)
13	54	22 (24)	2 (1/0)	22 (11/0)
12	22	6 (**14**)	4 (2/0)	**14**
11	18	6 (6)	2 (1/0)	
10	4	2 (4)	4 (2/0)	
9	6	2 (2)	2 (1/0)	
8	1	- (**1**)	**1**	

Column:
(1) Ply number grouping, n.
(2) Number of stacking sequences with ply contiguity ≤ 2.
(3) Number of n ply laminates from (2) matching n-1 ply laminates after single ply termination. Number of n ply laminates matching n+1 ply laminates are shown in parentheses.
(4) Number of solutions within each ply number grouping (n) and orientation (○ or ●/+ or –) of corresponding ply terminations, for 8-16 ply tapered laminate.
(5) Number of solutions within each ply number grouping (n) and orientation (○ or ●/+ or –) of corresponding ply terminations, for 12-16 ply tapered laminate.

Table 10 – Sub-sequence symmetries in compatible *Simple* laminate stacking sequences for single ply terminations, corresponding to the results of the: (a) top down procedure of column (3) and; (b) bottom up procedure of column (5) of Table 9.

	(a)									(b)				
	Ply contiguity = 1/Ply contiguity = 2									Ply contiguity ≤ 2				
n	8	9	10	11	12	13	14	15	16	12	13	14	15	16
AC									8/4					
AS	(-/1)	-/2	-/2	4/2	-/6	8/16	-/24	28/42	-/70	(14)	22	22	22	22
NN									-/6					
SS								-/4	-/4					

Table 11 – Results from the single ply termination algorithm applied to *B-T* coupled laminates.

(1)	(2)	(3)	(4)	(5)
16	5927	2,844 (3,066)	36 (18/0)	752 (286/0)
15	6054	1,496 (2,976)	36 (18/0)	286 (143/0)
14	980	484 (954)	36 (18/0)	588 (294/0)
13	878	332 (492)	36 (18/0)	294 (147/0)
12	203	96 (**203**)	36 (18/0)	**198**
11	148	62 (98)	36 (18/0)	
10	42	22 (42)	36 (18/0)	
9	28	18 (22)	18 (9/0)	
8	12	- (**12**)	**12**	

Column:

(1) Ply number grouping, *n*.

(2) Number of stacking sequences with ply contiguity ≤ 2.

(3) Number of *n* ply laminates from (2) matching *n*-1 ply laminates after single ply termination. Number of *n* ply laminates matching *n*+1 ply laminates are shown in parentheses.

(4) Number of solutions within each ply number grouping (*n*) and orientation (○ or ●/+ or –) of corresponding ply terminations, for 8-16 ply tapered laminate.

(5) Number of solutions within each ply number grouping (*n*) and orientation (○ or ●/+ or –) of corresponding ply terminations, for 12-16 ply tapered laminate.

Table 12 – Sub-sequence symmetries in compatible *Bending-Twisting* coupled laminate stacking sequences for single ply terminations, corresponding to the results of the: (a) top down procedure of column (3) and; (b) bottom up procedure of column (5) of Table 11.

	(a)									(b)				
	Ply contiguity = 1/Ply contiguity = 2									Ply contiguity ≤ 2				
n	8	9	10	11	12	13	14	15	16	12	13	14	15	16
NN					4/24	8/84	34/170	74/490	81/1,654	(44)	58	58	58	58
NS						-/16	-/16	-/152	-/152	(6)	12	12	12	12
SC					2/2	-/12	-/-	-/-	8/25	(2)	4	4	4	4
SN							-/24	-/76	-/96					
SS	(-/6)	12/10	-/22	32/44	-/72	102/170	-/274	286/676	-/968	(146)	220	212	212	212

Table 13 – Results from the single ply termination algorithm applied to *E-S-B-T* coupled laminates.

(1)	(2)	(3)	(4)	(5)
16	20,363	20,329 (20,355)	3,582 (934/930/784)	7,911 (2,086/2,020/1,719)
15	43,035	11,273 (21,243)	1,791 (467/465/392)	4,391 (1,156/1,118/961)
14	3,551	3,167 (3,551)	1,274 (340/324/270)	3,274 (948/714/664)
13	6307	2,111 (3,645)	637 (170/162/135)	1,637 (474/357/332)
12	675	623 (**675**)	462 (122/126/92)	**675**
11	1088	463 (673)	231 (61/63/46)	
10	149	137 (149)	174 (52/36/34)	
9	197	107 (141)	87 (26/18/17)	
8	35	- (**35**)	**35**	

Column:
(1) Ply number grouping, *n*.
(2) Number of stacking sequences with ply contiguity ≤ 2.
(3) Number of *n* ply laminates from (2) matching *n*-1 ply laminates after single ply termination. Number of *n* ply laminates matching *n*+1 ply laminates are shown in parentheses.
(4) Number of solutions within each ply number grouping (*n*) and orientation (○ or ●/+/–) of corresponding ply terminations, for 8-16 ply tapered laminate.
(5) Number of solutions within each ply number grouping (*n*) and orientation (○ or ●/+/–) of corresponding ply terminations, for 12-16 ply tapered laminate.

Table 14 – Sub-sequence symmetries in compatible *Extension-Shearing* and *Bending-Twisting* coupled laminate stacking sequences for single ply terminations, corresponding to the results of the: (a) top down procedure of column (3) and; (b) bottom up procedure of column (5) ofTable 13.

	(a)									(b)				
	Ply contiguity = 1/Ply contiguity = 2									Ply contiguity ≤ 2				
n	8	9	10	11	12	13	14	15	16	12	13	14	15	16
AC								4/20	-/-					
AN								8/8	-/-					
AS		-/6	-/-	-/24	-/-	24/74	-/-	40/186	-/-				16	
NC								-/32	-/-					
NN	(-/2)	6/28	4/12	24/364	8/128	218/3,056	64/1,240	1,184/28,212	59/10,454	(136)	280	280	1,165	1,147
NS		-/4	-/2	-/68	-/32	32/442	16/212	120/3,074	8/1,394	(32)	60	60	216	232
SC			-/2	8/20	2/4	24/108	8/50	100/336	6/118	(6)	14	10	85	57
SN		-/4	-/-	-/8	-/4	-/148	-/48	64/1,316	16/616	(4)	4	8	85	113
SS	(-/33)	69/80	-/129	211/361	-/497	627/1,554	-/1,913	1,893/6,438	-/7,367	(497)	1,279	1,279	2,824	2,824